ASTRONOMY

A POPULAR HANDBOOK

By HAROLD JACOBY

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Photo by Barnard, Oct. 19, 1911. Exposure, 46 min. (see p. 14).

PLATE 1. Comet c 1911, discovered by Brooks.

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BY

HAROLD JACOBY

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IN COLUMBIA UNIVERSITY

WITH THIRTY-TWO PLATES AND MANY FIGURES IN THE TEXT

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"To know
That which before us lies in daily life
Is the prime wisdom."

— *Paradise Lost*, VIII, 192.

PREFACE

The present volume has been prepared with a double purpose, and upon a plan somewhat unusual. First, an effort has been made to meet the wishes of the ordinary reader who may desire to inform himself as to the present state of astronomic science, or to secure a simple explanation of the many phenomena constantly exhibiting themselves in the universe about him; and the further purpose has been to produce a satisfactory textbook for use in high schools and colleges.

Thus, for the general reader, it has been thought necessary to eliminate all formal mathematics; for the student, on the other hand, the occasional use of elementary algebra and geometry are essential. To satisfy these two apparently contradictory conditions, the book has been written in two parts; the first free from mathematics, the second a series of extended elementary mathematical notes and explanations to which appropriate references are made in the first part of the book. Thus the general reader may confine his attention to the non-mathematical part; the student should master the whole volume.

Attention is directed especially to Chapter I, in which is presented a brief summary of the entire science. It is hoped that this will serve to strengthen in most readers a desire for further and more detailed information. To the student this chapter should furnish as much knowledge as he must have in his possession before beginning a direct

study of the sky with a telescope. In the author's extended experience as a college teacher of elementary astronomy, he has found it most desirable to give life to the subject by requiring frequent evening visits of students at the observatory. These should begin almost immediately upon commencing the study of the science; and the first chapter is therefore intended to give the students something to work upon, even in their earliest observatory visits. At Columbia and Barnard colleges, these visits are required on frequent dates, regularly assigned throughout the year, without regard to the state of the weather. When clear, the telescope is used; when this is impossible, oral and informal discussion takes place upon the work done in the classroom. Attention is also given to the daylight study of solar shadows, all students being required to construct a practical sundial, as explained in Chapter V.

The author has, of course, drawn freely upon many other books, especially in the preparation of numerous diagrams, and in arranging the various parts of the subject in order. But most of the diagrams are new, and all have been simplified as much as possible. In a few cases, illustrations were copied from very old astronomic textbooks: references are then always given, in the hope that some readers, at least, will be led to examine these fine venerable classics of the science.

Almost all the inserted plates are photographic reproductions of actual photographs. For these the author is under deep obligations to Professor E. E. Barnard, of the Yerkes Observatory, and to the astronomers of the Lick Observatory.

H. J.

Columbia University,
May, 1913.

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CHAPTER I

THE UNIVERSE

If a company of men and women should chance to be gathered together on some clear, quiet evening under the dome of the sky, and if there should happen to come into that company one known to possess an acquaintance with the facts and the theories of astronomic science, — if these things should occur, inevitably there would descend upon that astronomer a shower of questions. These he would answer in simple language, after a kindly fashion for many centuries the habit of his guild; and, as he passed on, he would once more marvel, as he had done many times before, at the changelessness of man's desires. For these questions of the multitude, welcome ever to the star-man, to-day still resemble those that were laid of old as problems before his predecessors at the side of the pyramids.

Why, for instance, does the moon appear at times as a full round disk, at others as a tiny crescent? Why do certain bright stars called planets seem to wander about among the multitude of their fellows? Why is summer hot and winter cold? How do navigators find their way across the trackless ocean by observing the heavenly bodies?

Let us begin by attempting to set forth as best we may the answer to some of these many eternal questions from the skies. For the astronomer is not always present; and even

if he were, it is often better to gather our information in silence, by means of the process called reading. The very name Astronomy tells us what our science is. Derived from two Greek words, Astronomy means "the law of the stars." Where does the law of the stars hold sway? Throughout all space. What is space? Space is the place where astronomy has its being. When does astronomy enforce its laws? Throughout all time. What is time? Time is the period during which astronomy has its being. Astronomy needs no logical definitions of space and time. They belong to it; they are part of it.

Somewhere, then, in the endless void of space our universe is suspended; the visible universe. Is that visible universe but one of many? Are there invisible universes without number scattered through the vastness of space like continents in an endless ocean? The human mind loses itself in speculations such as this: nor do such speculations here concern us; for astronomers consider only the ascertainable phenomena of the universe that unfolds itself to our senses.

There is in existence a vast quantity of *matter* and a vast quantity of active *energy*, or force. It is not necessary at this point to define these terms; but we should remember that according to accepted theory the total of matter and the total of energy in the universe do not change. Matter is never destroyed; and the accepted law of the conservation of energy tells us that the quantity of energy in the universe is likewise constant and unvarying in amount. None ever disappears out of existence. But both matter and energy may and do undergo changes in form and appearance. Thus, water may appear as steam or as ice; and the energy of a moving body may be transformed into heat, light, or electricity.

THE UNIVERSE

When we examine this visible universe of ours at night with the unaided eye, we see several different kinds of objects: nebulæ, or small luminous clouds; star clusters, like the famous group called the Pleiades; individual stars; the moon; and, occasionally, comets or meteors. In the day we see the sun; sometimes the moon; and very rarely indeed a particularly bright star or comet. We shall give here a brief outline of existing knowledge concerning these various celestial objects, leaving a detailed description of their peculiarities to later chapters. They are all composed of matter; all, if in motion, move in accordance with the laws of mechanical science which govern the operation of energy; and all, if they change, undergo only changes such as accord with the laws of physics and chemistry.

First, then, the nebulæ. We shall begin with these because they probably represent the form in which matter shows itself to us in its most primitive stage of development. Only one or two can be seen with the unaided eye; and these only on very clear nights when the moon is invisible. In the telescope they appear as patches of luminous cloud, often more or less irregular in form. They were once thought to be simply conglomerations of small stars, so close to each other that the optical powers of existing telescopes were unable to separate them into constituent units. This view gained in probability for a long time, because, as the power of telescopes increased with the increase of skill among opticians, astronomers were constantly *resolving* new nebulæ, as they used to call it; separating them into simple close clusters of faint stars.

But the invention of an instrument called the spectroscope, in the middle of the nineteenth century, put us in possession of a means, previously non-existent, for dis-

tinguishing with certainty between the light of incandescent gases and that derived from incandescent or luminous matter in the liquid or solid stage. With the spectroscope astronomers have been able to ascertain that there are many nebulæ in a truly gaseous condition; that probably most of these objects are gaseous bodies; that they could not be resolved into stars, even if terrestrial man possessed to-day telescopes more powerful than he is likely ever to have at his command.

According to many modern theorists, we may take the nebulæ to be matter not yet fashioned into stars. This means, of course, that certain forces are at work in the nebulæ; forces of irresistible power, slow in action, as all cosmic changes must be slow when measured by the life of human generations; but sure in action, too, with that infinite sureness which belongs in celestial spaces. These forces doubtless produce motions of vast import within the body of the nebula; heat is doubtless engendered; condensations occur at certain points; nuclei are formed; probably, finally, one or more stars take the place of these nuclei; and so, perhaps, is the original nebular material transformed into stars such as men see clustered upon the sky of night.

Certainly the force of gravitation must be active. Since the time of Newton, in the seventeenth century, it has been known that there is a force of gravitation; that under the influence of that force every particle of matter in the universe attracts or pulls every other particle of matter; that the combined effect is always motion of some sort, each particle pursuing in space some determinate path or orbit under the influence of gravitational attraction exerted by all the particles.

The most recent observations of nebulæ have brought out

Photo by Keeler, May 10, 1899. Exposure, 4 hours.

PLATE 2. Spiral Nebula.

the fact that they are extremely numerous; probably many hundreds of thousands exist, although only about ten thousand have been catalogued. This fact is of importance; for if we are to regard the stars as a product of development or evolution from the nebulæ, we should expect these gaseous bodies to exist in numbers comparable with the number of the stars themselves.

But of even greater interest is another recent observation of the nebulæ. The predominant type seems to be spiral in form; a species of central hub, carrying two attached curved spires, like a whirling wheel with two very flexible spokes but no rim. There can be little doubt that these nebulæ are subject to internal motions, probably rapid in themselves, but appearing infinitely slow to us because of the almost inconceivably vast distance by which we are sundered from them.

According to the foregoing theory, which admits the existence of irregular as well as spiral nebulæ, we should expect to find the stars in groups, a certain number assembled comparatively close together near certain former nebulous regions within the sidereal universe. And this is precisely what we do find. Usually the number of stars thus belonging together is small; very frequently but a single star can be detected with the telescope. But many of the constituent stars of a group may be too faint to show themselves on account of their distance; often they are all probably too faint, except the large one that may have resulted from the former hub or center of the parent nebula, if it was one of true spiral form. Furthermore, gravitational attractions and orbital motions may have commenced among the stars of every group even before they had become separate bodies. While they were still numerous, frequent collisions must

have brought about the coalescence of two or more into a single larger unit. In short, we have here an outline of a fairly consistent explanation to carry us forward from the nebular stage to that of stellar development, a theory that leads us to expect star-groups ranging all the way from a single visible luminous object to a detached assemblage closely packed in a globular cluster.

And the stars are simply suns; our sun is a star. There can be no doubt that the stars are not the tiny twinkling points of light they seem to be. Their apparent lack of size or volume is simply a result of the distance by which they are separated from us; their twinkling comes from undulations or other irregularities in the ocean of terrestrial atmosphere, or "air," through which we are compelled to view them. We know the stars to be self-luminous, masses of glowing incandescent solids, liquids, or gases. We know the stars to be composed of chemical elements practically identical with those found on the earth. We know the stars to be subject to the law of gravitation; that every particle of matter in each one of them is endowed with that mysterious quality postulated by Newton, the power of pulling all other matter in the universe. From all these known facts, and reasoning by analogy, we are led to believe the stars to be suns, more or less like our own sun, though by no means necessarily in the same stage of cosmic development. All are doubtless cooling gradually and steadily by the constant radiation of heat into space; some have probably reached temperature conditions similar to those existing in our sun; and there may very probably be some that are attended by planets like our earth.

The stars are classified according to their so-called magnitudes, by which astronomers mean simply their lucidity or

brightness, not their actual dimensions; the first-magnitude stars are the brightest, the fifth-magnitude stars the faintest usually visible to the unaided eye under ordinary conditions. There are in all about sixteen hundred stars of the first five magnitudes; and only about one half of these can be seen at any one time, when the sky is perfectly cloudless, because the other half are always concealed from view below the horizon. The stars are also divided into a series of so-called constellations; very irregular, even grotesque imaginary figures of men, animals, and other objects, placed in the sky by the astronomers of old, and retained there in a somewhat simplified form by the moderns, principally on account of an unwillingness to destroy the ancient landmarks of this venerable and venerated science.

The stars so far described are called fixed stars, which means that they do not change their relative positions in space; that any two of them now close together have been thus in proximity from the beginning, and will remain so to the end. But modern researches have brought out the fact that these apparently fixed stars are not really fixed absolutely. They have motions in space; these motions seem to us extremely slow and minute simply because the stellar distances are so vast. For at a sufficiently great distance, even large and rapid motions will necessarily appear reduced and retarded. And it is, in fact, quite inconsistent with what we know of the laws governing gravitational attraction to suppose any particle of matter in the universe to be really fixed in position absolutely. Everything must move; must be following some duly appointed path, ever contrasting the intricate complexity of nature with the wondrous simplicity of nature's order and nature's law. Even our sun, regarded as a star, cannot be fixed in space, but must be moving majestically

through the void, drawing with it our attendant earth, and ourselves upon it.

And if the stars are incandescent suns, we must expect to find, and we do find, that some among them undergo internal changes that make their visible brightness vary. In certain cases slowly, in others more rapidly, their luminosity waxes and wanes with a more or less periodical regularity. Now and again, rarely and at long intervals, some special catastrophe takes place; some convulsion of nature, whereby a new star is made to blaze forth into view where previously had been only darkness. Possibly we witness in such cases the result of a sudden collision in space between two ancient suns previously cooled through the ages, and long since bereft of luminosity and of life. The stars that change their brilliancy are called variable stars; those that blaze forth suddenly are "new stars," or *novæ*.

As we have already stated, the sky contains stellar systems other than those involving but a single visible object. Of these probably the most interesting are the double stars, composed of two individuals, often of different colors. These double stars appear but single to the unaided eye; only when the powers of a telescope of some size are brought into play, is it possible to resolve them into their component parts. In the field of view of such an instrument the stars all appear as brilliant points of light, occasionally glittering and sparkling, but the glitter and sparkle are imperfections caused by terrestrial atmospheric effects, and by the impossibility of constructing telescope lenses whose surfaces are ground to the right theoretic shape with absolute exactness. In other words, the stars appear in the telescope much as they do to the eye: only when the star is a double, the telescope often shows it as such, while the eye is unable to

see between the two components. And it is a very impressive sight, when we turn a telescope upon one of these double stars, to see the two tiny points of light projected on the deep, fathomless background of the night sky, and to realize that the speck of darkness between them is a bit of abysmal space.

Sometimes the close proximity of the components of a double star is fortuitous merely. The two objects may simply *appear* close together through happening to lie in almost exactly the same direction from us. But one of them may in reality be behind the other, and at a distance from us immeasurably greater than the first. In this respect astronomic observation differs from the viewing of ordinary objects on the earth. If, for instance, we should happen to notice two men, both standing at points almost exactly north of us, but one ten times as far away as the other, we would at once detect a difference of distance from the fact that the distant man would appear much smaller than the near one. But in the case of the stars, which we see as points of light merely, we could gather no such information. Even if one of the stars should be brighter than the other, this extra brilliancy might be due to a higher intrinsic light-giving power, and in no sense a result of greater proximity.

When two stars thus appear close together, though in reality separated by a great distance, they probably have nothing in common, and are of lesser interest. But in certain cases the two stars will appear close together through really being near each other in space. Then they must belong to a single system; have probably originated in a single nebula; true twin suns, bound one to the other and the other to the one; held by the invisible, intangible, but indestructible power of gravitational attraction.

In addition to these fixed stars, whose motions were unknown to the ancients; whose motions are so slow that generations of men must come and go before they can reveal themselves to the unaided eye, — in addition to these fixed stars, the night sky contains five other bright stars called of old the planets, from the Greek word πλανήτης, the wanderer. They have been named Mercury, Venus, Mars, Jupiter, and Saturn. The most conspicuous thing about them, when viewed without a telescope, is their peculiar and rapid motion among the fixed stars. They can be seen to make an entire circuit of the heavens, traveling apparently among the fixed stars, in brief periods of time ranging from about a year to about thirty years. Of course we now know the cause. These planets are not properly stars at all; they are like the earth, attendants of our sun, revolving around the sun in perfectly definite paths or orbits, and in perfectly definite periods of time. Compared with the fixed stars, they are all extremely near the sun. And being all thus comparatively near the sun, they are of course also all comparatively near each other; and our earth being one of the number, they are all comparatively near the earth, too. But we have just seen that the extreme apparent slowness of stellar motion is really only a result of the extraordinarily great distance by which we are separated from the stars; as this immensity of distance does not exist in the case of the planets, of course their apparent motions must and do appear to us comparatively rapid.

Their apparent motions are also complex in a high degree. Two of them, Mercury and Venus, move around the sun in orbits smaller than that of the earth, and therefore entirely within the earth's orbit; the other three, Mars, Jupiter, and Saturn, are exterior to the earth. Mercury has the

smallest orbit of all. It is always actually quite close to the sun, and therefore always appears near the sun when seen projected on the sky. Of course, it cannot be seen when the sun is visible on account of the overwhelming luminosity of the sun itself. Therefore we can observe Mercury occasionally only, just after sunset, near the point of the horizon where the sun has disappeared; or just before sunrise, near the point of the horizon where the sun is about to make its appearance. It is thus always seen in the evening or morning twilight, and was called of old the evening star or the morning star. The same is true of the planet Venus, which attains, however, a much greater apparent distance from the sun.

The exterior planets, Mars, Jupiter, and Saturn, may be seen at certain times throughout a wide range of space on the sky, and at any hour of the night, all of which phenomena will be explained in detail in a later chapter. Still other planets exist; but they are mostly too faint for the unaided eye; they have been discovered telescopically in modern times. All, together with our sun itself, are probably the result of gradual changes in a parent nebula.

The planets are unlike the stars in still another important particular. We have seen that the stars are self-luminous, incandescent; the planets are quite different, and give out no light of their own. They shine only by reflected light which they receive from the sun. The light goes from the sun to the planet; illumines it; and then we see the planet by solar light, just as we see objects in a room by reflected solar light, which we call daylight. This produces a rather curious telescopic planetary phenomenon called *phase*, a phenomenon which is most conspicuous also in the case of our moon. The planets are globular in shape, and

therefore only one hemisphere can be illumined by the sun at any one time. But the planet does not usually happen to turn its illuminated hemisphere directly towards the earth. Therefore we usually see only a part of the bright hemisphere, and this often looks more or less like what is called a half-moon. In other words, we always see a hemisphere of the globular planet, but it is not the same hemisphere which is turned toward the sun, and which is therefore bright. If the hemisphere we see and the bright hemisphere are mutually exclusive, we see a dark or "new-moon" phase. If the bright hemisphere and the one we see overlap, we see a crescent, half-moon, or other phase, as the case may be. Among the planets, Mercury, Venus, and Mars show the most conspicuous phase phenomena.

Sir John Herschel has given a good illustration of dimensions in our solar and planetary system. Represent the sun by a globe two feet in diameter. Then Mercury will be a grain of mustard seed on a circle 164 feet in diameter with the sun near its center; Venus, a pea, 284 feet distant; the earth, also a pea, 430 feet away; Mars, a pin's head, 654 feet; Jupiter and Saturn, oranges, distant respectively half a mile and four-fifths of a mile. The nearest fixed star, on the same scale, would be distant about 8000 miles, not feet. This illustration brings out clearly the comparatively minute dimensions of the solar system in relation to the vastness of stellar distances.

In actual appearance the planets differ greatly in the telescope; and they differ especially from the fixed stars. For even our most powerful optical apparatus will not suffice to magnify the latter so as to make them appear otherwise than as minute points of light. Many of them doubtless possess globular dimensions greatly exceeding any-

thing we find in the solar system; but the vast distances cause these dimensions to shrink into mere nothingness, even in our largest telescopes.

But in the case of the planets these great distances do not exist, and therefore the telescope shows their spherical size in the plainest possible way. But the planets differ greatly one from the other. Jupiter shows a bright, nearly round disk, crossed by a few dark straight lines or bands. It is accompanied, in small telescopes, with four satellites or moons, which can be seen to revolve around the planet. At times they pass behind the planet and disappear; and again, one or other of them is so placed that the planet interposes between it and the sun. Then, too, it disappears; for the satellites also shine by reflected solar light; and, of course, they receive none when Jupiter is placed between them and the sun. Finally, at certain other times a satellite may pass between Jupiter and the sun; and then it can be seen to cast a small round shadow dot on the bright surface of the planet. Such phenomena are called eclipses.

Saturn is the most beautiful of the planets, viewed with a telescope of moderate size. It has a number of moons or satellites, mostly too small to be seen in a glass of low power; but its most conspicuous feature is the famous ring of Saturn. This is a flat disk surrounding the planet, and, in the words of Huygens, who was the first to explain it correctly, nowhere "sticking to" the planet. The ring, like the other bodies of our system, shines by reflected solar light; and it is always distorted in appearance, as seen from the earth, into a flattened oval or ellipse, like a cart-wheel seen nearly edgewise. At certain times we actually do see it exactly edgewise, and then it appears, of course, like a thin, straight, bright line against the dark sky background. And when the ring

appears opened up to a considerable extent, we can see this dark background of the sky by looking through the openings between the ring and the ball of the planet.

Mars and Venus show us plain bright disks of moderate size, exhibiting in small telescopes little or no detail of any kind in the way of markings or bands. Their most conspicuous feature is the phase, which is much more marked than it is in the case of Jupiter and Saturn, whose phase phenomena are practically altogether unnoticeable. This follows, of course, from the fact that the quantity of visible phase is due to proximity; and Mars and Venus, being the planets nearest to our earth, must, of course, show more phase than the distant planets Jupiter and Saturn.

Mercury, as we know, is seen only in the twilight, showing in the telescope a small disk with marked phases.

Comets are occasional visitors to the solar system. They come presumably from outer space in the course of their orbital motions under the influence of gravitational and perhaps other forces; remain for a time in the vicinity of the solar system; are consequently visible to us; and finally retire again into the depths of space whence they came. When bright enough to be observed without the telescope, they commonly exhibit to our view a brilliant cometary "head," containing a central condensation or nucleus surrounded by a mass of tenuous luminous haze, and to it often attached a long visible streamer or tail, in olden times dreaded by all as a possible harbinger of wars and pestilence.

All these cometary phenomena are well seen in the photograph reproduced as a frontispiece in the present volume. The tail in this case has more than one streamer; and its length, as photographed, is about 11°, or nearly one-eighth the distance from the horizon to the zenith.

The tail actually seen by astronomers was at one time twice as long. The little curved lines on the photograph are star-images. We should of course expect these to be round dots in the picture; but in photographs of this kind they are almost always drawn out into little curves, for a very simple reason. The telescope is aimed accurately at the comet when the exposure of the photographic plate is commenced, and it is kept thus pointed at the comet during the whole duration of the exposure. This of course makes a "moved picture" of the stars, as photographers would call it. For the comet will "wander" among the stars, like a planet, in consequence of its orbital motion in space; and if the telescope's movement upon its stand is adjusted correctly to allow for the comet's motion, the photographic images of the stars must suffer.

The earth, considered as an astronomic body, is but one of the smaller planets; yet in one respect it is the most important of all, since it is the one upon which we live. Astronomers have been able to ascertain many facts about the earth, which we shall for the present summarize with the utmost brevity, postponing all detailed description to a later chapter. We know, first, that the earth rotates once daily on an axis; that this rotation carries us around, too; that in consequence of it the sun, stars, and other heavenly bodies seem to rise in the east, climb upward in the sky, and finally sink down again and set in the west. We also know that our earth, like the other planets, travels around the sun in an orbit; that it requires a whole year to complete a circuit of that orbit; that in consequence of the daily axial rotation and the yearly orbital revolution, we experience the phenomena of night and day, summer and winter, — phenomena to be explained fully

later; finally, we know from actual measures made upon the surface of our planet that the earth is a slightly flattened globe about 8000 miles in diameter.

The moon is the only satellite of our earth, and by far the most conspicuous object in the night sky; the most beautiful and interesting of all the heavenly bodies when observed through small telescopes; and important especially as being our nearest neighbor in the whole wide domain of cosmic space. Again summarizing existing knowledge as briefly as possible, the moon is now thought by astronomers to have once formed a part of the earth; to have been set free in some very distant age in the past by the action in some way of gravitational and possibly other forces. It revolves around the earth in an orbit somewhat similar to the earth's own annual orbit around the sun; completes such an orbital revolution in about twenty-seven and one-quarter days; and, in consequence thereof, appears to make a complete circuit among the far more distant fixed stars and planets in the same period, traveling around from a position of apparent proximity to any given fixed star back to the same star again in the twenty-seven and one-quarter day period. It is not self-luminous or incandescent, but shines by reflected sunlight like the planets; in consequence of its nearness to the earth, it exhibits the most pronounced phase phenomena, varying all the way from the full-moon, down through the half-moon stage, to actual invisibility at the time of new-moon. It is about 240,000 miles distant from the earth; is about 2000 miles in diameter; and the gravitational attraction of its mass upon the waters of terrestrial oceans gives rise to the ebb and flow of the tides.

The physical or actual appearance of the moon is not unlike that of the earth. The surface, as seen in the tele-

Photo at Lick Observatory.

PLATE 3. The Moon in the First Quarter Phase.

scope, is very much broken; there are several mountain ranges, and, especially prominent, a great number of large craters, apparently of volcanic origin, and usually having a mountain peak in the center. Extremely conspicuous features of the lunar surface, as seen in small telescopes, are the very black shadows which are cast on the surface when sunlight falls obliquely on the mountains and craters. There is no air or other atmosphere, and no water; nor have we any reliable evidence that any perceptible changes have taken place in the volcanic surface features since accurate records of telescopic observation were begun by men.

To complete this preliminary brief outline survey of our subject, it remains to add a few words about the sun, the central body of our solar system. The sun is our source of light and heat; without it life, as we know it, would be impossible on our earth. It is about ninety-three million miles distant from us, and nearly a million miles in diameter; within its vast bulk might be placed the earth and moon, together with the entire lunar orbit in which, as we have said, the moon revolves around the earth in twenty-seven and one-quarter days. The sun turns on an axis in a period of about twenty-five terrestrial days; its surface is usually marked by the well-known sun spots, visible in small telescopes plainly, and first seen by Galileo, when he turned upon the sun probably the first telescope ever made. These spots are now known to have periods of special frequency. Every eleven years they occur in greater numbers than usual; and this period of eleven years is in some mysterious way connected with the known frequency periods of auroras and magnetic storms on our earth. The bulk and mass of the sun are so great that its gravitational attraction far exceeds that of all the planets combined. It thus becomes

the gravitational ruler of the whole solar system; around it all the planets may be said to revolve in their duly appointed paths or orbits.

It is hoped that the foregoing brief summary of astronomic science may help to awaken a desire in the reader to possess more detailed knowledge; and this we shall endeavor to give in later chapters; perhaps we may be permitted to conclude the present one by calling attention to the value of astronomy for practical purposes as well as for mental discipline and study. It is often said that astronomy is a somewhat detached subject; of interest certainly, but having little or no close and intimate relation to the everyday affairs of human life. But in reality the converse is the truth. Probably no other of the more abstruse sciences enters so directly and so frequently into our daily affairs as does astronomy. There are at least three services performed by astronomy that are essential, and without which civilization, as we know it, would be impossible. These things are: first, the regulation of time; second, the execution of boundary surveys and the making of maps and charts; third, navigation.

Few persons stop to think when they enter a jeweler's shop to correct their watches by comparison with the jeweler's "regulator," or when they communicate by telephone with a central telephone station to ask for the correct time, that both the jeweler and the telephone operator must themselves have some source of correct time by which to regulate their regulators. This source of correct time is the astronomical observatory. The standard observatory clock is itself but a fallible piece of machinery fabricated by fallible human hands, and it can be kept right only by constant comparisons, made on every clear night, with the

unvarying time standards provided by nature, the stars themselves in their courses. For instance, time observations of the stars are made regularly and nightly in the United States Naval Observatory at Washington, the chief official astronomic station of the United States government. With these observations the standard clocks in the clock room of the observatory are corrected and timed; and from these standard clocks electric signals are sent out daily in accordance with a pre-arranged schedule so that time-balls can be made to indicate the exact instant of noon to the people, and jewelers and others may correct their regulators. Thus is every citizen in touch with the astronomic observatory almost daily and of necessity, although he does not generally realize the fact until it is brought specially to his attention.

And the matter of mapping and charting is equally dependent upon astronomy. Ordinary small surveys of farms or towns may be made by ordinary surveyor's instruments without constantly having recourse to astronomers. But of what value would be a map of an entire continent unless the customary latitude and longitude lines were inscribed upon it? And these essential lines cannot be so inscribed without astronomic observations. Such observations must necessarily be made specially for the purposes of each survey, and the consequent calculations always depend, too, upon certain prior astronomic data contained in published astronomic "tables" or printed books, themselves in turn based on average or mean results obtained in the great observatories of the world during the last couple of centuries by steady continuous systematic study and observation of the stars.

Even more important than continental maps for the progress of civilization are the coast charts published by the various governments of the maritime nations. These also

require very precise latitude and longitude lines; and here, as before, recourse must be had to astronomic observations and accumulated astronomic results.

Finally, navigation itself, upon the open sea, could not proceed successfully without astronomy. Those of our readers who have crossed the ocean in a magnificent modern steamer may have seen at times the captain or navigating officer "take the sun," as it is called, with a sextant. Possibly they have thought that after making such an observation the navigator could read on the face of the sextant the exact position of the ship at the moment, its latitude and its longitude on the earth, as ordinarily understood in geography. But such is by no means the fact. Before they can be made to yield this essential information, sextant observations must be subjected to a somewhat laborious process of numerical calculation, or "reduction," as it is called. This is an astronomic process; and in carrying it to completion the navigator again requires certain printed tables of a purely astronomic character. These are contained in a book called the "nautical almanac," which is published annually in various languages by the several civilized governments of the world. And again, as before, for the preparation of such nautical almanacs, these governments must maintain, and do maintain, astronomic computing bureaus, manned by astronomers, and employing in their calculations once more the published results obtained by astronomers of the past in the various great fixed observatories. The details of all these astronomic activities must, of course, be postponed to later chapters; but it is hoped that enough has been said here to remove from the reader's mind the possible notion that astronomy is of little or no practical utility in the ordinary affairs of men.

But far beyond and above all this, the study of astronomy possesses a value peculiarly its own, as a means of mental training. On account of venerable age and consequent approximate perfection of knowledge, this science is characterized especially above all others by the peculiar intricacy of the elementary problems it presents, and by the unusual exactness of which their solutions admit. Furthermore, notwithstanding the importance of its direct practical applications, which have been mentioned, the study of astronomy is peculiarly free from any materialistic tendency,— from any connection, in short, with utilitarian motives. It is not a vocational study, giving knowledge which can be sold for money by the young college graduate upon his entry into practical affairs. But it is preëminently a study which will give a clearer outlook upon the universe in which we pass our lives, preëminently one that will make that universe seem a pleasanter place in which to live. So that if a certain portion of our time is to be devoted to studies that are not strictly vocational, astronomy will surely be found a profitable and desirable subject. And surely also there is much to be gained in our choice of studies from the selection of such as are likely to arouse a real interest in the student; to arouse that desire for knowledge which, once awakened, will make the task of the teacher an easy one. Here again astronomy holds a most favorable place. That which has its being within the confines of a single drop of water is as wonderful as are the motions within a planetary or sidereal system. But the animalcules within that drop of water, though their number be myriad, can never stir our deepest interest, for they are without that strong appeal to the imagination, without those vast distances and mighty forces, the materials of astronomic study alone.

CHAPTER II

THE HEAVENS

PROBABLY the best method of approaching the study of astronomy is to begin with those observations and problems that do not require the use of any instruments whatever. These problems are surely the earliest problems, since men of old must have begun to discuss the mysterious events they could see about them in the universe long before they had invented even the rudest instruments of measurement.

Astronomy is a study of the sky; and the first thing to be noticed in a study of the sky is the sky itself. To us it appears at night like a great, round, blue, hollow dome within which we are standing. To its interior surface seem to be attached the apparently numberless bright twinkling points of light we call stars. In the day it carries only the sun, and perhaps, too, the moon rather faintly visible; and in the intermediate periods which we call twilight, and which occur at dawn and at dusk, we can see perhaps two or three dim stars, called morning and evening stars. We know that these morning and evening stars are certain of the planets, which, as we have already seen, are members of the solar system like our earth, circling around the sun, each in its proper path or orbit.

But there is no real dome of the sky above and around us; it is simply an optical illusion, a creation of our own imagination. Nevertheless, it is most convenient to imagine it to

be real, because we can thus fix our first astronomical ideas to something tangible; and by a consideration of this round dome as if it actually existed, we shall be able to clarify and to solve many interesting problems. Granting, then, that there is such a dome above us, we have no reason to imagine it other than perfectly round. Let us regard it as a great hollow ball or sphere; astronomers have given it the name Celestial Sphere.

The next question is whether this celestial sphere is the same sphere everywhere. Is the celestial sphere surrounding New York identical with that surrounding the city of Capetown, South Africa? The answer is: yes. The sphere is the same sphere everywhere. Theoretically, the center of the sphere is at the center of the earth; and since the diameter of the earth is about eight thousand miles, an observer on the earth's surface will be distant about four thousand miles from the true center of the sphere. But such a distance as four thousand miles is literally a mere nothing compared with the infinitely vast distance of the celestial sphere. The whole planet earth shrinks into a mere dot in comparison. It makes absolutely no difference whether you are on the earth's surface, or could be transferred to its center, you would see identically the same imaginary celestial sphere. The stars and other heavenly bodies, wherever they may be situated around us in space, seem to be projected upon that distant celestial sphere, and attached to its interior surface. Even if you could make a sudden jump of about ninety-three million miles from the earth to the sun, you would still see the same identical sphere, much too far away to be affected by such a little change in the observer's position. Not only the earth, but its entire orbit, including the sun, shrink into a dot. Astronomy is

truly a science of vast distances. But there is this essential difference between the distance of the celestial sphere and all other distances in the science. The far-ness (if we may use such a word) of this imaginary sky sphere is infinitely greater than any other actually known and measured by men.

The accompanying Fig. 1 is intended to illustrate this notion of the celestial sphere. The large circle is supposed to represent the sphere; only, of course, its size cannot be made big enough; the reader must imagine it extended to infinity. The dot E at the center of the big circle is the earth; the reader and the author are supposed to be standing on the surface of that dot. The tiny circle represents the earth's annual path around the sun, the sun itself being the larger dot at the center of the tiny circle. The crosses represent stars scattered through sidereal space at all sorts of distances from the earth. The lines with arrows passing through the crosses indicate the points on the interior surface of the celestial sphere where the stars will appear to be projected, and where they will seem to be attached to the interior or supposedly visible surface of the sphere. The longest arrow indicates the point on the sphere where the sun will appear projected, that arrow

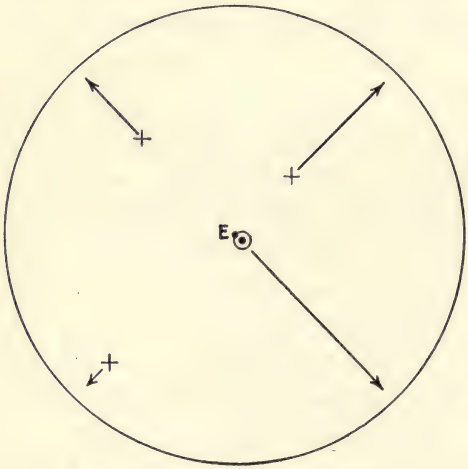

Fig. 1. The Celestial Sphere.

being, of course, merely a straight line passing from the earth to the sun and thence continued outward to the sphere. For the sun will also appear to us as if attached to the interior surface of the sphere, like the stars, at the point indicated by its arrow. This elementary notion, that the various celestial bodies will appear to be located on the sphere at the points shown by their arrows is an important idea, and one that is not at all difficult to grasp. We must not forget that the arrows are all supposed to be infinitely long; even the solar arrow is infinite, although the sun dot and the earth dot are very near each other, cosmically speaking.

Having thus fixed our ideas as to the celestial sphere, we must next study it in its relation to the various objects that appear projected upon it; and the first important thing to consider more in detail is the position of the sun on the sphere. We have already seen that the earth travels around the sun once in a year. The path or orbit in which the earth thus travels is an oval or ellipse; but for the purpose of a first approximation such as we shall here consider, we can take this path of our earth to be a circle, with the sun at its center. Now this circular orbit, like every circle, must lie entirely in a single plane or flat surface. The accompanying Fig. 2 shows this circular approximate orbit of the earth E moving around the sun at the center S in the direction shown by the arrow. The single plane or flat surface in which the entire orbital path lies is here of course the flat plane of the paper on which this page is printed.

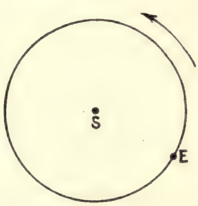

FIG. 2. The Earth's Orbit.

It is evident that the earth, being always in its orbit,

must likewise always be situated in the plane of the paper. And the sun, being at the center of the circular orbit, must also be in the same plane. From these considerations follows the important preliminary principle that earth and sun are both constantly in a single plane. To this important fundamental plane has been given the name Plane of the Ecliptic.

The plane of the ecliptic is defined, then, as the plane in which are situated at all times the sun, the earth, and the earth's orbit around the sun. Now let us extend our ideas so as to include the celestial sphere in our consideration of the earth's orbit. Imagine the orbital plane, but not the orbit, extended or stretched outward, indefinitely, farther and farther, approaching gradually an infinite bigness, until at last it meets the imaginary celestial sphere. Evidently, it will cut out a circle on the celestial sphere, just as though one were to slice a round orange with a flat cut. The line in which the rind of the orange would be severed by such a cut would then be a circular line; and so also must the line cut out on the celestial sphere by the ecliptic plane be a circle. The fact that the sphere is an immense globe and the orange a small ball here makes no difference. The principle is the same.

It is possible to draw a little more information from the analogy of the orange. Wherever we slice the orange, we obtain a circle; but if it was sliced through the center, the orange would be cut in two equal halves, and then the circle would be the largest circle that could possibly be drawn around the rind of the orange. Applying this to the case of the celestial sphere cut by the ecliptic plane, we see at once that here also the sphere is cut in two equal halves. For the earth, as we have seen, is at the center of the celes-

tial sphere; and therefore the ecliptic plane, which passes through the earth, is also a cut or slice through the center of the celestial sphere. Consequently, the circle cut out on the celestial sphere by the ecliptic plane produced to infinity is a circle as large as can possibly be drawn on the celestial sphere, and it divides that sphere in two equal halves. Such a circle drawn on a sphere, dividing it into halves, is called a Great Circle of the sphere. The particular great circle of the celestial sphere, cut out by the plane of the ecliptic produced to infinity, is called simply the Ecliptic.

The ecliptic, then, is defined as a great circle of the celestial sphere cut out by the plane of the earth's orbit around the sun, produced to infinity. It would be a convenience if some one could go up to the sky and mark out the ecliptic circle upon it with a big paint-brush. While this is impossible, it is perfectly easy to mark it upon a celestial globe; and the reader is advised to examine such a globe, when he will surely find the ecliptic plainly drawn upon it.

The important peculiarity of the ecliptic circle is this: the sun must always at all times appear to lie in that circle. And the reason is quite simple, as shown again in Fig. 3. Here we have once more drawn a large circle to represent the infinite celestial sphere; and the dot which should represent the combined sun, earth, and earth's orbit around the sun is shown at the center, magnified into a circle. The observant reader will notice, upon comparing Figs. 1 and 3, that in the former figure the earth occupies the center of the sphere, whereas in Fig. 3 the sun is at the center. But the figures are interchangeable, as we already know, because of our having assigned *infinite* size to the celestial sphere.

In Fig. 3 the smaller circle represents the earth's orbit around the sun, E' and E'' being two positions of the earth

in its orbit. The corresponding apparent positions of the sun, as projected on the celestial sphere, are shown at S' and S''. For, as we already know, if an imaginary line be drawn from the earth to the sun, we must necessarily see

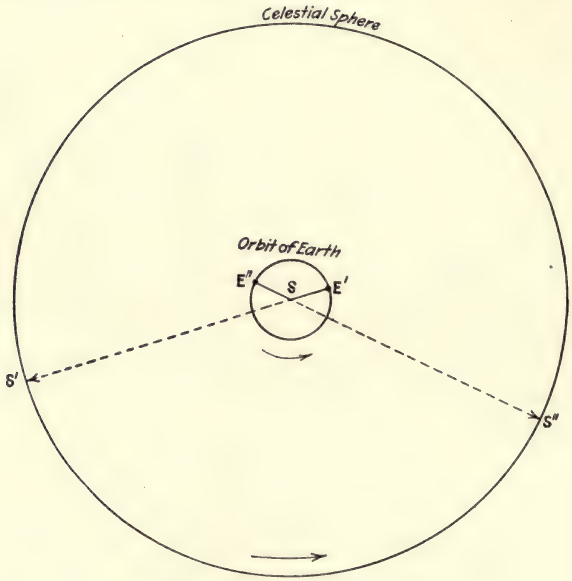

FIG. 3. The Ecliptic Circle.

the sun from the earth along the direction of that imaginary line; and if the line be extended outward until it pierces the celestial sphere, the sun will appear to us projected on the sphere at the point where the sphere is pierced by the line.

Now this sight line from the earth to the sun will necessarily lie entirely in the plane of the earth's orbit, for in that plane both the earth and the sun are at all times situated. Consequently, the sight line, when extended to pierce the celestial sphere, must necessarily always pierce that sphere somewhere on the circle cut out on the sphere by the plane

of the earth's orbit produced outward to infinity. But this circle is the ecliptic; and thus we have a proof that the sun must always appear on the sky projected upon the ecliptic circle. And it is certainly a most remarkable thing that it should thus be possible to draw an imaginary circle on the sky such that at all hours of the day, on every day of the year, and of every year, when we look at the sun, it will appear to be situated at some point of that circle. Yet it all follows quite simply from the above elementary considerations concerning our earth's orbital motion around the sun. And it is furthermore already equally evident that as the earth progresses around its orbit, as shown by the curved arrow, the sun will appear to progress around the ecliptic circle with a rate of motion corresponding to the earth's own motion in its orbit.

Figure 3 also gives a good opportunity to explain the meaning of the terms "angle" and "angular distance," which we shall have frequent occasion to use. An angle is defined as the difference in direction between two lines. Thus, if we consider the lines SS' and SS'' in Fig. 3, the angle between them is indicated by the combination of letters $S'SS''$. Every angle is thus indicated by a combination of three letters; the middle letter of the three always indicating the point of the angle, or its so-called "vertex." The corresponding angular distance on the celestial sphere between S' and S'' is the arc $S'S''$; and such angular distances must of course be measured in degrees. In Fig. 3 the angular distance $S'S''$ is about 120°, or one-third of an entire circumference of 360°.

These facts about the ecliptic constitute one of the most important discoveries of the very earliest astronomers. The hazy records of extreme antiquity indicate that the Chinese

knew the ecliptic and had measured its position on the sky as early as 1100 B.C. The early Greek astronomers of Alexandria certainly knew of it; for instance, we have fairly reliable records showing that Eratosthenes (276–196 B.C.) measured its position quite accurately.

The next important phenomenon to which our attention must be directed results from still another motion of our earth; namely, its axial rotation. As we all know, the earth turns on its axis once daily; a motion which is quite distinct from its orbital revolution around the sun. Both motions take place simultaneously, the earth traveling around the sun in its orbit while it is at the same time spinning on its axis, much as a couple of waltzing dancers move from end to end of the room while at the same time spinning rapidly around each other.

This terrestrial rotation has an immediate effect upon the celestial sphere and all the heavenly bodies which appear projected upon it. For the astronomer, being fastened to the earth, turns around with it, perforce. And as the earth turns, with the astronomer attached, it is constantly presenting him to a new part of the celestial sphere. Just so a dancing couple face every point of the compass in succession, in consequence of their spinning motion, and quite independent of the fact that they are also moving about in the room at the same time.

This turning of the astronomer successively toward different parts of the celestial sphere makes that sphere appear to him as though it were turning around the earth instead of the earth turning within it, precisely as a railway passenger sees fields and trees apparently flying past his train, although he knows these objects are really fixed in position, and himself in rapid motion.

The axial rotation of the earth takes place from west to east; and the consequent seeming rotation of the celestial sphere is from east to west. Objects projected on the sphere partake of this seeming motion; the sun, the moon, the morning and evening stars, and all the other stars.

This is the cause of day and night; of the rising and setting of all heavenly bodies, including the sun. As the earth rotates from west to east, they all seem to revolve in the opposite direction daily, rising from beneath the eastern horizon, slowly climbing the sky, and again sinking down to set in the west. These facts are quite generally known with respect to the sun and moon, but comparatively few are aware that the stars also rise and set. It is reported that Sir George Airy, a recent astronomer royal of England, used to say that not more than one person in a thousand knows that the stars, like the sun, rise and set. Most people think the stars are always the same, simply a uniform countless assemblage of thickly clustered luminous points.

Having thus explained the earth's rotation, we must next consider its rotation axis. Our planet earth, in its rotation, turns about an imaginary line or axis passing through its center and meeting the earth's surface at the north and south poles of the earth. Now imagine for a moment this rotation axis extended outward in both directions, farther and farther, until at last the two ends pierce the celestial sphere itself. They would, of course, mark out on the sphere two points corresponding exactly to the two terrestrial poles. These two points are called the north and south Poles of the heavens, or the celestial poles. The long line joining them is the axis of the celestial sphere, and a very short bit near the middle of the line is the terrestrial rotation axis. Figure 4 again shows the celestial sphere, this time with the earth

at the center, magnified from its proper size of a mere dot, so as to exhibit the earth's rotation axis and its prolongation, the axis of the celestial sphere. N and S are the north and south poles of the earth; NS is the terrestrial rotation axis; and its prolongation to the celestial sphere marks out N' and S', the north and south poles of the celestial sphere.

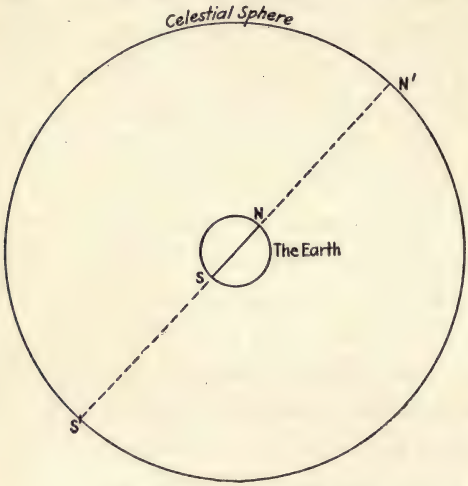

FIG. 4. The Celestial Poles.

Now since the apparent rotation of the celestial sphere is merely a result of the earth's turning, and since the latter takes place around the axis, so also the great sphere's seeming turning must take place about this same axis. In other words, all the stars must seem to revolve nightly around the two poles of the heavens. Stars very near the poles on the sky will seem to turn in little circles; those farther from the pole will seem to turn in larger and larger concentric circles.

Figure 5 shows a few of these circles in which the stars appear to revolve nightly, and indicates that those near the two poles of the heavens are small. As we go farther from the poles the circles become larger, until at last we come to stars halfway between the two celestial poles, where the largest of all the circles occurs. The circles are of course all parallel; and they are concentric in the sense

that their real centers all lie on a single straight line, the axis of the celestial sphere. The stars, as they appear to revolve in the circles, of course complete a revolution every twenty-four hours, since the axial rotation of the earth within the sphere is the true cause of the whole phenomenon; and this axial rotation occupies exactly one day of twenty-four hours. And because of this daily period, the circles are called Diurnal Circles of the celestial sphere. Diurnal circles are defined, then, as parallel circles on the celestial sphere in which the stars complete their daily apparent rotation around the celestial poles.

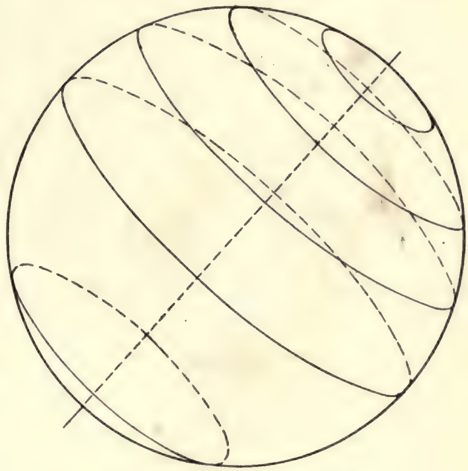

Fig. 5. Diurnal Circles.

We have just seen that the largest of all the diurnal circles is the one halfway between the two celestial poles; and it is a particularly important one. It of course divides the entire celestial sphere in two halves, which are called the northern and southern celestial hemispheres, and this largest diurnal circle is itself called the Celestial Equator. It corresponds exactly to the equator on the earth, which similarly divides our planet into northern and southern hemispheres. In fact, it is clear that as the terrestrial and celestial poles correspond exactly, so also the terrestrial and celestial equators must correspond exactly. And it is therefore also possible to define the celestial equator in a

manner quite analogous to the definition of the other important great circle of the celestial sphere, the ecliptic. For if, as in the case of the ecliptic plane, we imagine the plane of the earth's equator stretched out and extended until it finally reaches the celestial sphere, it will cut out a great circle on the sphere, and this great circle is the celestial equator. So we might define the celestial equator as a great circle on the celestial sphere cut out by the plane of the earth's equator produced to infinity, and this definition is equivalent to the former one, which describes the celestial equator simply as the largest of all the diurnal circles.

Having thus defined the celestial poles and equator, it is easy to carry analogy a little farther, and inquire what corresponds on the sky to latitude and longitude on the earth. The reader will recall from geography that when we desire to define the position of a place on the earth we do so by giving its latitude and longitude. Terrestrial latitude is defined as the angular distance of a place north or south of the earth's equator, and terrestrial longitude is its angular distance east or west from some so-called "prime meridian," such as that of Greenwich, England.

Exactly analogous methods are used for defining a star's place on the sky, or the location of the point where it appears to us projected on the celestial sphere. Unfortunately, the terms celestial latitude and longitude have not been used for this purpose. Instead of these terms, astronomers use the words "declination" and "right-ascension"; which bear the same signification with respect to the celestial equator that terrestrial latitude and longitude bear to the equator on our earth.

It is of interest to consider here the initial point from which astronomers reckon right-ascensions; for there is no prime

meridian on the sky like that of Greenwich on the earth. Instead, astronomers use an initial point on the celestial equator, and from it the right-ascensions of all celestial objects are counted. This point is called the Vernal Equinox, and its location will be understood easily from the following considerations.

We have so far defined two great circles of the celestial sphere, each dividing the sphere in two halves. They are the ecliptic circle and the celestial equator. Now these two circles, as shown in Fig. 6, must intersect at two opposite points on the sphere; for any pair of great circles on any sphere must evidently do this. These two opposite points are called equinoctial points; one is the Vernal Equinox, the other the Autumnal Equinox. We shall have occasion farther on to explain the importance of these two points a little more in detail; for our present purpose we need merely remember that the vernal equinox point is by universal convention selected as the initial point for measuring all right-ascensions.[1]

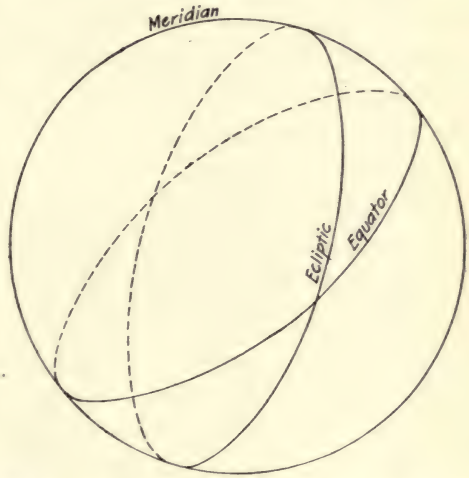

Fig. 6. Two Great Circles intersecting at Opposite Points of the Sky.
(After Cassini's *Astronomie*, p. 78. Paris, 1740.)

At the risk of seeming somewhat tiresome, we must still add to these rather prolix preliminary explanations a very

[1] Note 1, Appendix.

few more necessary definitions. For there is still another important great circle on the celestial sphere, again dividing it into a pair of halves, but a different pair from the two hemispheres north and south of the celestial equator. This important great circle is the Horizon. In astronomy the horizon is precisely the same thing as the horizon in ordinary life. It is defined accurately as a great circle on the celestial sphere cut out by an infinitely extended level plane touching the earth at the point where the observer stands.[1] Of course, in the interest of exactness, we should note in passing that the same horizon circle would be cut out on the sky by a plane parallel to the first, but passing through the earth's center beneath the observer's feet. This is, of course, again a result of the fact that the earth's radius of four thousand miles, by which distance these two planes are separated, is a perfectly negligible quantity in comparison with the infinite distance of the celestial sphere.

Having defined the horizon, it is easy to add two other definitions, both of which refer to astronomical terms having also the same signification precisely that they bear in ordinary English. These are the Zenith, which is simply the point of the celestial sphere directly overhead, and therefore exactly 90° distant from every part of the horizon; and Altitude, or angular elevation above the horizon. Altitude is defined accurately thus: the altitude of a celestial body is its angular distance (p. 29) above the horizon. The altitude of the zenith is thus evidently 90°.

As we now know the meaning of the two points on the celestial sphere called the celestial north pole and the zenith, it is possible to define next the Celestial Meridian. This is a great circle drawn on the sphere from the celestial north pole

[1] This plane, in mathematical language, is a plane tangent to the earth.

to the zenith, and thence extended completely around the sphere until it returns again to the pole. Very simple considerations show that the celestial meridian must pass through the north and south points of the horizon.[1]

The accompanying Fig. 7, representing a celestial globe, may make the foregoing description clearer. The circle *HVO* is generally made of wood, and represents the celestial horizon. *HPZAO* is usually made of brass, and represents the celestial meridian, passing through the celestial pole *P*, the zenith *Z*, the north point of the horizon *H*, and the south point of the horizon O. The circle *ASQ* is the celestial equator, everywhere 90° distant from the pole *P*. The circle *BC* is a diurnal circle. *ZV* is a flexible strip of brass marked with degrees and pivoted at *Z*. It can be turned to any part of the horizon, and, by means of the degree divisions marked upon it, we can measure the altitude or angular elevation of any star above the horizon. Some of the constellation figures (p. 7) are also drawn on the globe.

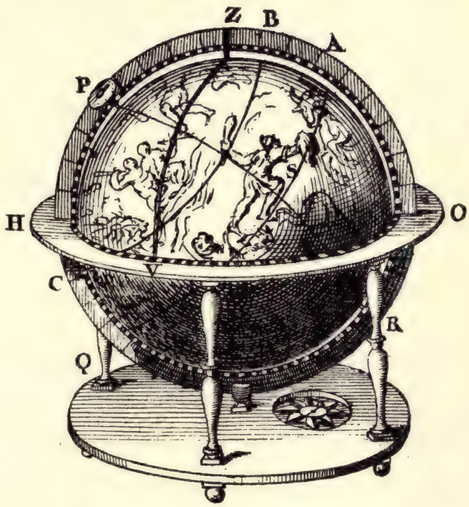

FIG. 7. The Celestial Globe.
(From Lalande's *Astronomie*, 3 ed., Tome 1, p. 74. Paris, 1792.)

Having now defined the principal circles and points upon the celestial sphere, let us next investigate the position of the

[1] For additional definitions and explanations, see Note 2, Appendix.

north pole of the sphere with respect to our horizon. We shall first imagine an observer standing at the north pole of the earth. It is evident from the accompanying Fig. 8 that such an observer would see the celestial pole directly overhead, in the zenith. For P being the observer's position at the north pole of the earth, PS will be the earth's rotation axis, passing through the two poles of the earth. And if this axis is lengthened out to an infinite size, it will meet the celestial sphere at P', the north pole of the sphere, which will clearly be directly overhead. PH is a level plane touching the earth where the observer stands; consequently H is a point of the horizon, in accordance with our definition (p. 36). OH' is a plane passing through the earth's center parallel to the level plane PH; and the points H and H' will coincide on the celestial sphere because the distance PO is absolutely negligible in comparison with the infinite distance of the sphere. These considerations show that to an observer at the pole of the earth the celestial pole will be at the zenith, and its altitude, or angular elevation above the horizon, will be 90°.

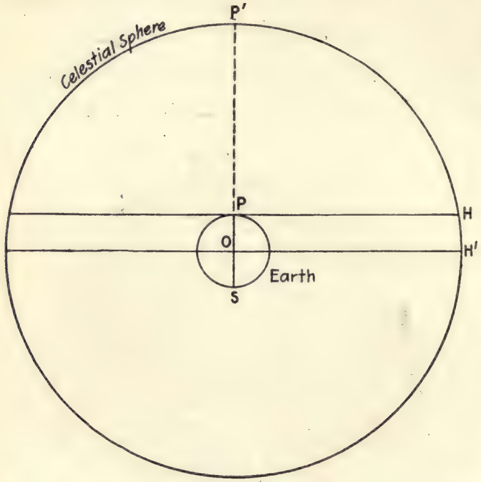

Fig. 8. Observer at the North Pole; Celestial Pole in the Zenith.

To an observer standing at the terrestrial equator the position of the pole will be quite different, as shown in Fig. 9.

If we place the observer on the earth at *E*, and call *E* a point of the equator, the terrestrial rotation axis will be at *PS*, because any point on the terrestrial equator must be 90° distant from the terrestrial poles. This puts the celestial north pole at *P′*, which coincides with *K*, a point on the horizon of an observer at *E*. It follows from this that if we go to the equator of our earth, we will there see the celestial pole in our horizon, distant 90° from our zenith at *Z*, directly overhead.

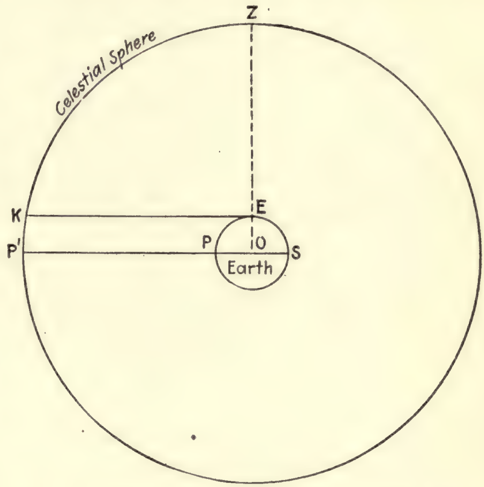

FIG. 9. Observer at the Equator; Celestial Pole in the Horizon.

Having thus ascertained that to an observer at the pole of the earth the celestial pole appears overhead, and to one at the equator in the horizon, it is not difficult to realize that an observer traveling from the pole to the equator will see his celestial pole gradually seem to move down from his zenith to his horizon. For if the celestial pole occupies two extreme positions in the zenith and horizon when the observer is in two extreme terrestrial positions at the pole and equator, it is clear that as the observer occupies successive intermediate terrestrial positions, the celestial pole will seem to occupy successive positions also, intermediate between the zenith and horizon. This is the reason why travelers going south, and noting the pole

star night after night, see that star gradually sinking lower in the sky; and if they continue southward quite to the equator, they see the pole star actually disappearing at the horizon. For the pole star is so placed in space as to be projected on the sky very near the imaginary celestial pole; and consequently the visible pole star partakes of the changes which we have just explained.[1] In fact, the altitude, or angular elevation of the celestial pole above the horizon, is everywhere equal to the observer's terrestrial latitude, or angular distance from the terrestrial equator.

This very important theorem enables us at once to study the very different appearance of the celestial sphere and its diurnal circles as seen from different places on the earth. At the equator, where the pole is in the horizon, the celestial sphere looks like Fig. 10, called the Right Sphere. Here the diurnal circles (p. 33) are all perpendicular to the horizon, and they are all bisected or halved by the horizon. Consequently, as the celestial bodies perform their daily apparent rotation with the sphere, in consequence of the corresponding daily axial rotation of the earth inside, — their diurnal circles being all halved by the horizon, — all the celestial bodies will be above the horizon just as long as they are below it. They will be "up" twelve hours, and "down" (or "set") twelve hours.

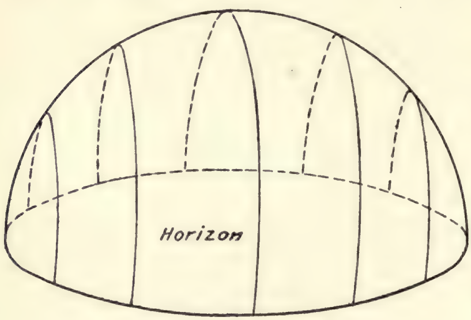

Fig. 10. The Right Sphere.
(After Long's *Astronomy*, Vol. 1, p. 91. Cambridge, 1742.)

[1] Note 3, Appendix.

Now we have seen (p. 29) that as the earth travels around the sun in its annual orbit, the sun seems to travel around the ecliptic in a corresponding manner. But, wherever it may be projected on the ecliptic, it must always be on a diurnal circle; in the light of what we have just learned about the right sphere, this diurnal circle must be halved by the horizon; therefore, to an observer at the equator, the sun will be above the horizon twelve hours every day in the year. We therefore see that at the equator day and night are always equal throughout the whole year.

Quite a different state of things holds at the pole, where we see what is called the Parallel Sphere, as indicated in Fig. 11. Here the celestial pole is at the zenith, and the diurnal circles are all parallel to the horizon. If a celestial body is above the horizon at all, its entire diurnal circle is above the horizon; it will remain "up" twenty-four hours during each axial rotation of the earth. The largest diurnal circle, the equator, here coincides with the horizon; to an observer in the northern hemisphere, stars between the celestial equator and the north pole never set; those between the equator and the south pole never rise.

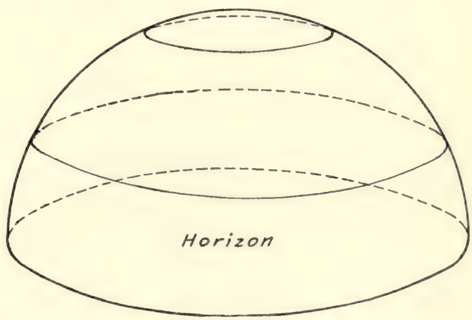

FIG. 11. The Parallel Sphere.
(After Long's *Astronomy*.)

How would these facts affect the sun, which is always seen in the ecliptic, as we know? We also know that the ecliptic is halved or bisected by the celestial equator (p. 35).

Therefore, during half the year the sun will be between the equator and the north pole. During that half-year its successive diurnal circles on the parallel sphere will be entirely above the horizon, and the sun will not set. This explains the important and well-known fact that at the north pole the sun remains above the horizon six months, and day, as well as night, is six months long.

To observers situated on the earth in places like New York, intermediate between the pole and the equator, the sky appears in the form called the Oblique Sphere, shown in Fig. 12. Here the diurnal circles are neither perpendicular to the horizon, nor parallel to it. Being parallel to each other, they all make the same angle with the horizon, an angle which is different in different terrestrial latitudes.

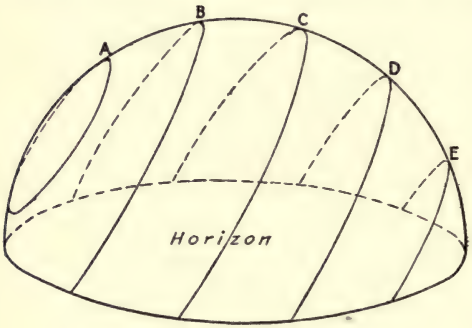

FIG. 12. The Oblique Sphere.
(After Long's *Astronomy*.)

And the diurnal circles are not halved by the horizon, either. Each such circle is divided by the horizon in two unequal parts. If the circle is between the celestial equator and the north celestial pole, as B, Fig. 12, the part above the horizon is the longer. If the circle is between the equator and the south pole, as E, the part below the horizon is the longer. Thus it follows that stars projected on the sky between the equator and the north celestial pole are above the horizon each day longer than they are below it, and *vice versa*. Only stars on the celestial equator itself have a

halved diurnal circle, and are above and below the horizon equal twelve-hour periods. Some of the diurnal circles quite near the north pole, as A, do not reach the horizon at all. Stars projected on these diurnal circles will therefore never set; and stars with corresponding diurnal circles near the south pole will never rise.[1] Observers in the southern hemisphere of the earth, of course, have these conditions reversed.

The sun, always projected on the ecliptic, may have its diurnal circle divided either way. We have seen (Fig. 6, p. 35) that the ecliptic is bisected or halved by the equator. Consequently, when the sun is seen in one half of the ecliptic, it is between the equator and the pole, and therefore above the horizon longer than below it; and when it is seen in the other half of the ecliptic, it is below the horizon longer than above it. In the one case, the days are longer than the nights; in the other, the nights are longer than the days. As the sun is seen in one half of the ecliptic during about half of each year, it follows that during half of each year our days are longer than our nights in the temperate regions of the earth, where the oblique sphere prevails. Only when the sun is exactly on the equator, at one of the two points where it is intersected by the ecliptic, does the sun have a halved diurnal circle, giving us equal periods of light and darkness, — equal days and nights. We have already seen that these two points of intersection of the equator and ecliptic are called equinox points. We now know the origin of the name; when the sun is seen projected at either of these points of the ecliptic, we have equal days and nights. It may facilitate the comprehension of these facts if the reader will again examine Fig. 7, p. 37, the Celestial Globe.

[1] Note 4, Appendix.

From the above elementary considerations follows at once a preliminary understanding of the phenomenon called the Seasons. For it is clear that the half-year during which our days are longer than our nights will be a summer or hot half-year, since we obtain our heat from the sun; and the half-year with the long nights will be a cold or winter half-year. Near the terrestrial equator, where the right sphere gives constantly equal days and nights, there must be, and is, a complete uniformity of seasons. Near the pole, with its parallel sphere, there is a long six months' summer day, and a corresponding winter night. But the polar summer is itself cold, because even in summer the sun never rises to a great altitude above the polar horizon.

CHAPTER III

HOW TO KNOW THE STARS

Any one beginning the study of astronomy quite naturally desires to proceed as quickly as possible from the reading of books about the stars to an examination of the stars themselves in the sky. And in a first preliminary survey it is of interest to learn the names of the principal stars and constellations as they have been handed down to us from olden times. It is not at all difficult to acquire this knowledge, now that we have become acquainted (Chapter II) with the celestial sphere, and the more important lines and circles which astronomers imagine to be drawn upon that sphere.

There are four objects that often puzzle beginners, when they attempt to compare star maps with the night sky for the purpose of identifying the more important bright stars. These are the same four things that so puzzled the ancients, the four bright planets, Venus, Mars, Jupiter, and Saturn. Mercury, the only other bright planet, is rarely seen; but one or more of the above four is almost sure to be conspicuous in the sky, to a certain extent impairing the correctness of our star maps.

For these star maps do not show the planets; and for a very simple reason. We know (p. 10) that the planets seem to wander among the fixed stars; they appear, now here, now there, at very widely varying points of the sky. On the other hand, the fixed stars (p. 7) retain relative posi-

tions practically unchanging; if, for instance, any three are located in a straight line, they will continue to lie on that straight line for centuries, so far as observations with the unaided eye can ascertain. It would be possible to make a star map showing both stars and planets as they appear on any given date. But such a map would not be correct six months later; while it would still show the fixed stars in their proper relative positions, the planets would be wrongly placed, on account of their wanderings. For this reason, astronomers omit the planets altogether from their star charts; and beginners are puzzled.

For the beginner, upon looking at the sky, always observes the planets first of all, because they appear as bright as, or brighter than, the most brilliant fixed stars on account of their proximity to the earth. For instance, the beginner may see three lucid stars forming a small triangle, with the brightest star of the three at the apex of the triangle. He at once looks at the star map, to identify this triangle. Finding none in the proper place, he always concludes that he has misunderstood the printed directions; packs up his books and lantern, and returns indoors, discouraged. The beginner in astronomy is always modest as to his abilities, and blames himself if the universe fails to fit the printed directions. Nor does any real astronomer ever lose this modest characteristic of the beginner; for he who has studied this science most deeply is ever most of all convinced that he is still a beginner.

Of course the absence of the triangle from the star map was simply due to the extremely brilliant object at the apex being a planet, and therefore properly absent from the map. The triangle on the sky appears on the map as a simple straight line with but two stars upon it.

Therefore the beginner should first of all learn to know the planets, so that he can eliminate them in comparing his star map with the sky. And it is fortunately easy to become familiar with the planets, perhaps even easier than to learn the stars. We have merely to take advantage of the planets' superior brilliancy in order to identify them. The best way is to make observations in the dusk, after sunset, before the stars begin to become visible. If there is any bright planet above the horizon at that time, it will be the first to show itself in the twilight; it will be the evening star.

But this priority of appearance in the evening is not necessarily a sure test for distinguishing the planets; for if no planet is above the horizon at the moment of sunset, the first object seen in the dusk of the twilight sky will, of course, be the brightest of the fixed stars then above the horizon. Therefore it is important to have another criterion for identifying the planets. It is a fact that the planets always appear projected on the sky rather near the ecliptic circle (p. 27). Therefore, if we could locate the position of the ecliptic circle on the celestial sphere, we should have additional evidence as to whether the evening star appearing first is really a planet. If a planet, it must be near the ecliptic circle.

The following method will enable the beginner to locate the ecliptic circle approximately on the sky. One point of the circle is, of course, determined by the position of the sun, which, as we know (p. 27), is always seen projected on that circle. Consequently, as we are making these observations in the evening twilight, it follows that one point of the ecliptic is near that point of the horizon where the sun has just set.

If we can now locate on the sky one other point of the

ecliptic, we can determine roughly the location of the entire ecliptic circle; for two points are sufficient to locate any great circle on the sky. This can be done best by making use of the celestial meridian, which we recall as a great circle drawn on the sky from the zenith directly overhead down to the south point of the horizon (p. 36). The point of the meridian crossed by the ecliptic can be ascertained from the following little table, which gives the roughly approximate altitude, or angular elevation above the horizon, of that point on the meridian which is crossed by the ecliptic.

To use this table, it is merely necessary to face the south point of the horizon, and imagine the meridian drawn vertically upward on the sky from that point to the zenith overhead. Next we must imagine the entire distance on the meridian from horizon to zenith divided into ninety equal degrees or spaces. Then the table gives us for various terrestrial latitudes, and for various dates, the number of degrees between the horizon and the point of the meridian at which the ecliptic crosses it. To facilitate the practical use of the table we have placed in it, next to each number of degrees, a simple fraction which will perhaps be more convenient in making actual observations. Thus, where the table gives 46°, we find also the fraction $\frac{1}{2}$, meaning that the ecliptic crosses the meridian approximately halfway up from the south point of the horizon to the zenith. The fraction $\frac{1}{2}$ belongs with 46°, because 46 is approximately half of 90°, the total angular distance from horizon to zenith.

For example, if we should observe at New York (approximate latitude 40°) at sunset on January 1, we would imagine a great circle drawn around the sky from the sunset point of the horizon to a point on the meridian halfway between

Table for Finding the Ecliptic at Sunset

Angular Altitude of its Intersection with the Meridian above the South Point of the Horizon

	Latitude 30°	Latitude 40°	Latitude 50°
January 1 . .	$58^\circ\ \frac{2}{3}\ \substack{u \\ n}$	$46^\circ\ \frac{1}{2}\ \substack{u \\ n}$	$32^\circ\ \frac{1}{3}\ \substack{u \\ n}$
February 1 . .	$73\ \frac{4}{5}\ \substack{u \\ n}$	$62\ \frac{2}{3}\ \substack{u \\ n}$	$49\ \frac{5}{9}\ \substack{u \\ n}$
March 1 . . .	$82\ \frac{8}{9}\ \substack{u \\ n}$	$71\ \frac{4}{5}\ \substack{u \\ n}$	$61\ \frac{2}{3}\ \substack{u \\ n}$
April 1 . . .	$83\ \frac{8}{9}\ \substack{d \\ n}$	$73\ \frac{4}{5}\ \substack{d \\ n}$	$63\ \frac{2}{3}\ \substack{d \\ n}$
May 1 . . .	$78\ \frac{6}{7}\ \substack{d \\ n}$	$66\ \frac{3}{4}\ \substack{d \\ n}$	$54\ \frac{3}{5}\ \substack{d \\ n}$
June 1 . . .	$75\ \frac{6}{7}\ \substack{d \\ -}$	$52\ \frac{3}{5}\ \substack{d \\ -}$	$39\ \frac{2}{5}\ \substack{d \\ -}$
July 1 . . .	$52\ \frac{3}{5}\ \substack{d \\ s}$	$40\ \frac{4}{9}\ \substack{d \\ s}$	$26\ \frac{2}{7}\ \substack{d \\ s}$
August 1 . .	$42\ \frac{4}{9}\ \substack{d \\ s}$	$31\ \frac{1}{3}\ \substack{d \\ s}$	$20\ \frac{1}{5}\ \substack{d \\ s}$
September 1 .	$38\ \frac{2}{5}\ \substack{d \\ s}$	$28\ \frac{1}{3}\ \substack{d \\ s}$	$18\ \frac{1}{5}\ \substack{d \\ s}$
October 1 . .	$37\ \frac{2}{5}\ \substack{u \\ s}$	$27\ \frac{2}{7}\ \substack{u \\ s}$	$17\ \frac{1}{5}\ \substack{u \\ s}$
November 1 .	$39\ \frac{3}{7}\ \substack{u \\ s}$	$28\ \frac{1}{3}\ u$	$18\ \frac{1}{5}\ \substack{u \\ s}$
December 1 . .	$45\ \frac{1}{2}\ \substack{u \\ s}$	$34\ \frac{2}{5}\ \substack{u \\ s}$	$22\ \frac{1}{4}\ \substack{u \\ s}$

the zenith and the south point of the horizon. This imagined line would be part of the ecliptic; extending it beyond the meridian, and around the sky to the eastern horizon, would give us the remaining visible portion of the ecliptic. And any object suspected of being a bright planet would necessarily be found very near this ecliptic circle. If the moon should chance to be visible at sunset, it would give us an additional point near the ecliptic; for the moon likewise always appears in the immediate vicinity of that circle.

Actual observations of this kind will of course be extended in the twilight for about an hour after sunset. As the position of the ecliptic changes somewhat during that hour, we have added two letters to each number in the table. One of the letters is either a *u* or a *d*, and shows whether the ecliptic point on the meridian is moving *up* or *down* at the moment of sunset. The other letter is either an *n* or

an *s*, and indicates whether the ecliptic point on the horizon is moving *north* or *south* at the moment of sunset. Thus, an hour or more after sunset on January 1 at New York (latitude 40°), we should draw the ecliptic a little to the north of the observed sunset point in the horizon, and a little above the 46° point on the meridian.

If we find a brilliant object in the dusk in this way on the ecliptic, we may still further test its planetary character by the absence of twinkling, for planets do not twinkle as much as stars. If the suspected object shines quietly, serenely, almost without scintillation, we may be tolerably sure it is a planet.

Still another important aid is at the service of the beginner in his planetary search, — the ordinary almanac. This will tell him what "evening stars" or planets are visible on the date when he makes his observations; and it is certainly a great help to know in advance whether any planets are to be in sight. The almanac will also inform him as to the names of the planets he may expect to see.

But even without an almanac it is generally easy to distinguish between the different planets. Mercury, when visible, always appears very near the horizon, close to the point where the sun has set. The best date to look for it may be found by adding successive periods of one hundred and sixteen days to the initial date, Nov. 2, 1913. The planet can usually be seen for a few days before and after the dates obtained in this way, if the horizon is unusually free from cloud or mist. Conditions are most favorable when the computed dates occur in the early part of the year, from January to May. And in these months especially it is important to begin looking for Mercury at least a week before the predicted dates.

Venus should be sought after sunset on the ecliptic; its angular distance from the sun is never more than 47° (about one-quarter of a great semicircle of the sky); and it may be much less. In looking for it, about an hour after sunset, we must remember that in an hour the sun will have moved a considerable distance below the horizon; therefore, even if Venus is 47° distant from the sun, we must expect its distance from the sunset point of the horizon to be considerably less. An initial date when Venus attains its greatest distance from the sun is Feb. 12, 1913. Subsequent occurrences of the same phenomenon may be expected at intervals of 584 days thereafter (1.60 years). These dates are, of course, highly favorable for observing the planet. Both Mercury and Venus are extremely bright.

Mars, Jupiter, and Saturn also always appear near the ecliptic, but they may attain very great angular distances from the sun. They are, in fact, directly opposite the sun in the sky at certain dates, which are the most favorable dates for finding these planets. The dates are:

Mars, Jan. 5, 1914, and thereafter at intervals of 780 days
Jupiter, July 5, 1913, and thereafter at intervals of 399 days
Saturn, Dec. 7, 1913, and thereafter at intervals of 378 days

When thus opposite the sun, the planets are easily found. It is merely necessary to imagine a straight line drawn from the sun to the observer, and thence continued outward to the celestial sphere at a point opposite the sun. And if we imagine the line drawn an hour after sunset, we must not draw it from the sunset point of the horizon, but from the sun itself, making an approximate allowance for the sun's having moved some distance below the horizon during the interval of an hour since sunset. On these critical dates

Mars, Jupiter, and Saturn are on the meridian, due south, at midnight.

For some time after the critical dates, these three planets, always remaining near the ecliptic, diminish their angular distances from the sun at the approximate monthly average rate of 25° for Mars, 30° for Jupiter, and 32° for Saturn. In estimating such angular distances it is well to remember that the angular diameter of the full moon is about one-half a degree. Furthermore, all the above numbers vary somewhat in different years. The interval of 780 days between successive critical dates for Mars is especially variable: it is usually only about 750 days when the predicted date occurs in the early months of the year.

A final test as to the planets may be obtained if the observer has a small telescope or good field glass at his disposal. In such an instrument the planets show their round disks quite plainly, while the fixed stars appear in the field of view as mere points of light without any visible extension into disks. In a three-inch telescope Jupiter shows moons, usually four, and Saturn usually exhibits the ring. Most observers detect in Mars a sort of reddish or ruddy color.

Coming now to the identification of the fixed stars, we shall employ a method resembling somewhat our procedure in the case of the planets. It is not our purpose to include in the present volume detailed charts showing all stars visible to the unaided eye, but rather to confine our attention to the stars of especial brilliance, and the more conspicuous constellations with which every one should have an acquaintance.

The first things to find in the sky are the pole star and the constellation *Ursa Major* (Great Bear or "Dipper"). These objects are near the north celestial pole, and very

far from the ecliptic; consequently, the planets never appear among them to confuse the visible configurations of stars. The pole star, close to the north celestial pole, is always elevated above our horizon by an angular altitude very nearly equal to the observer's latitude (p. 40). To find it, we must therefore face the north, and imagine the celestial meridian drawn on the sky vertically upward from the north point of the horizon to the zenith. The pole star will then be found almost exactly on the meridian, and elevated above the horizon by an angle equal to the observer's terrestrial latitude. In New York, for instance, it will be elevated 41°, or about $\frac{4}{9}$ of the total angular distance from horizon to zenith. The pole star is not very brilliant; being of the second magnitude, it will be inferior to several of the brightest stars visible in various parts of the sky.

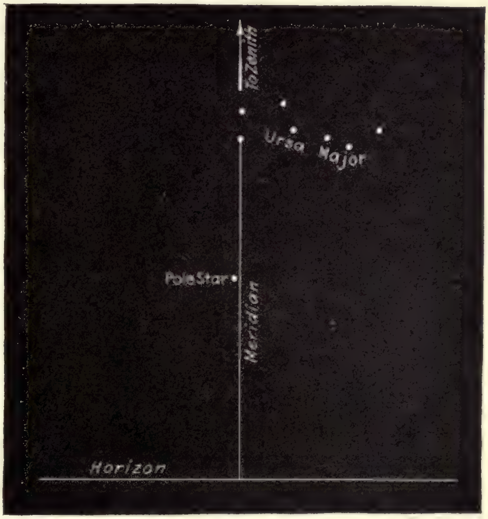

FIG. 13. The Pole Star and *Ursa Major* as seen at 9 P.M. on April 21.

To verify this identification of the pole star we make use of *Ursa Major*. This constellation contains seven stars, not of the first magnitude, arranged as shown in Fig. 13. This figure exhibits the constellation as it appears in the sky at 9 P.M. about April 21 in each year. The reader will notice that the two end stars of the seven are in the meridian directly above the pole star, and that they point almost

exactly toward the pole star. For this reason these two stars are called "The Pointers." If these seven stars appear on the sky occupying the position shown in Fig. 13 with respect to the pole star at 9 P.M. about April 21, there is no doubt that the pole star has been identified correctly. In using Fig. 13, the reader should bear in mind that the constellation *Ursa Major* will appear much larger on the sky than it does in the figure. The scale of the figure has been so chosen that the distance of *Ursa Major* from the pole star is proportioned correctly to the elevation of the pole star above the horizon; and this choice of scale makes the constellation appear rather small. The other constellation figures, 14, 15, 17, 18, 19, 20, 21, 22, are all drawn to the same scale, to avoid confusion; and the reader must expect all these constellations to be larger on the sky than they appear in the figures.

In consequence of the seeming rotation of the celestial sphere about the pole (p. 32), the pointers will further occupy the positions shown in Fig. 14, at 9 P.M. on the several dates indicated in the figure.

On intermediate dates the pointers will of course occupy intermediate positions; and with the help of these figures the reader should have no difficulty in finding the pole star and making certain of its identification by means of the pointers.

There is one other interesting constellation near the celestial pole: Cassiopeia, the "Lady in the Chair." It is found easily, also, by the aid of the pointers. Imagine a straight line drawn from the pointers to the pole star, and continued beyond the pole star an angular distance equal to the distance between the pointers and the pole star. The end of the line will then be in Cassiopeia, and the appear-

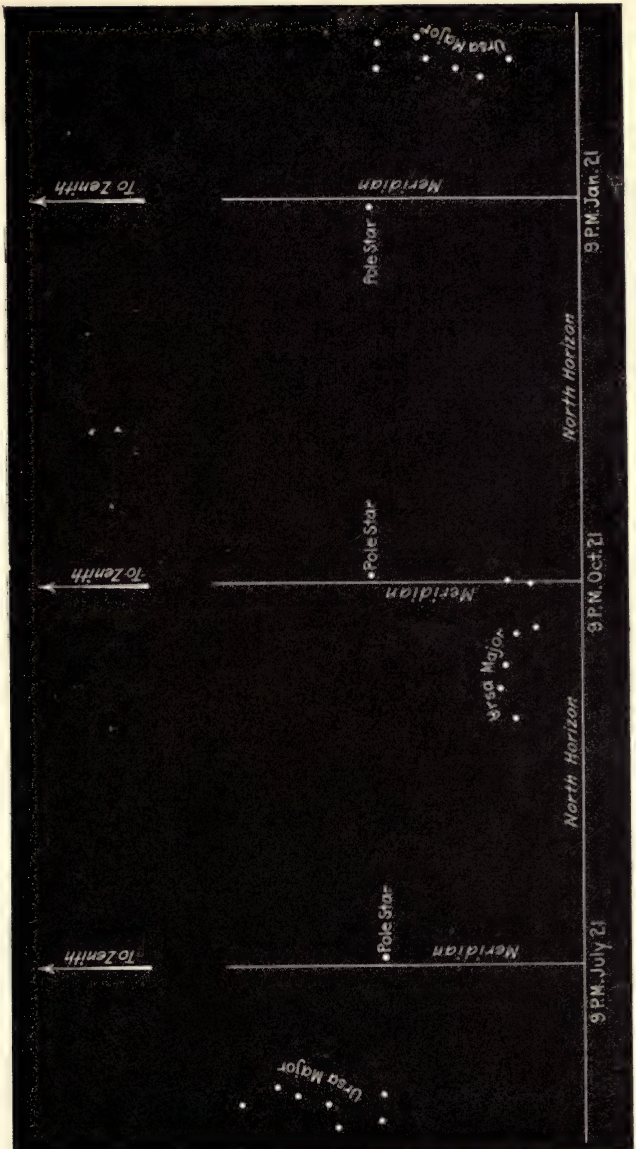

FIG. 14. The Pole Star and *Ursa Major* as seen at 9 P.M. on the Dates Indicated.

ance of that constellation is shown in Fig. 15. It looks like the letter *W*. The arrow shown in the figure indicates the direction of the pole star from Cassiopeia, and is approximately a continuation of the line by means of which Cassiopeia was found. In comparing Fig. 15 with the sky, it is therefore necessary to turn the book around until the arrow is nearly parallel to the direction of the pointers from the pole star. This would make the arrow vertical upwards, as shown in Fig. 15, at 9 P.M. on May 18, and vertical downwards at 9 P.M. on November 18. It would be horizontal to the right on February 18, at 9 P.M.; and horizontal to the left on August 18, 9 P.M. On intermediate dates the arrow would of course occupy positions intermediate between these vertical and horizontal ones; always, of course, at the hour of 9 P.M.

FIG. 15. Cassiopeia 9 P.M., May 18.

Having thus indicated a method of finding the two important polar constellations, we shall next show how to identify the brightest fixed stars of the first magnitude visible in the United States and Europe. They are fifteen in number; in the following list we have arranged them in the order of luminosity, the brightest of all being placed first.

To find these stars, we shall use a method similar to that employed for locating the ecliptic circle on the sky. Let the observer face the south at 9 P.M., and imagine the meridian drawn on the sky vertically upward from the south point of the horizon to the zenith, directly overhead. Let him once more imagine the meridian divided into ninety degrees or spaces, beginning at the south point of the horizon, and ending at the zenith. The following table will then tell him the dates on which the various stars in question

First-magnitude Stars

Name	Constellation		Color
Sirius	Canis Major	(Big Dog)	Blue-white
Vega	Lyra	(Harp)	Blue-white
Arcturus	Bootes	(Bear-keeper)	Orange
Capella	Auriga	(Charioteer)	Yellow
Rigel	Orion	(Hunter)	White
Procyon	Canis Minor	(Little Dog)	White
Betelgeuse	Orion	(Hunter)	Red
Altair	Aquila	(Eagle)	Yellow
Aldebaran	Taurus	(Bull)	Red
Antares	Scorpius	(Scorpion)	Red
Pollux	Gemini	(Twins)	Orange
Spica	Virgo	(Virgin)	White
Fomalhaut	Piscis Australis	(Southern Fish)	Orange
Regulus	Leo	(Lion)	White
Deneb	Cygnus	(Swan)	White

appear on the meridian, and their altitude or angular elevation above the south point of the horizon when they are thus situated on the meridian, always at the hour of 9 P.M. The date of reaching the meridian at 9 P.M. is the same for all terrestrial latitudes; but the altitudes vary in different latitudes, and are therefore given in the table for latitudes 30°, 40°, and 50°. If the observer's latitude is intermediate between 30° and 40°, or between 40° and 50°, he can of course use altitudes intermediate between those given in the table. Sometimes the tabular altitudes are a little greater than 90°. This indicates that the stars in question cross the meridian north of the zenith. To see them, an observer facing south would need to bend his head back so as to see a little beyond his zenith. A better way is to turn around and face the north, when the stars in question will be seen very near the zenith.

The identification of the bright stars will, of course, include an identification of the important constellations in which they are situated, as indicated in the preceding table.

TABLE TO BE USED IN FINDING FIRST-MAGNITUDE STARS ON THE MERIDIAN AT 9 P.M.

STAR	DATE ON MERIDIAN, 9 P.M.	ALTITUDE ABOVE SOUTH POINT OF HORIZON		
		Lat. 30°	Lat. 40°	Lat. 50°
Sirius . . .	Feb. 15	43°	33°	23°
Vega . . .	Aug. 15	99	89	79
Arcturus . .	June 10	80	70	60
Capella . .	Jan. 23	106	96	86
Rigel . . .	Jan. 23	52	42	32
Procyon . .	Mar. 1	65	55	45
Betelgeuse .	Feb. 2	67	57	47
Altair . . .	Sept. 3	69	59	49
Aldebaran . .	Jan. 13	76	66	56
Antares . .	July 13	34	24	14
Pollux . . .	Mar. 2	88	78	68
Spica . . .	May 28	49	39	29
Fomalhaut .	Oct. 20	30	20	10
Regulus . .	Apr. 8	72	62	52
Deneb . . .	Sept. 16	105	95	85

The above table is correct at 8 P.M. instead of 9 P.M. on dates two weeks later than those given in the table. It is correct at 10 P.M. on dates two weeks earlier than the tabular dates.

To facilitate finding the bright stars on dates other than those on which they reach the meridian at 9 P.M., we now give another table containing the dates when these stars rise and set at 9 P.M. as seen from the three terrestrial latitudes 30°, 40°, and 50°. In addition to the dates of rising and setting, the table contains the direction (as N.E., S.W.,

etc.), to which the observer must turn in order to see his star rise or set. In making these observations it is important to remember that the immediate vicinity of the horizon is usually obstructed by trees, houses, etc., and that even when these obstructions are absent, the horizon itself is seldom entirely free from clouds or mist. Therefore the observer should not expect a rising star to be visible for some time (possibly as much as an hour) after 9 P.M. on the tabular date of rising; and he may expect it to disappear from view some time before 9 P.M. on the tabular date of setting.

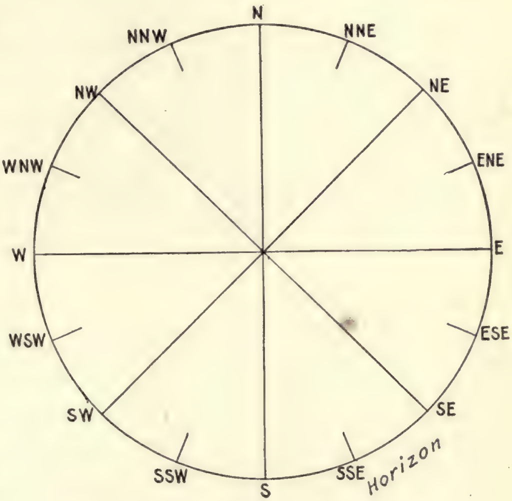

FIG. 16. The "Points of the Compass."

The directions N.W., S.E., etc., to which the observer must turn, are roughly approximate only; but accurate enough to facilitate finding the stars. The accompanying Fig. 16 shows the order in which these directions follow each other around the horizon.

The table on the next page is correct at 8 P.M. instead of 9 P.M. on dates two weeks later than those given in the table. It is correct at 10 P.M. on dates two weeks earlier than the tabular dates.

To aid still further in the identification of the finest con-

TABLE TO BE USED IN FINDING FIRST-MAGNITUDE STARS WHEN THEY RISE AND SET AT 9 P.M.

Name of Star	Latitude 30°				Latitude 40°				Latitude 50°			
	Rising 9 p.m.		Setting 9 p.m.		Rising 9 p.m.		Setting 9 p.m.		Rising 9 p.m.		Setting 9 p.m.	
	Date	Direction	Date	Direction	Date	Direction	Date	Direction	Date	Direction	Date	Direction
Sirius . . .	Nov. 26	ESE	May 7	WSW	Dec. 1	ESE	May 3	WSW	Dec. 7	ESE	Apr. 26	WSW
Vega . . .	Apr. 18	NE	Dec. 12	NW	Apr. 3	NE	Dec. 27	NW	Mar. 3	NNE	Jan. 27	NNW
Arcturus . .	Feb. 26	ENE	Sept. 21	WNW	Feb. 20	ENE	Sept. 27	WNW	Feb. 13	NE	Oct. 4	WNW
Capella . .	Sept. 17	NE	June 1	NW	Aug. 24	NNE	June 24	NNW	July 25	N[1]	July 25	N[1]
Rigel . . .	Oct. 29	E	Apr. 20	W	Oct. 31	E	Apr. 17	WSW	Nov. 3	E	Apr. 14	WSW
Procyon . .	Nov. 27	E	June 3	W	Nov. 25	E	June 5	W	Nov. 23	ENE	June 7	W
Betelgeuse .	Oct. 30	E	May 9	W	Oct. 28	E	May 11	W	Oct. 25	ENE	May 13	W
Altair . . .	May 29	E	Dec. 8	W	May 27	ENE	Dec. 10	W	May 24	ENE	Dec. 13	W
Aldebaran .	Oct. 4	ENE	Apr. 24	WNW	Sept. 30	ENE	Apr. 29	WNW	Sept. 24	ENE	May 5	WNW
Antares . .	Apr. 30	ESE	Sept. 26	WSW	May 8	SE	Sept. 18	SW	May 19	SE	Sept. 6	SW
Pollux . . .	Nov. 13	NE	June 20	WNW	Nov. 2	NE	June 23	NW	Oct. 22	NE	July 12	NW
Spica . . .	Mar. 4	ESE	Aug. 21	WSW	Mar. 7	ESE	Aug. 18	WSW	Mar. 10	ESE	Aug. 14	WSW
Fomalhaut .	Aug. 10	SE	Dec. 30	SW	Aug. 19	SE	Dec. 20	SW	Sept. 3	SE	Dec. 6	SW
Regulus . .	Dec. 30	ENE	July 16	WNW	Dec. 27	ENE	July 19	WNW	Dec. 22	ENE	July 23	WNW
Deneb . . .	May 12	NE	Jan. 20	NW	Apr. 19	NNE	Feb. 12	NNW	Mar. 17	N[1]	Mar. 17	N[1]

[1] Does not rise or set in latitude 50°. On the tabular date this star will be due north, near the horizon, below the pole star. (See p. 43.)

FIG. 17. Auriga, with Capella.

FIG. 18. Cygnus, with Deneb.

FIG. 19. Gemini, with Pollux.

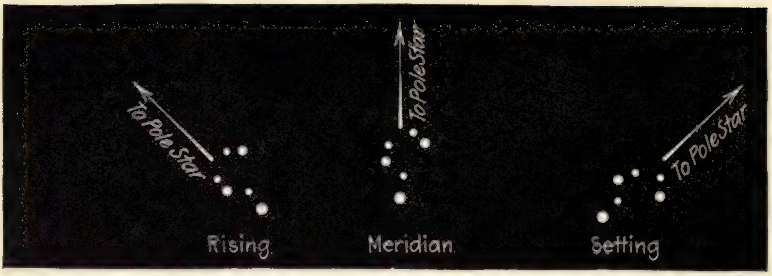

FIG. 20. Leo, with Regulus.

FIG. 21. Scorpius, with Antares.

FIG. 22. Orion, with Rigel and Betelgeuse.

stellations, we have prepared the preceding diagrams exhibiting their appearance when rising, when setting, and on the meridian. In each case the diagram contains an arrow showing the direction of the pole star; and the dates when the several constellations may be seen at 9 P.M. can be taken from the preceding tables.

Those of our readers who may desire to extend their knowledge to the less conspicuous constellations may now do so easily. It is merely necessary to proceed from the constellations already known to those not yet identified, by the aid of a star atlas. In doing this it will be best to look for the known constellations and first-magnitude stars on the maps, and proceed from them first to the neighboring unknown constellations. There is little difficulty in doing this; the knowledge of a few stars with which to begin is the only troublesome part of the problem. It is hoped that the tables and diagrams of the present chapter will suffice to remove this initial difficulty.

It is also possible to identify the stars by means of a globe such as that illustrated in Fig. 7 (p. 37), but it is not easy to learn the method of using a globe without the aid of oral teaching. A few minutes' explanation from some person who understands the use of the instrument is better than many printed pages in a book. There is also another contrivance, called a planisphere, which is simple in use, and much less costly than a celestial globe. This instrument represents the globe projected on a plane or flat surface; and by means of a rotating disk of cardboard, it shows at a glance what stars are visible above the horizon at any hour of the night and on any date in the year. Planispheres are always accompanied with printed instructions suitable for use by a beginner in astronomy.

In a study of the present chapter the reader will have noticed that we have given practical directions for finding the stars, without elaborate explanation of the principles upon which these directions are based. This will enable him to commence his study of the sky without waiting until he has mastered the later chapters of the book; it is hoped to increase his interest by thus allowing him to undertake practical work at the earliest possible moment.

CHAPTER IV

TIME

We have seen (p. 19) that it is one of the principal duties of the astronomer so to regulate clocks that they may indicate accurate time: let us now endeavor to explain the meaning of the word "time" in astronomy. We shall make use of our definition (p. 36) of the celestial meridian as a great circle of the celestial sphere passing through the celestial pole, the zenith, and the north and south points of the horizon. For in astronomy, this meridian plays a most important part in the explanation of no less than four different kinds of time. These are called: —

1. Sidereal time.
2. Apparent solar time.
3. Mean solar time.
4. Standard time.

A unit of some sort is necessary for measuring the duration of these various varieties of time: and for this purpose astronomers use the Day; though not the same "day" for the four different kinds of time. There is a sidereal day, for measuring sidereal time; an apparent solar day, for apparent solar time; and a mean solar day, used for both mean solar and standard time.

Let us consider first the simplest kind of time, sidereal or "star-time." We have had (p. 35) a definition of the vernal equinox as one of the points on the celestial sphere at which the ecliptic circle crosses the celestial equator; and we have already made some use of this important point. We shall now find that it is fundamental also in the measurement of sidereal time.

As the celestial sphere performs its diurnal seeming rotation, due to the real axial turning of the earth within it, the vernal equinox, like the stars, rotates with the sphere.[1] Consequently, once during each complete diurnal rotation of the sphere, the vernal equinox will cross the celestial meridian. At the precise instant when the vernal equinox thus crosses the celestial meridian, the sidereal day begins. As the seeming turning of the sphere proceeds from east to west, the vernal equinox will begin to move westward from the meridian as soon as the sidereal day has commenced; and after a complete rotation, it will again reach the meridian from the east. The sidereal day will then end, and at the same instant a new sidereal day will begin. The sidereal day is defined, then, as the interval of time between two successive returns of the vernal equinox to the meridian.

The sidereal day is divided into twenty-four sidereal hours; and these hours are counted continuously from 0 to 24, without using the letters A.M. and P.M. When the vernal equinox is exactly on the meridian, and the sidereal day begins, the sidereal time is $0^h\ 0^m\ 0^s$; and this would be the time indicated on the dial of a standard sidereal clock, if the clock were exactly right. Then, after the vernal equinox has passed the meridian, and has completed one twenty-fourth part of an entire diurnal rotation, it is $1^h\ 0^m\ 0^s$ sidereal time; 2^h, 3^h, 4^h, etc., follow in succession; until, at 23^h sidereal time, the vernal equinox lacks but one hour of reaching the meridian once more.

When the vernal equinox is 1^h west of the meridian, we say that its "hour-angle" is 1^h; and similarly for 2^h, 3^h, etc., up to 24^h. Thus the hour-angle of the vernal equinox at any moment may be defined as the quantity of rotation

[1] Cf. Note 2, Appendix.

made by the celestial sphere since the vernal equinox was last on the meridian, this rotation being measured in hours, minutes, and seconds, and an entire rotation of the sphere corresponding to 24 hours. And in the light of this definition we may define the sidereal time at any instant as the hour-angle of the vernal equinox at that instant.[1] Recurring to our definition of right-ascension (p. 34), it may be here stated as an additional fact that the right-ascension of any star appearing on the celestial meridian at any instant is always exactly equal to the sidereal time at the same instant.[2]

This last important fact calls attention to a simple and interesting relation between sidereal or star-time, and the stars themselves. If, for instance, we have at hand a correct sidereal clock, and that clock indicates 3^h sidereal time exactly, then any star whose known right-ascension is 3^h may be found at that moment on the meridian. Furthermore, sidereal time enables us to know at once how much time has elapsed since any given star was on the meridian. Thus, at 4^h sidereal time, we know that our star, whose right-ascension is 3^h, passed the meridian one hour ago. At 5^h we know it was on the meridian two hours ago, etc.; and thus we know approximately where to look for it in the sky.[3]

We must next consider the explanation of solar time, and its relation to sidereal time. Let us begin with apparent solar time, which is the kind of time kept by the actual sun, as we see it in the sky. The definitions are quite similar to those we have already given for sidereal time. The unit for measuring the duration of apparent solar time, the apparent solar day, is defined as the interval between two successive returns of the visible sun to the celestial meridian. The

[1] Note 5, Appendix. [2] Note 6, Appendix. [3] Note 6, Appendix.

day begins when the sun is exactly on the meridian; when the axial turning of the sphere has carried it one twenty-fourth part of an entire diurnal rotation westward from the meridian, astronomers say it is 1^h apparent solar time, etc. Following the analogy of sidereal time, we may define the hour-angle of the visible sun as that quantity of the celestial sphere's rotation which would carry the sun from the meridian to its actual position on the sky. And we may then define the apparent solar time at any instant as the hour-angle of the visible sun at that instant. Astronomers do not use A.M. and P.M.: apparent solar time is counted continuously from 0^h to 24^h, like sidereal time.[1]

We have seen that successive returns of the sun to the meridian, giving the solar day, and successive returns of the vernal equinox, giving the sidereal day, are both caused by the same apparent axial rotation of the celestial sphere. We are therefore confronted by the question: why are these two kinds of day not exactly equal? To answer this question, we recall (p. 27) that the sun appears at all times somewhere on the ecliptic circle in the sky; but that (p. 29) it never appears at the same point of that circle on two successive days.

The motion of our earth, in its annual orbit around the sun, makes us see the sun projected at opposite points of the ecliptic circle at intervals of about half a year. Opposite points of the ecliptic circle are 180° apart; and half a year contains 183 days. Therefore, the sun changes its apparent position on the ecliptic circle about 180° in 183 days, or one degree daily. Now, to simplify matters, let us imagine that the sun appeared at the vernal equinox exactly at noon on a certain day. We already know that the sun appears

[1] Note 7, Appendix.

at the vernal equinox once each year; let us now imagine that it did so exactly at noon on one of the days in some particular year. On that occasion, the apparent solar day and the sidereal day must have commenced at exactly the same instant. For the one kind of day begins when the sun is on the meridian; the other, when the vernal equinox is on the meridian. On the occasion when they were both on the meridian together, both days must have commenced together.

But while the next apparent diurnal rotation of the sphere was in progress, the sun did not remain at the vernal equinox. Its daily change of about one degree, as seen projected on the ecliptic circle, must have made it appear approximately one degree east of the vernal equinox on the ecliptic, by the time a single diurnal rotation had been completed. Therefore, at the instant when the vernal equinox again reached the meridian, thus completing the sidereal day, the sun must still have been a short distance east of the meridian. The diurnal rotation must have continued a little longer to bring the sun to the meridian, so as to complete the apparent solar day as well.

From these considerations it follows that the solar day is a little longer than the sidereal day. The difference is about four minutes: under the conditions imagined above, the sun would have reached the meridian at the end of the day about four minutes behind the vernal equinox. At the end of a second day it would have been about eight minutes behind the equinox, and so continuing on succeeding days.

Thus there is a constantly increasing difference between solar and sidereal time, sidereal time gaining about four minutes daily on solar time. If a solar clock and a sidereal clock are placed side by side, it is easy to follow this con-

tinually increasing gain of sidereal time by simply making a daily comparison between the two clocks.

It is evident that this difference of the two clocks will amount to 24 hours in a year, since $4^m \times 365$ is approximately 1440 minutes, or 24 hours. And the actual lag of the sun is a little less than 4^m, just enough to make the yearly gain exactly 24 hours. It is, in fact, evident that as the sun's apparent motion in the ecliptic circle is due to the earth's annual orbital motion around the sun, and as this orbital motion is completed in a year, it must happen at intervals of one year that the sun must return again to the vernal equinox, and everything repeat itself once more. The sidereal clock will gain just one day in the year; and if it agreed with the solar clock at the beginning of the year, the two clocks must again be together at the end of the year. Accordingly, the number of sidereal days in the year is one greater than the number of solar days. And the whole difference between sidereal and solar time is due to the fact that the sidereal day depends on the earth's axial rotation alone, while the solar day depends on both the axial rotation of the earth and the daily fraction of its annual orbital motion around the sun.

This lagging of the sun behind the vernal equinox amounts to 4^m approximately each day, as we have seen, but this approximate quantity of 4^m is itself variable, within certain limits, throughout the year. The reasons for this variation will be explained in detail in a later chapter; but one reason is quite obvious. The earth does not move uniformly in its annual orbit around the sun. And since the sun's apparent motion, as projected on the ecliptic circle, is simply a result of the earth's orbital motion, it follows that the sun's daily change of position in the ecliptic circle is not uniform either.

Consequently, the lag of the sun behind the vernal equinox will not be the same each day, and as the sidereal days are all equal, because the earth rotates uniformly on its axis, the solar days are unequal.

There are various inconveniences resulting from this inequality of solar days: prominent among them is the difficulty of making solar clocks that will run with other than uniform motion. A clock keeping pace accurately with the inequalities of the solar day would be almost a mechanical impossibility.

Therefore astronomers have adopted an imaginary conventional mean solar time, and a conventional unit for it, the mean solar day. These are so arranged that they correspond accurately to the average performances of the actual visible sun and the apparent solar day, and differ as little as possible from them. The mean solar days are all of equal length. We can, if we choose, even think of an imaginary mean sun in the sky, whose hour-angle from the meridian at any instant will be the mean solar time at that instant. Such a mean sun would occasionally have a greater hour-angle than the actual visible sun, and then the mean solar time would be later than the apparent solar time. The mean solar clock would be fast of an apparent solar clock, if there were such a thing. And when the mean sun's hour-angle was less than that of the visible sun, the mean solar clock would be slow. We shall return later to the difference between these two kinds of solar time more in detail: the above explanation is sufficient for our present purpose.

These differences between mean solar time and apparent solar time are never greater than about one-quarter of an hour. But the difference between either kind of solar time and sidereal time of course ranges all the way from zero up to

24 hours. It is zero, as we have seen, when sun and vernal equinox are together. Then solar time lags behind sidereal time continuously about four minutes daily, until in a year the accumulation totals one day, and the two kinds of time are together again. We call the date in each year when the two kinds of time agree, March 21, or thereabouts. This is therefore the date where the sun appears in the vernal equinox.

These facts explain clearly the varying aspect of the stellar heavens night after night. The fixed stars, as seen projected on the sky, maintain positions practically unchanging with respect to the vernal equinox. Any fixed star will therefore rise, pass the meridian, and set a certain definite number of hours and minutes after the vernal equinox. In other words, it will do these things every night at the same sidereal time. Consequently, as the sidereal time gains about four minutes daily on solar time, each star will rise, pass the meridian, and set about four minutes earlier each night by solar time.

For instance, referring to our table (p. 60), we find that at New York (approximate latitude 40°) Arcturus rises at 9 P.M. on February 20. On February 21 it will therefore rise at 8.56; on February 22, at 8.52; etc. Two weeks after February 20, Arcturus will rise 56 minutes earlier, or approximately one hour. This explains the statement (p. 59) that all the stars in the table will rise at 8 P.M. instead of 9 P.M. two weeks after the dates given in the table.

Having now explained the meaning of time, it becomes possible to set forth very simply the astronomic signification of the time differences existing between different places on the earth. Why does Chicago time differ from New York time or London time? Recurring to our definition of the celestial

meridian (p. 36), we remember that it passes through the zenith, or point directly overhead. But the point overhead in London does not coincide with the point directly overhead in New York. Therefore London and New York will have different zeniths, and different celestial meridians.

Furthermore, we have just explained solar and sidereal time to be the hour-angles of the sun and the vernal equinox from the celestial meridian. It follows that if London and New York have different celestial meridians, all hour-angles must be different at any instant in the two cities. Consequently, neither sidereal nor solar time at London will be the same as New York sidereal or solar time at the same moment. How much will they differ?

To answer this question we must have recourse once more to geography. The reader will remember that the surface of the earth is supposed to be divided by a series of lines called terrestrial meridians of longitude, great circles drawn on the earth from the north to the south terrestrial pole. We have already mentioned (p. 34), for instance, that the terrestrial meridian of Greenwich, England, is the prime meridian for reckoning terrestrial longitudes. And the longitude of New York is simply the angle at the north pole of the earth between the terrestrial meridians of Greenwich and New York.

Now the celestial meridians of these two places correspond on the sky to their terrestrial meridians on the earth.[1] Therefore the angle between their celestial meridians at the north celestial pole will be the same as the angle between their terrestrial meridians at the north pole of the earth. In other words, it will be the same as their terrestrial difference of longitude. And since time at Greenwich

[1] Note 8, Appendix.

or New York is simply an hour-angle measured from the celestial meridian of Greenwich or New York, it follows that the difference in time will be equal to the longitude difference of these two places on the earth.

Many beginners grasp this matter of time differences more easily in another way. Because the sun rises in the east, and moves westward in the sky, and because New York is west of Greenwich, the sun must pass the celestial meridian over Greenwich before it reaches that over New York. Therefore, when it is noon in New York, noon has already occurred in Greenwich, and it is already afternoon in the latter place. Consequently, Greenwich time is later than New York time; and Greenwich clocks are fast of New York clocks. So of any two places, *east* clocks are always *fast* clocks: both words end in *ast*.

To complete this part of our subject it is still necessary to explain what is meant by standard time, the ordinary time in actual use in our everyday affairs. It has no direct connection with astronomy, but is a mere conventional arrangement designed to prevent the inconvenience due to the fact that astronomical mean solar time, as we have seen, is practically never the same in any two places on the earth. It is not possible to avoid large time differences, such as exist, for instance, between Greenwich and New York. But there is no reason for the public to be troubled with minor time differences of a few minutes only.

The plan actually adopted is as follows: Greenwich is taken as the initial point for reckoning all standard time. The earth is then divided by a series of standard meridians 15° or 1^h apart, and everywhere the time of the nearest standard meridian is adopted arbitrarily for use instead of the mean solar time formerly employed. Thus our ordinary

clocks not only fail to conform to the motions of the actual visible sun; they no longer even run in conformity with the imaginary mean sun. But the standard time for which they are regulated differs from mean solar time by a constant difference only in each locality. This constant difference is the time difference already explained, as it exists between the terrestrial meridian of the locality and the nearest standard time meridian.

The great advantage of this system arises from the standard meridians having, by definition, time differences that are exact multiples of 1^h. The standard times of any two places must therefore differ by an exact number of hours, without minutes or seconds; whereas the true mean solar time difference will practically always be an odd fraction of hours, minutes, etc. It follows, for instance, that a traveler going from New York to Chicago can set his watch on arrival by merely turning it back one hour. To make his watch accord with Chicago standard time, he does not need to consult any timepiece in Chicago. If his watch was correct in New York by New York standard time, it will be similarly correct in Chicago, if it be set one hour slow of New York time.

We shall close the present chapter with a brief explanation of the International Date Line. This is another conventional arrangement intended to prevent certain difficulties arising from the time differences that confront travelers who circumnavigate the entire earth. A person going eastward from Greenwich, for instance, will set his watch one hour faster for every 15° of longitude he traverses, in accordance with the explanations we have already considered.

But if he should travel entirely around the earth, and continue the same treatment of his watch, he would find,

upon his return to Greenwich, that the watch had been set fast a total of 24 hours during the trip. The traveler would apparently have gained a day; and if he kept a daily journal or diary, he would find the current date in his journal one day later than the date printed in the London morning papers issued on the day of his return to Greenwich. And in a similar way, if the traveler had proceeded westward from Greenwich, his diary would have been one day "slow" of the London papers on his return.

Of course there is no real gain or loss of a day. If the traveler went around the earth with uniform velocity, and made the circuit in 24 days, for instance, he would have changed his longitude 15° daily, since $15° \times 24 = 360°$. This would make his daily time difference just one hour. Therefore, while he would appear to gain a day in 24 days, yet each of these 24 days would be only 23 hours in length: his apparent gain of one day would be offset exactly by his loss of one hour on each of 24 consecutive days.

The above inconsistency is not convenient, even though it is apparent merely, not real. Therefore it has been agreed that navigators shall change their date arbitrarily by one day when circumnavigating the earth; and that they shall make this change when they reach a certain longitude, also arbitrarily chosen on the earth. The terrestrial meridian of longitude thus chosen is 180° distant from Greenwich. This meridian passes through the Pacific Ocean; it is most appropriate for the purpose because comparatively few ships navigate that part of the earth, and so the arbitrary change need be made but rarely.

But it has not been found possible to confine this change of date accurately to the 180° meridian of longitude. There

are certain groups of islands crossed by this meridian, and it would obviously be most confusing to have different dates in force in neighboring islands of the same group. Therefore an arbitrary irregular line has been drawn on the map of the Pacific Ocean, and called the international date line. Navigators are all instructed to change their date by one day when crossing this line; skipping a date if they are proceeding westward, and counting a date twice if they are moving eastward. And the arbitrary line is drawn in such a way as to avoid as far as possible confusing changes of date in neighboring islands or in the possessions of a single nation. It may be remarked also that some of the ordinary standard time meridians have been similarly bent a little at certain points, so as to avoid having two kinds of standard time in two parts of a single city, or in two cities very near each other.

CHAPTER V

THE SUNDIAL

By means of the definitions and explanations contained in Chapter IV, we can now solve a very interesting practical problem. The sundial is no longer an instrument of essential importance in everyday affairs, since time is now universally measured with mechanical clocks and watches; but it still remains a most instructive toy, and is as much as ever a desirable ornamental monument in gardens and other public places.

We shall confine our attention to one of the simplest forms of the instrument, — the dial drawn on a horizontal flat surface. Upon that surface is erected a vertical *gnomon;* and the shadow of this gnomon falling on the dial indicates the hour of the day by its position among the dial lines. Our problem is to design the correct shape of the gnomon and to draw the lines properly upon the dial itself.

In Fig. 23 we give a sketch of a complete horizontal sundial. The gnomon *abc* is made of a piece of flat brass plate firmly fastened to the base *ABCD*, upon which the dial itself is drawn. The edge *ab* casts the shadow by means of which the dial measures time.

It is necessary that the angle *bac* at the base of the gnomon be equal to the terrestrial latitude of the place in which the dial is to be used. And the gnomon may be designed easily so as to have the correct angle by the method shown in Fig. 24.

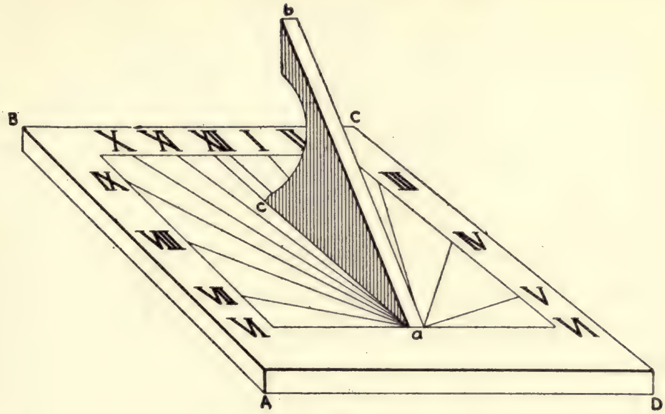

FIG. 23. Horizontal Sundial.

Draw the line *ac* of any desired length, according to the size of dial it is intended to construct. At the point *c* draw the line *cb* perpendicular to *ac*. The proper length of *cb* may be found by multiplying the length adopted for *ac* by the factor[1] given in the following little table for various terrestrial latitudes:

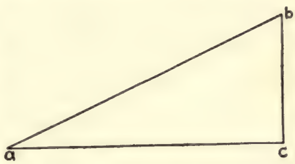

FIG. 24. Drawing the Gnomon.

TABLE FOR CONSTRUCTING THE GNOMON

LAT.	FACTOR
25°	0.466
30°	0.577
35°	0.700
40°	0.839
45°	1.000
50°	1.192
55°	1.428

Thus, in latitude 40°, if *ac* has been made 10 inches long, *cb* would be 8.39 inches.

[1] Note 9, Appendix.

This having been done, the gnomon will have the proper angle at its base. The construction of the dial itself is shown in Fig. 25. The double line *ac* corresponds to the line *ac* in Fig. 24, the two lines composing the double line *ac* in Fig. 25 being separated by the exact thickness of the brass plate used in making the gnomon. The gnomon must afterwards be fastened to the dial in such a way that *ac* of Fig. 24 will come exactly upon *ac* of Fig. 25.

The hour lines of Fig. 25 are drawn as follows: continue the double line *ac* to a point *M*, and make the distance *cM* of such a length that it will be equal to the length of *ac* multiplied by the factor[1] given in the following little table for various terrestrial latitudes:

Table for Constructing Dial Lines

Lat.	Factor
25°	0.423
30°	0.500
35°	0.574
40°	0.643
45°	0.707
50°	0.766
55°	0.819

Now draw the long line *PcQ* of indefinite length, perpendicular to *ac*; and draw the two lines *MN* parallel to *PQ*. Draw the two circular arcs *cc′* with centers at *M*, and divide each arc into six equal parts, giving the points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11. Draw lines as shown: *M* 1, *M* 2, *M* 3, *M* 10, *M* 11, etc., and continue them to the line *PQ*, giving the points I, II, III, IV, 5′, XI, X, IX, VIII, 7′. Then the lines *a* I, *a* II, *a* III, *a* IV, *a* V, *a* XI, *a* X, *a* IX, *a* VIII, *a* VII, as shown, will be the hour-lines of the dial for the several hours of the day. The six o'clock line is drawn from *a* to VI, parallel to *PQ*.

[1] Note 10, Appendix.

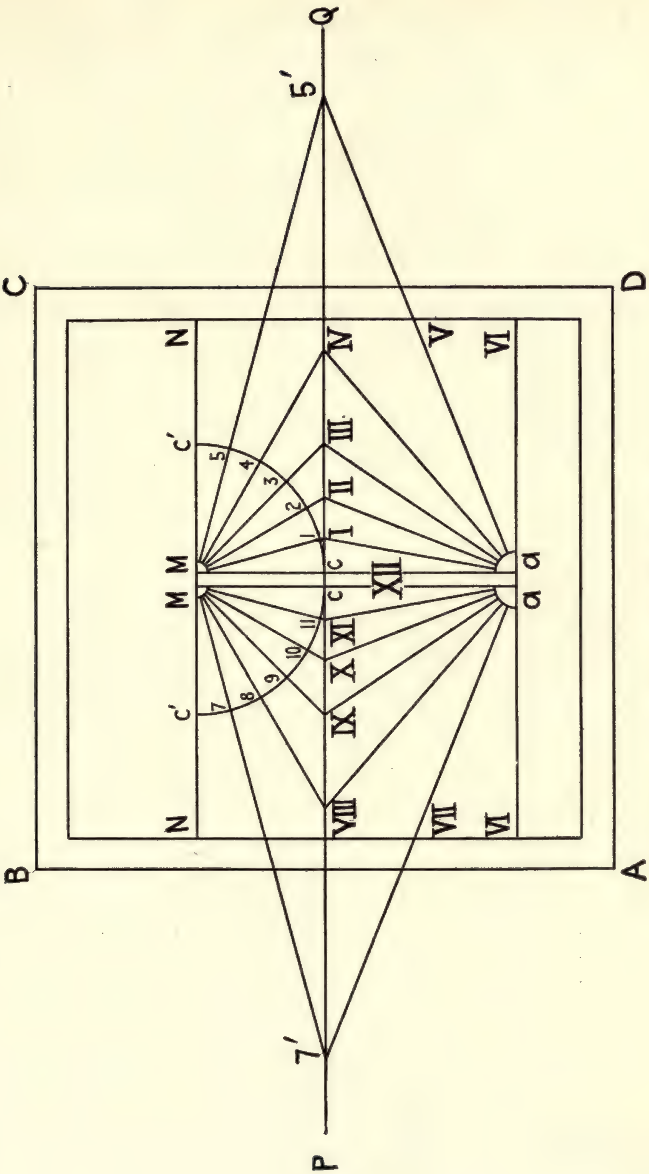

Fig. 25. Drawing the Sundial.

The hour-lines having been drawn in this way, and the gnomon fastened to the base as already indicated, the whole instrument is ready for use. When setting it up in the sunshine, however, it must be properly "oriented," or turned around to the correct position. This will be the case if the line *ac* is made to point in the exact north-and-south direction, the end *c* being toward the north. And the easiest way to orient the dial is to turn it until the shadow of the gnomon indicates the time in accord with a good watch previously set to correct time.

But it must not be expected that the sundial will keep pace accurately with the watch. For the dial shows the shadow cast by the actual visible sun. And as the actual visible sun gives us apparent solar time (p. 67), the sundial must also give apparent solar time.

The difference between this kind of time and mean solar time (p. 71) is shown in the following table for various dates in the year; and this difference should, of course, also be considered when orienting the dial by means of a watch.

TABLE OF DIFFERENCES BETWEEN SUNDIAL TIME AND MEAN SOLAR TIME

Jan.	1	Dial slow 4^{m}.	July	1	Dial slow 3^{m}.
Jan.	15	" slow 10	July	15	" slow 5
Feb.	1	" slow 14	Aug.	1	" slow 6
Feb.	15	" slow 15	Aug.	15	" slow 4
March	1	" slow 13	Sept.	1	" correct
March	15	" slow 9	Sept.	15	" fast 5
April	1	" slow 4	Oct.	1	" fast 10
April	15	" correct	Oct.	15	" fast 14
May	1	" fast 3	Nov.	1	" fast 16
May	15	" fast 4	Nov.	15	" fast 15
June	1	" fast 3	Dec.	1	" fast 11
June	15	" correct	Dec.	15	" fast 4

We have already stated (p. 71) that the detailed explanations of these varying differences between the two kinds of

solar time will be found in a later chapter; for our present purpose it is sufficient to use the foregoing tabulation without further comment.

But we must not expect sundial time to agree exactly with our watches, even after we have made allowance for the above table of time differences. For our watches indicate standard time (p. 74), whereas the foregoing table merely corrects sundial time to make it accord with mean solar time. To ascertain the additional correction required to transform the mean solar time into standard or watch time, we must know the longitude difference of the place where the dial is located from the nearest standard meridian.

For instance, New York, in longitude 74° west of Greenwich, is 1° east of the nearest standard meridian, which is in 75° west longitude from Greenwich. Therefore New York local mean solar time is later (or fast) of the nearest standard meridian (p. 74). The difference will be 4^m, since 1° must correspond to 4^m if 15° of longitude correspond to 1^h. It follows that the sundial, even after the correction from our table has been applied, will still always be 4^m fast of standard time as used in New York. This final difference of 4^m should again also be considered in orienting a dial by means of a watch.

The foregoing directions for making a sundial have been put in such form that any one can use them, even if entirely ignorant of astronomic principles. But the knowledge we have gained in Chapter IV should enable us to understand the sundial much more thoroughly. In the first place, we recall (p. 40) that the altitude, or angular elevation, of the north celestial pole above the horizon is exactly equal to the terrestrial latitude of the observer. Now we have made the

surface of our dial level, and constructed the base angle of the gnomon such that the time-measuring edge *ab* is likewise elevated by an angle equal to the latitude. And in orienting the dial we also turned it around until the gnomon pointed exactly north.

In other words, the dial and its gnomon are so arranged that the edge *ab* of the gnomon points exactly at the north pole of the celestial sphere. The gnomon's edge is therefore parallel to the axis of the celestial sphere; since, as usual, we may neglect the tiny radius of the earth in comparison with the infinite distance of the sphere. It follows that the diurnal rotation of the celestial sphere will seem to take place around the edge of the gnomon.

So the sun each day will also seem to perform its diurnal rotation around the edge of the gnomon. Now we have seen (p. 68) that the apparent solar time at any instant, or the hour-angle of the visible sun at that instant, is simply the quantity of rotation made by the celestial sphere since the sun was on the meridian. The sundial merely measures this quantity of rotation; and thus becomes a measurer of apparent solar time. When the visible sun is on the meridian, the shadow of the gnomon falls, as it should, on the north-and-south line of the dial, marked XII. When the quantity of diurnal rotation is 15°, or one hour, the shadow falls on the line marked I; etc.[1]

The accompanying Plate 4 is a photograph[2] of the largest sundial ever built. It was erected about 1730 by Jai Singh II, Maharaja of Jaipur, and restored in 1902 by order of the Maharaja Sawai Madho Singh. The huge gnomon,

[1] Note 10, Appendix.

[2] From *The Jaipur Observatory and its Builder;* by Lieutenant A. ff. Garrett, R. E., and Pundit Chandradhar Guleri. Allahabad, 1902.

PLATE 4. The Samrat Yantra, "Prince of Dials," at Jaipur.

containing stone stairs, is 90 feet high, and its base is 147 feet long. The shadow falls on a great stone quadrant instead of a level surface; and the radius of the quadrant is 50 feet. The shadow moves on the quadrant at the rate of two and one-half inches per minute.

CHAPTER VI

MOTHER EARTH

THERE was once an old professor of astronomy who used to begin a lecture on "the earth" by telling his students that the old Greek astronomers always assigned to the earth the gender feminine, probably because she was constantly leading them astray in their scientific investigations. And it must be conceded that any one beginning to study the earth in its astronomic relations with the rest of the universe would find it almost impossible to avoid being misled by his early observations. In fact, the very first thing we must learn about the earth is to unlearn almost everything we ascertain by the actual use of our eyes.

For instance, if an ignorant person — a person ignorant of astronomy — were asked to examine the earth and to describe it, he would say it is a flat plain, roughened with hills and valleys, but still in the main a great plain. But an astronomer would be compelled to ask him to unlearn this at once, because the earth is really a big round ball or globe.

And further direct examination of the earth by this ignorant person would lead him to another fact which he would consider certain. He would say the earth is stiff and steady, and that it does not move. Another thing for him to unlearn as quickly as possible; for here again is mother earth a deceiver, for she is really whirling around on an axis once a day, and also speeding along in her annual orbit around the

sun at the rate of about eighteen miles per second. And she has a number of motions and wobbles in addition to these.

Now such an imaginary person is by no means to be regarded as an impossibility. Probably a majority of those who have inhabited the earth since the beginning have been thus ignorant; possibly a majority of those now living are nearly as ignorant. Some of the greatest names of antiquity are linked with conceptions of the universe quite at variance with facts now known; many of the ancient philosophers were quite without knowledge of the earth's true motions. Pythagoras, who lived in the sixth century before Christ, or some of his disciples, were perhaps the first to introduce the idea of terrestrial motion into science. Copernicus, in his great work *De Revolutionibus* (published 1543), quotes the Pythagorean philosophers in support of his new theories.

But it is not our purpose to trace the development of modern accepted ideas as to the earth's motions through the vast literature of the last four or five centuries; we shall confine our attention to an explanation of things as they are. In the first place, let us consider the rotundity of the earth. There are a number of convincing arguments to prove that the earth is curved, and not a flat plain such as it appears to be. It has been circumnavigated many times, for one thing. And an even stronger proof of the earth's curvature is furnished by the appearance of ships at sea. When we examine a vessel approaching us from a distance (Fig. 26), we always see the masts and sails before the

FIG. 26. Curvature of the Earth. (From Sacroboscus' *Sphaera*, Edition of 1564.)

hull becomes visible; and this quite irrespective of the direction from which the ship is coming toward us. This proves that the earth's surface is curved — is convex — in all directions. It proves that the surface of the earth slopes downward, as it were, in every direction from the point where the observer stands. And once granting that the earth is convex, its approximate sphericity is proven beyond a doubt by the shape of the shadow it casts into space on the occasions when eclipses of the moon occur. A vast number of such eclipses have been observed; and always, without exception, the edge of the obscured part of the lunar surface is curved, and curved as only the shadow cast by a spherical earth could possibly be curved.

Next, as to the earth's axial rotation: how do we know that it turns daily on an axis passing through the terrestrial poles? Strong doubts existed on this point at least as late as the time of Galileo, early in the seventeenth century. Thus we may quote the following from p. 244 of Salusbury's quaint translation of Galileo's *Dialogue on the Two Chief Systems of the World* (published by Galileo in 1632; Salusbury's translation published in 1661):

"Salviati: 'As in the next place, to the instance against the perpetual motion of the earth, taken from the impossibility of its moving long without wearinesse, in regard that living creatures themselves, which yet move naturally, and from an inborn principle, do grow weary, and have need of rest to relax and refresh their members —'

"Sagredus (interrupts): 'Methinks I hear Kepler answer him to that, that there are some kind of animals which refresh themselves after wearinesse, by rolling on the earth; and that therefore there is no need to fear that the terrestrial Globe should tire, nay it may be reasonably affirmed, that it

enjoyeth a perpetual and most tranquil repose, keeping itself in an eternal rowling.'"

To-day, as in the time of Copernicus or Galileo, the obvious astronomical arguments are not logically conclusive. There is nothing to determine whether the diurnal rotation of the heavens, sun, moon, and stars, is produced by the sky turning around the earth, or the earth itself turning in the opposite direction inside the sky.

Fortunately we have now good experimental proof that the earth really turns on its axis once in twenty-four sidereal hours. But, strange to say, this experimental proof did not exist until 1851. In that year the physicist Foucault performed a most striking experiment in the Panthéon at Paris, whereby it became possible for the spectators to see the earth, as it were, actually turning under their feet. This Foucault experiment, as it has since been called, is not difficult to perform; it has been repeated by many astronomers and physicists since the original observation was made, and always with the same result, favorable to the hypothesis of terrestrial axial rotation.

Foucault suspended a very long pendulum consisting of a heavy ball attached to a wire free to swing in any direction. The only object in using a pendulum of great weight and length is to diminish the disturbing effects of possible air-currents in the room, and of other undesirable causes which might make the oscillations of a smaller pendulum vary from their theoretically correct position.

When such a perfectly free pendulum is set swinging very carefully, it will continue to vibrate back and forth, until it is finally brought to rest by the friction of the surrounding air, and the resistance to bending of the wire by which it is suspended. But the direction in space of the plane of vibra-

tion (the direction in which the wire moves back and forth) will tend to remain constantly the same, because no forces are applied to the pendulum at right angles to the direction of its swing; and it would require the application of such forces to alter the direction in space of the plane of oscillation. This principle, that a free-swinging pendulum will tend to oscillate in an unvarying direction, is the fundamental principle of the Foucault experiment.

Now let us suppose for a moment that the experiment could be performed at the north pole of the earth. Suppose we could there set the pendulum swinging in the direction of the star Arcturus, for instance, and that we marked on the floor, under the swinging ball, the direction in which the oscillations commenced. Then, if there were no axial rotation of the earth, the pendulum would continue to swing back and forth, exactly over the same mark until it stopped. And it would always swing in the direction of the star Arcturus.

But if the earth is turning under the pendulum, it will carry the mark on the floor around with it. And the pendulum still constantly continuing to swing toward Arcturus, there must result a visible rotation of the mark on the floor with respect to the direction of the pendulum's swing. This motion of the mark will keep pace exactly with the terrestrial axial rotation; and after the earth has made a complete rotation in twenty-four sidereal hours, the mark must once more come exactly in line with the direction of the pendulum's oscillation.

In any latitude other than that of the north pole, the state of affairs is not quite so simple. But it is certain that in any latitude whatever, if the earth is perfectly immobile, and has no rotation of any kind, there can result no motion

whatever of the mark on the floor with respect to the pendulum. Once started over the mark, the pendulum must continue to oscillate over it. Yet whenever and wherever this experiment has been tried, large motions of the mark have been observed. Moreover, and most important of all, the rate at which the marks have been observed to move has always been found to agree accurately with the rate calculated by theory[1] on the supposition that the earth rotates on its axis once in twenty-four hours. The conclusion is irresistible that our earth is really subject to such a rotation.

We are not limited to the Foucault pendulum for an experimental demonstration of terrestrial axial rotation. It was pointed out by Newton that we can test this question by the simple experiment of dropping a heavy object from the top of a tall tower, and noting exactly where it falls upon the earth beneath. Newton had received a letter (December, 1679) from Hooke, asking for some "philosophical communication." In his reply he suggests the above experiment and says the falling body "will not descend in the perpendicular, but, outrunning the parts of the earth, will shoot forward to the east side of the perpendicular."

It is obvious that if terrestrial rotation really exists, the top of the tower will move faster than the bottom because it is farther from the center of the earth, and so moves on a longer radius. Therefore, a body dropped from the top retains an extra eastward impetus in descending, and must strike the earth a little to the east of the spot directly under the point from which it was allowed to fall. It would not fall parallel to the string of a plumb-bob.

Modern experiments on this principle, performed in 1831, were on the whole inconclusive in their results because it

[1] Note 11, Appendix.

was found impossible to avoid the interfering effects of air currents, and because the metal balls that were allowed to fall could not be prevented from being deflected a little one way or the other as a consequence of friction with the air. The errors introduced by these disturbing causes were large enough to mask almost completely the eastward deflection predicted by Newton; but this deflection undoubtedly exists to the extent required by theoretic calculations based on the accepted hypothesis of terrestrial axial rotation.

Having thus described the evidence which leads us to believe in the sphericity and diurnal rotation of the earth, let us next consider the methods by which its size and weight have been determined. Up to the present we have assumed as a first approximation that the earth is exactly spherical in form. Though this assumption is not quite accurate, we shall continue it a moment longer, and use it to explain a simple method of measuring the earth's size approximately.

We have but to return to the process of Eratosthenes of Alexandria (250 B.C.), one of the ancients who believed the earth to be round. Eratosthenes used a method practically equivalent to setting up a vertical post, and observing each day the length of its shadow cast upon a level surface. He was especially careful to measure the shadow when it was shortest each day. This occurs, of course, at noon, when the sun is on the meridian. Furthermore, the length of the short noon-shadow is not the same every day, for a very simple reason. We recall that the sun always appears at some point in the ecliptic circle (p. 27), and that during about half the year that point is located between the celestial equator and the north celestial pole (p. 43). During that half-year there must come a day when the sun appears in that point of the ecliptic which is farthest north from the

celestial equator. This point is called the Summer Solstice; the sun reaches it on or about June 21 of each year; on that date we have the longest day of summer; the sun rises higher in the sky at noon than it does on any other date, and the noon-shadow of a post is the shortest of all the noon-shadows during the year.

While the noon-shadow will thus be the shortest possible on June 21 everywhere in the northern hemisphere, it will not be equally short in all places. For, as shown in Fig. 27, the length of the shadow will depend on the angular distance of the sun from the zenith at noon. In the figure, Z is the zenith, S the sun on the meridian at noon, BC the post, and AB the length of the shadow. In a place where the sun is exactly overhead, in the zenith, the post will cast no shadow; but with the sun at S, the shadow has the length AB.

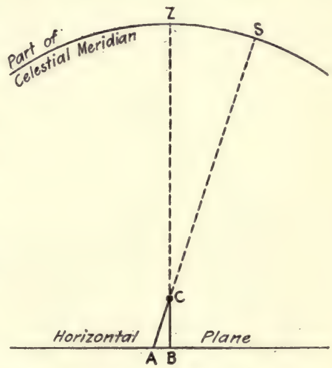

FIG. 27. Length of Short Noon-shadow.

And the angular distance of the sun from the zenith, or the angle ZCS, can be found easily by measuring the noon-shadow length AB together with the height of the post BC, and then constructing a diagram like Fig. 27.

Now Eratosthenes not only made observations of this kind at Alexandria, but he caused similar observations to be made simultaneously at another place called Syene. He was able to assure himself that the corresponding observations were really made on the same day by using in both places the date when the short noon-shadow was the shortest of the whole year.

The line joining Syene and Alexandria was a north-and-south line, or terrestrial meridian of longitude, as we would call it to-day. Eratosthenes measured on the surface of the earth, with measuring rods, the linear distance between the two places, and found it to be, in Greek measure, 5000 *stadia*. By combining this linear measurement with his shadow observations, he was able to ascertain the size of the earth, supposed to be spherical. Figure 28 shows how this was done. The circle represents the earth, with Alexandria and Syene situated at A and S. The zeniths of Alexandria and Syene lie in the directions of Z' and Z, respectively. The shadow observations showed that the sun, on the day when its shadow was shortest at noon, was exactly in the zenith at Syene, while on the same day at Alexandria the angular distance of the sun from the zenith was one-fiftieth part of a circumference, or 7° 12,′ as we should call it in modern angular measure.

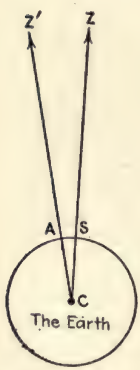

FIG. 28. Eratosthenes' Measurement of the Earth.

Now the sun's distance from the earth is so great that its rays falling on Alexandria and Syene may be regarded as parallel. Therefore these rays would come down to the point A in a direction parallel to ZS; and so the angular distance 7° 12′, measured at A, is equal to ACS, the angle at the earth's center between terrestrial radii drawn to Syene and Alexandria. In other words, Eratosthenes found that 5000 linear stadia, measured on the surface of the earth, correspond to one-fiftieth of the entire circumference. Consequently, the linear length of the earth's whole circumference must be 50×5000 stadia, or 250,000 stadia. And from this

measurement of the circumference Eratosthenes could find the length of the earth's radius, also in stadia. Unfortunately, we do not know the length of his stadium in modern measures, and are therefore unable to judge the precision of his result.

But this old method of Eratosthenes is to-day still in principle the method used for measuring the earth; though modified, of course, by modern instruments of precision, and modern methods of observing. Accurately stated, the process of measuring the earth, which is called Geodesy, consists of two separate and distinct operations. The first corresponds to the measurements Eratosthenes made on the surface of the earth between Syene and Alexandria for finding the linear distance between these two places.

Two suitable fundamental points on the earth's surface are selected, and their relative positions, as well as the linear distance between them, are measured with the utmost precision. This is accomplished by means of a survey called a geodetic triangulation. First, a chain of triangles is laid down on the earth, as shown in Fig. 29; and then, with very accurate surveying instruments, all their

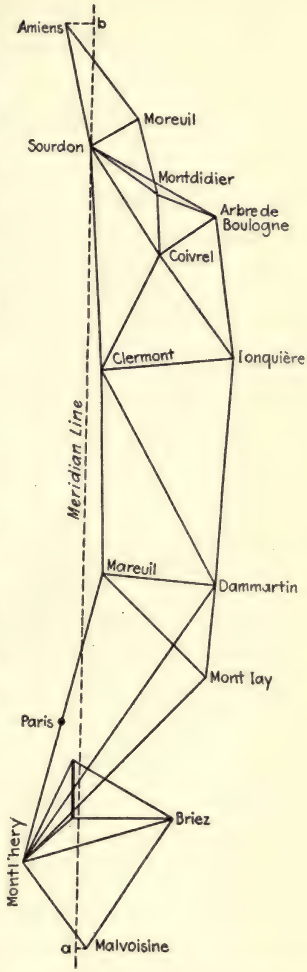

FIG. 29. Geodetic Triangulation.

(From Picard's *Dégré du Méridien entre Paris et Amiens,* Plate II, p. 116. Paris, 1740.)

angles, and at least one side of one triangle, are measured with the greatest care.

The triangles are usually laid down in such a way that the two fundamental points originally chosen are situated near the two ends of the chain of triangles, and preferably near the north and south ends. Then a north-and-south line is inserted in the survey; and thus the process of geodetic triangulation finally furnishes us with the precise linear distance by which one of the original points is north of the other; or, in other words, the modern equivalent of the 5000 stadia of Eratosthenes. This distance is now usually expressed in meters or in feet.

In addition to the triangulation, the other operation, which corresponds to Eratosthenes' post and shadow observations, is completed with precise astronomical instruments such as will be explained in a later chapter. For our present purpose, it is sufficient to remark that with the astronomical instruments in question it is possible to determine by observation of the stars, and with very high precision, the exact terrestrial latitudes of our two fundamental end points. This having been done, the difference of the two latitudes, so determined, gives us, in degrees, an arc corresponding to the arc Eratosthenes measured with his shadows.

Recurring to Fig. 28, we may now let the points A and S represent the two end points of the triangulation, supposed situated on a north-and-south line, or terrestrial meridian. The survey gives the linear distance AS; and the astronomical observation of the latitude difference gives the corresponding angle ACS at the earth's center. It is then easy to form the following proportion: Angle $ACS : 360° ::$ linear distance AS : linear length of entire circumference.

By the aid of this proportion we can calculate the length

(or number of feet) in the earth's circumference, and thence obtain the length of the terrestrial radius.[1] It is 3959 miles long.

It is scarcely necessary to remark that operations of this kind for determining the size of the earth have been repeated frequently at many different parts of the earth's surface. Indeed, the importance of the problem warrants the expenditure of almost endless time and trouble for its solution with the highest possible precision.

And a most interesting result has been found from this frequent repetition of Eratosthenes' method. The radii obtained in different parts of the earth are not in exact accord. The earth may be considered spherical as a first approximation, but as a first approximation only.

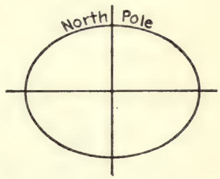

FIG. 30. Flattening of the Earth.

When we measure, for instance, the number of feet in an arc corresponding to 1° of latitude difference near the equator of the earth, and again in a very high latitude near the north pole, we find the two numbers different. The polar degree is longer in feet than is the equatorial degree. This can be explained in one way only. The earth is not an exact sphere, but is flattened somewhat at the poles, so that the meridian section is shaped somewhat as shown in Fig. 30 (greatly exaggerated).

It is obvious, of course, that the more flattened a circular arc is, the longer must be the radius of the circle. A little circle with a radius of one inch will exhibit considerable curvature even in a very short arc; but a large circle, with a

[1] The radius of a circle can, of course, be computed easily from the circumference by well-known mathematical methods.

radius of 100 yards, will show but very little curvature in a short piece of it. So the curvature of our earth at the poles is like that of a large circle; near the equator it is like that of a smaller circle.

Now this flattening of the earth at the poles is exactly what we should expect if the earth's form has been influenced by its daily axial rotation; and it is certain to have been so influenced. The rotation must produce a centrifugal force which would tend to make the particles of matter composing the earth move from the polar to the equatorial regions. The quantity of such motion, and the consequent quantity of flattening, must depend on the velocity of rotation. If the earth rotated several times as fast as it actually rotates, we should expect a considerably larger difference between the polar and equatorial diameters of the earth.

Newton made an attempt to calculate the flattening of the earth by means of his newly discovered law of gravitation. But his result was not accurate; on account of certain inherent difficulties of the problem, it can be solved best by actual observations rather than theoretical computations. In 1672, the astronomer Richer had already made a scientific expedition to Cayenne, and there found that his astronomical clock, which ran correctly at Paris, lost about two minutes daily. This was mainly due to the same centrifugal force by which the flattening of the earth is produced. Richer's clock was a pendulum clock. At Cayenne, near the equator, the centrifugal force must be near its maximum. For this force being due to the earth's motion of rotation, it will be greatest in places near the equator, which are whirling around rapidly in a large circle. Places near the pole are near the rotation axis, and have

therefore comparatively slow motion and moderate centrifugal force.

So the centrifugal force at Cayenne, being large, and acting contrary to the gravitational force of the earth as a whole, diminished that pull upon Richer's pendulum, and therefore made it oscillate slower, so that the clock "lost time."

In spite of Richer's observation and Newton's calculation, many scientific men doubted the polar flattening of the earth; especially as certain French geodetic results did not accord with this theory. But in 1735–1744 Maupertuis measured a meridianal arc in Lapland, and Bonguer and La Condamine one in Peru; the comparison of these arcs left no doubt of Newton having been right.

In comparatively recent years our knowledge of the earth's true shape has been extended greatly by entirely new methods which we have not yet described. The modern applications of Eratosthenes' plan have all involved triangulations extending in a north-and-south direction only. But it should be possible to employ with equal advantage similar geodetic surveys extending in an east-and-west direction. Only, in the latter case, the purely astronomic observations would involve a determination of the longitude difference between the two end stations of the survey, instead of their latitude difference.

But astronomers had no means of measuring longitudes with a precision comparable to their measures of latitude until the introduction of the electric telegraph. If the two end stations are telegraphically connected, it is easy to send practically instantaneous signals from one station to the other. By means of these signals, accurate clocks, regulated by observations of the stars, and mounted at the

two stations, can be compared, and thus the time difference (p. 72) of the two stations determined within a small fraction of a second. And the time difference once known, the corresponding longitude difference is at once obtained, since 15° of longitude correspond to each hour of time difference. Furthermore, since it has become possible to determine the terrestrial radius by east-and-west triangulations, it follows that we can now use equally well triangulations extending in any direction whatever, provided we measure both the latitudes and the longitudes at the two end stations.

Still another and quite different method of verifying the precision of results obtained from triangulations has been introduced in recent years. We have seen that the increased centrifugal force near the earth's equator, acting against the earth's gravitational attraction, tends to diminish the effect of the latter, and that a pendulum will therefore swing more slowly near the equator than it will near the poles. The quantity of this retardation can be calculated accurately from the known approximate dimensions of the earth, and its known velocity of axial rotation.

But when such calculations are compared with actual observations of pendulums carried to different places on the earth, it is found that the retardation near the equator is larger than can be explained as a result of centrifugal force. The reason is obvious. On account of the earth's flattening at the pole, the pendulum is actually farther from the earth's center when carried to the equator than it is in high northern latitudes, near the pole. As gravitational attraction, according to Newton's theory, diminishes with any increase of distance from the attracting body, it follows that the earth's pull upon a pendulum will be a minimum at the equator.

Consequently, we need merely carry a pendulum of unvarying length to high northern and to equatorial latitudes; and compare with great accuracy its time of vibration. The difference, after correction for the effects of varying centrifugal force, will be a measure of the variations in the earth's gravitational attractive force, and will thus become a measure of existing variations in the length of the earth's radius. Very elaborate "pendulum surveys" of this kind have been made in recent years, and these verify the results of our latitude and longitude geodetic triangulations.

We may therefore regard the earth's true shape as now known with considerable accuracy. But as this accuracy has increased, with the introduction of modern precision, minor irregularities in the earth's shape have been brought to light. The meridians of our planet are in the main ovals, such that, approximately:

$$\frac{\text{Equatorial diameter } \textit{minus} \text{ polar diameter}}{\text{Equatorial diameter}} = \frac{1}{295}.$$

But these meridians are not ellipses of exact form. In recent years a new mathematical term has been introduced by geodesists to describe the true shape of the earth. They call the earth a Geoid; and a geoid is defined as a surface everywhere perpendicular to the direction of the plumb-bob string, or the pull of gravity, and therefore everywhere coinciding with the mean surface of the ocean. The geoid surface coincides theoretically with the earth's surface; for it includes the effects of centrifugal force, as well as all possible variations of the direction in which terrestrial gravity acts, and of the pull which it exerts.

Having thus indicated the methods employed by astronomers to measure Mother Earth, let us next consider the process of weighing her. And when we begin to speak of

weighing the earth, it becomes necessary to emphasize the distinction existing between the so-called *mass* of a body of any kind and its *weight.* Bodies have weight on the earth simply because of the gravitational pull of the earth upon them. And we have already seen that this gravitational pull is not everywhere the same, being greatest near the poles, where the flattening of the earth brings us nearest to the center. Consequently, we need some kind of a unit, analogous to a unit of weight, but one that is everywhere the same.

The unit of mass is such a unit. The weight of a body is variable in different places, but its mass is everywhere the same. If we can determine its mass in one place, we know its mass everywhere. For instance, if we adopt as our unit of mass a certain standard pound that is preserved in the United States Government Bureau of Standards in Washington, and if we wish to know the mass of a certain stone, we might carry it to Washington, and there weigh it in comparison with the standard pound.

If it weighed exactly as much there as the standard pound, we should say it had a mass of one pound. Now anywhere else on the earth it would still weigh very nearly one pound, because the gravitational attraction exerted by the earth varies but little in different localities on its surface, the earth being so very nearly an exact sphere. But if that stone could be carried to the surface of the sun, where the solar gravitational attraction is about 28 times as great, on account of the sun's vast bulk, it would then weigh 28 pounds; but its mass would still be only one pound, as before. Its mass having been found to be the same as that of the standard pound in Washington, it would be the same everywhere in the universe.

So when we speak of weighing the earth, we mean, in precise language, determining its mass. So far as terrestrial man is concerned, there is no exactness in speaking of the earth's weight, since there is no such thing as weight, except in the case of bodies situated on the earth's surface and attracted by the earth. This state of affairs is by no means objectionable, because, for all practical purposes in astronomy, it is really the mass of the earth that we need to know.

We need to know how strongly the earth exerts a gravitational pull upon the other planets in the solar system. And under Newton's law of gravitation this pull is proportional to the earth's mass. No such thing as weight enters into Newton's law anywhere. According to that law, two bodies whose masses are M and M', and whose distance asunder is D, — two such bodies attract each other with a force of attraction which may be indicated by the following simple formula:

$$\text{Force of attraction} = \frac{MM'}{D^2}.$$

Stated in words, this formula means that between these two bodies exists an attractive force which is proportional to the product of their masses, and inversely proportional to the square of the distance between them.

Various experimental methods have been used to measure the earth's mass, all depending on the following principle: we take some small object on the earth's surface, and compare the attractive force exerted by the earth upon that object with the corresponding attractive force exerted upon it by some other large terrestrial body of known mass. The attractive force exerted by the earth can, of course, be measured by weighing the chosen small object with an ordi-

nary balance; that exerted by the large object of known mass must be ascertained by means of special experiments. But when we thus know the relative attractive forces exerted upon the same small object by the earth and the terrestrial body of known mass, we know the relative masses of the earth and that body, since attractive force is always proportional to the mass of the attracting body. Thus we arrive at a knowledge of the mass of the earth in terms of the large body of known mass.

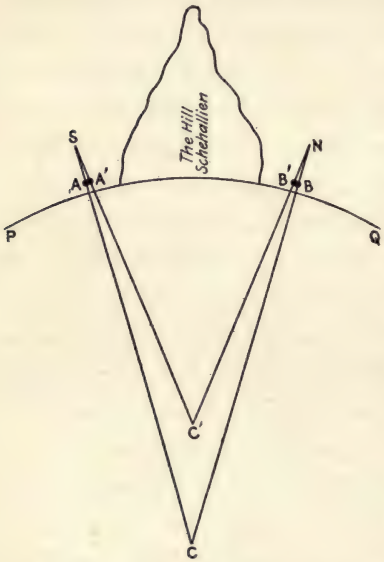

FIG. 31. Mountain Method of Maskelyne.

We shall first describe the so-called "Mountain Method," used successfully by Maskelyne in 1774 in Scotland.[1] He selected for his terrestrial body of known mass a certain hill called Schehallien,[2] and made a very careful survey of the region surrounding it. Figure 31

[1] Maskelyne, "Account of Observations made on the Mountain Schehallien for finding its Attraction," Phil. Trans. Roy. Soc. LXV, Part II, p. 500. "Redde, July 6, 1775."

Hutton, "Calculations from the Survey and Measures taken at Schehallien, etc.," Phil. Trans. Roy. Soc. LXVIII, Part II, p. 689.

Maskelyne and Hutton carried out their calculations in such a way that the density or specific gravity of the earth was made the principal object of their researches. We have modified slightly their presentation of the subject, so as to make the earth's mass or weight the object sought. The two problems are identical, as we shall see further on.

[2] To find a hill suitable for his purpose, Maskelyne sent into Scotland a certain Charles Mason, who selected Schehallien after a long search.

shows his method of procedure. PQ is a portion of the earth's surface, here supposed spherical, and C is the center of the earth. SA and NB are two plumb-bobs hung on opposite sides of the mountain at two principal stations of the survey. The station N was chosen nearly due north of the station S.

Owing to the gravitational attraction of the hill, both plumb-bobs were deflected toward it. Instead of pointing toward the center of the earth at C, they pointed toward C', a point situated between the center C and the surface PQ.

Now it was possible to ascertain by observation both the angle C and the angle C'. For the angle C is simply the latitude difference of the two stations N and S, since they are on the same terrestrial meridian, or north-and-south line. And this latitude difference would be one of the results furnished by the survey, which must make known the number of feet N was north of S. Then, knowing the terrestrial radius, the number of feet corresponding to one degree of latitude was known, and so the exact number of seconds of arc in the latitude difference was also known.

On the other hand, the angle C' was ascertained by means of astronomical observations at the two stations N and S. It was merely necessary to make the observations usual in astronomic determinations of terrestrial latitude. It is sufficient for our present purpose to mention here one peculiarity of observations of this kind. It is always necessary, in adjusting our instruments, to make use of a plumb-bob, or its equivalent, a spirit-level, to ascertain the direction of the zenith (p. 36) directly overhead.

This was the same Mason who was employed in 1763 by Lord Baltimore and Mr. Penn to survey the famous Mason and Dixon line to settle the boundary between Maryland and Pennsylvania in the American colonies.

Ordinarily, results obtained in this way are correct: but, in the present case, they were rendered incorrect by the presence of the neighboring hill Schehallien. The astronomical latitudes determined at N and S were necessarily both erroneous, and the errors were equal to the deflections of the plumb-bobs at the two stations. Then, when the latitude difference was derived from the astronomical observations, it came out as the angle C', instead of the correct latitude difference C. In other words, the astronomic observations gave the latitude difference referred to the false zeniths indicated by the plumb-bobs deflected by the mountain, while the survey gave the correct latitude difference C.

In the actual experiment, the difference between C and C' came out $12''$ of arc, a quantity large enough to admit of easy measurement; and thus the angular deflection of the plumb-bobs became known. It was next necessary to ascertain by measurement the mass or weight of the hill itself. This was accomplished by first computing its vólume approximately from the data furnished by the survey. Then borings were made into the hill, and specimens brought to the surface. These were tested to ascertain their "specific gravity," or weight per cubic foot, as compared with water. With this information at hand it was easy to find the mass of the hill.

The distance of the hill from the plumb-bobs being also known, it now became possible to calculate how great must be the attractive force exerted by the hill on the plumb-bobs to produce the observed deflection of $12''$ appearing in the difference $C' - C$. The attractive force exerted by the earth on the plumb-bobs was ascertained by weighing them in an ordinary balance; and thus Maskelyne found the

relative attractive forces of the earth and of the hill upon the same plumb-bobs. And the ratio of these two attractive forces then made known the relative masses of the earth and of the hill. We have just seen that the mass of the hill was ascertained from the borings, etc.; and so the mass of the earth finally became known, too. This great classic experiment gave the first knowledge as to the mass of our planet.

Unfortunately, the result was not very accurate; the difficulties inherent in the measurement and testing of the hill precluded the possibility of high precision. Consequently, a few years later (1798), Cavendish[1] employed a method which can be entirely completed in a laboratory, and which, with various minor modifications, has since given us all the information we possess as to the earth's weight or mass. Indirectly, yet just as surely as if the earth could be placed in a gigantic scale-pan, is it possible to weigh the planet.

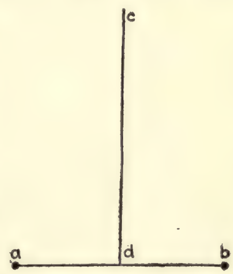

FIG. 32. Torsion Balance.

The principal part of the Cavendish apparatus is called a Torsion Balance, shown in Fig. 32. A very light rod *ab* carries a small metal ball at each end. The rod is suspended at its middle point *d* by means of a very fine silk thread *cd* from a fixed support *c*. In recent instruments the silk thread is replaced by a fiber of quartz made by fusing the quartz and drawing it out to a microscopic fineness.

The balance can be set in rotation about the supporting fiber *cd*, and will then oscillate backwards and forwards like an ordinary pendulum, until it is gradually brought to rest

[1] Phil. Trans. Roy. Soc., 1798, p. 469.

by the continued friction of the surrounding air. During these oscillations the rod, of course, remains horizontal, being exactly balanced at its middle point *d*.

Before explaining the use of such a balance in weighing the earth, it is necessary to show how the so-called "constant" of the balance itself may be determined. This constant may be called the "torsional constant" of the balance; it is a measure of the quantity of force which must be applied to the balance in order to make it turn about the support *cd*. This quantity of force will, of course, depend on the thickness and stiffness of the fiber suspension *cd*. For when the balance turns, the fiber is twisted, and therefore the torsional constant will be large if the fiber is of such a kind as to resist a twisting effort quite strongly.

The letter T is used to designate the torsional constant of any given balance. Accurately stated, T is the quantity of force required to turn the balance through unit angle, the said force being applied to the balance at unit distance from the center *d*. In our modern system of units:

Unit of length is the centimeter,

Unit of angle is 57.3°,

Unit of weight is the gram,

Unit of time is the mean solar second.

Now it is possible to determine the constant T for any given torsion balance by observing its time of vibration;[1] and this having been done, we may apply the balance to our problem. For this purpose, it must be mounted in such a way that its oscillations can be observed while it is under the influence of the gravitational attraction exerted by a couple of heavy lead balls brought very close to the little balls which are on the ends of the torsion balance rod.

[1] Note 12, Appendix.

Figure 33 shows the apparatus, the reader being here supposed to examine it from above, looking down upon it along the direction of the supporting fiber *cd* (Fig. 32).

In Fig. 33 the line *ab* shows the position in which the small balls *a* and *b* would finally come to rest after oscillating, if the balance were allowed to oscillate quite undisturbed by the proximity of the big lead balls. But if these latter are placed in the position A' and B', their gravitational force will attract the little balls *a* and *b*, so that the final position of rest will be $a'b'$ instead of *ab*. And if the big lead balls are placed at A'' and B'', the final position of rest will be $a''b''$.

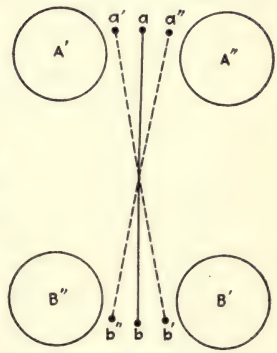

FIG. 33. Cavendish Experiment.

In addition to the torsion balance the apparatus for the Cavendish experiment must therefore include two big lead balls, together with suitable mechanical arrangements for transferring them conveniently from the position $A'B'$ to the position $A''B''$. This having been provided, it is possible to ascertain by observation the distances $a'a''$ and $b'b''$; and this, together with our knowledge of T, will tell us the quantity of gravitational attractive force exerted by the big lead balls upon the little balls *a* and *b*.

But the corresponding attractive force exerted by the earth upon these little balls *a* and *b* may be ascertained easily by weighing them in an ordinary balance, since weight is merely a result of the earth's gravitational attraction. Thus we return to the principle used by Maskelyne, and which we have already explained to be fundamental in all experiments of this kind. Having ascertained separately

the attractive force exerted on the little balls by the big ones and by the earth, we have once more the ratio between the masses of the big balls and the earth, since these attractions are proportional to the masses according to Newton's law. And since masses are in a sense only another name for weights, we have the ratio of the earth's weight to that of the big lead balls.[1]

The best result obtained in this way for the mass of the earth, from the average of several modern repetitions of Cavendish's experiment, is:

$$6 \times 10^{27} \text{ grams.}$$

The size, shape, and mass of the earth having been determined, it is easy to calculate its average density or specific gravity. This is, of course, simply the average weight of a cubic centimeter of terrestrial material as compared with a cubic centimeter of water. We have merely to calculate the earth's volume from its radius, which is extremely simple if we regard the earth as a sphere, and not very difficult, even if we take account of the polar flattening.

Knowing the earth's volume, we can then compute the weight of an equal volume of water, and the ratio of the weight of this volume of water to the weight of the earth will be the earth's average density. Thus we see that the problem of weighing the earth is really equivalent to the problem of determining the earth's density. (Cf. p. 104, footnote.)

In this way the earth's density is found to be about 5.5, which means that a cubic foot of average terrestrial material weighs 5.5 times as much as a cubic foot of water.

Having now discussed the methods of ascertaining the

[1] Note 13, Appendix.

mass of our earth, and the average density of the materials composing it, we shall next consider for a moment the structure of the earth's core. Our knowledge is here necessarily based on theoretical considerations only, it being obviously impossible to penetrate the earth's interior for the purpose of making actual observations. The deepest existing mines and borings pierce but a very short part of the outer terrestrial crust, when we consider that the radius of the planet is about 4000 miles.

But such as they are, these mines and borings indicate a decided increase of temperature as we go deeper into the earth. The fact that such temperature increases are always found shows that there must be a steady supply of heat from the interior; if there were not, the outer shell containing the borings would speedily acquire a uniform temperature. And we have further conclusive evidence of great interior heat from the volcanoes.

Many theorists have held in the past that there is a central molten nucleus in the earth; we now believe that the hot nucleus is solid. It is doubtless quite hot enough to be fused under ordinary circumstances; but at the enormous pressure existing inside the earth, it is probably impossible for any substance to melt, even at a very high temperature. The strongest argument for believing in a solid earth, as against a molten earth having a thin solid exterior shell, is derived from the phenomena of the tides. The tidal rise and fall of the oceans is caused by the gravitational attraction of the moon. If there were but a thin shell of solid earth, it would be forced to rise and fall also, for it could slide on the interior fluid mass. And if the shell rose and fell with the water, we would not have observable tides along the coast lines of the continents. The earth's in-

terior is therefore probably solid, with a rigidity about equal to that of steel.

Under this theory we must regard the fluid lava ejected by volcanoes as derived perhaps from minor "pockets," in some way protected from the usual pressure, and therefore containing molten matter. Or we may imagine that the pressure of the crust may be diminished materially at some point for a time, whereby the solid matter immediately under that point might suddenly fuse and give rise to an eruption.

A very remarkable phenomenon having a certain bearing upon the above theories is the Variation of Latitude. This was first proved observationally by Küstner in 1888, when he found that the latitude of the Berlin observatory was subject to slight changes. In the following year an expedition was sent to Honolulu, while careful observations were continued simultaneously in Germany. It was found that the latitude of Honolulu increased when the German latitudes decreased, and *vice versa.*

Since terrestrial latitude is merely angular distance from the terrestrial equator, it follows from the above that the earth's equator must be swinging in some way. And as the equator is everywhere 90° distant from the terrestrial poles at all times, it follows that the earth's polar axis must also be in motion.

Later elaborate observational researches have shown that such is really the case. The earth's pole is really in motion, though the motion is quite small. A circle with a radius of 50 feet would include all the pole's wanderings so far observed. Mathematical investigations show that this phenomenon indicates a solid but not quite absolutely rigid earth, thus affording a further verification of the accepted theory as to the solidity of our planet.

We now pass from the interior of the earth to the part which is above the surface, — the atmosphere. This is a mixture of various gases, principally nitrogen and oxygen, with small amounts of carbon dioxide, water vapor, and various rare gases in most minute quantities. The entire atmosphere is part of the earth, and moves with it in its diurnal rotation and annual orbital revolution.

Perhaps the most important function of the atmosphere is the distribution of sunlight in all directions by reflection from the tiny particles in the atmosphere. This explains our being able to see objects on the earth by the help of sunlight. We cannot see such objects unless sunlight falls on them in the right direction to be reflected back from the object to the observer's eye. And as the atmospheric particles reflect sunlight in *all* directions, it follows that some light is sure to fall on all surrounding objects in such a way as to be reflected to our eye and make the objects visible.

This same cause produces the apparent bright background of the sky in daytime. Were it not for the atmosphere, the sky would be dark in the daytime, as it is at night; and we should see the stars at all hours. And the blue color of the sky, as well as the other colors seen at sunset, etc., are doubtless a result of prismatic effects produced by atmospheric particles.

Twilight is another important phenomenon due to the atmosphere. After the sun has set below the horizon, it continues to illuminate particles of the upper atmosphere. These particles once more reflect the light, so that a certain diminishing quantity of atmospheric illumination continues until the sun has sunk about 18° below the horizon.

Another function of the atmosphere is to act as a kind of blanket to retain solar heat upon the earth. The sun

sends us rays that are practically all light-rays. Rays of this kind pass quite easily through the atmosphere, and heat the earth's surface. Then, at night, when the earth begins to radiate heat into space, it sends out a kind of heat-rays that pass through the atmosphere with the greatest difficulty only. Consequently, the earth remains much warmer than it would otherwise do; and this action of the atmosphere has much to do with making the earth habitable. The phenomenon is due to a transformation of the character of solar rays by being first absorbed and then radiated by the terrestrial surface. The water vapor in the atmosphere is particularly effective in this matter.

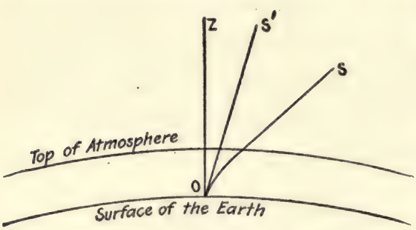

FIG. 34. Refraction.

Another, less important, atmospheric effect is known as Refraction. Light-rays coming from any celestial body must pass through the air before they reach the observer. As shown in Fig. 34, these rays are bent, or refracted, as they pass from the outer, and less dense, parts of the atmosphere to the lower and denser strata. The light of a star in the zenith at Z would come straight down, without change. For it is a principle of refraction that in passing from any stratum to a denser one, light is not bent when it is perpendicular to the strata. But if it makes an angle with the surfaces of the strata, it is bent toward the perpendicular.

Thus light coming from a star at S would travel through the air in a curve, and would finally reach the observer at O as if it had come in a straight line from S'. The figure is, of course, greatly exaggerated; but the effect of refraction

is to make all the heavenly bodies appear to us nearer the zenith than they really are. The effect is greatest when we observe near the horizon. Thus the sun, when setting, will still be entirely visible after it has passed below the real horizon. At such a time, too, the lower edge of the sun, being nearest the horizon, is refracted more than the upper edge. And so the setting sun usually appears of an oval shape instead of round, as it should be.

CHAPTER VII

THE EARTH IN RELATION TO THE SUN

In the last chapter we have discussed the earth as a separate astronomic body, to be measured and weighed without special reference to any other object in the universe. We have also (Chapter II) considered the earth to some extent in its relation to the celestial sphere, and found how various important points and circles on that sphere correspond to the terrestrial poles, equator, etc. Finally, we have made use of the plane of our earth's annual orbit around the sun, extending it outward to the celestial sphere, to gain a definition of the ecliptic circle (p. 27), and, for the purpose of a first approximation, we have taken the earth's orbit around the sun to be a circle, with the sun at its center (p. 25).

But the real terrestrial orbit around the sun is a slightly flattened oval or ellipse, with the sun at a point situated near the center of the oval, and called the Focus of the ellipse. These facts were first discovered by Kepler, who used a method to be described in a later chapter; if it were necessary to establish their correctness to-day, by means of observations, it would be possible to do so in a very simple way.

The necessary observations would consist in ascertaining, on many different dates, the exact position of the point at which the sun appears projected on the celestial sphere. In other words, we should measure frequently, with suitable instruments to be described later, the sun's declination and

right-ascension (p. 34); these, as the reader will remember, define the sun's apparent position on the celestial sphere, precisely as latitude and longitude define the position of any city on the earth's surface.

Now if we locate on a celestial globe (p. 37) these successive points occupied apparently by the sun on various dates, we shall find that they all lie on a single great circle of the celestial sphere, which, as we have already seen (p. 27), is called the ecliptic circle. And the fact that the sun's observed positions on the celestial sphere thus all lie on a single great circle, constitutes an observational proof that the earth's orbit around the sun is really contained in a single plane, or flat surface.

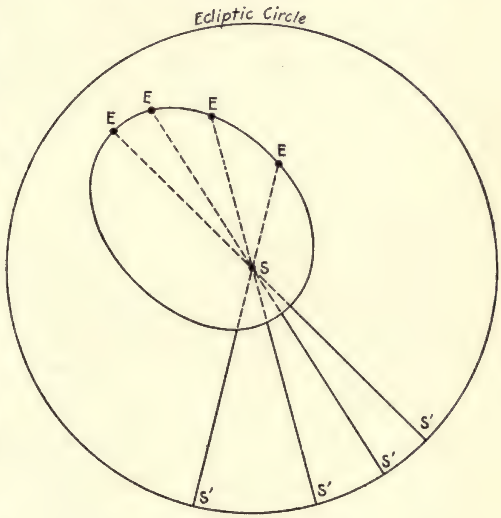

Fig. 35. Orbit of Earth.

Let us next, in Fig. 35, resume Fig. 3 (p. 28), drawing it, however, in a slightly modified way, with the earth's orbit greatly enlarged. But in spite of this enlargement, the reader must remember that the earth, sun, and entire terrestrial orbit together represent a mere dot in comparison with the infinitely distant ecliptic circle on the celestial sphere.

Now, in this Fig. 35, let the large circle represent the ecliptic circle on the celestial sphere, and let S' represent

various points at which the sun appears projected, when observed on different dates. The true position of the sun in space is always at S. Now draw straight lines from these observed points S' through S, and continue them to certain other points E.

We know that the sun is projected on the ecliptic circle at the points S' because the earth, in its orbital motion, occupies successively the points E. If we take S as the true position constantly occupied by the sun, it follows that when the apparent positions of the sun on the ecliptic circle are at the points S', the earth's positions E will all be somewhere on the extended lines $S'S$. But as yet we do not know where the points E are situated on those extended lines $S'S$. We know they are somewhere on those lines, but to know the true shape of the earth's orbit we must ascertain the relative distances of the various points E from S by a different kind of observation.

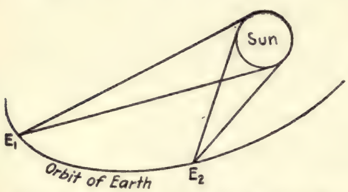

FIG. 36. Sun's Angular Diameter.

This can be accomplished by measuring, with a suitable instrument, the Angular Diameter of the sun on the various dates when the positions S' were observed. To understand what is meant by angular diameter, let us imagine two straight lines, drawn from the earth to two opposite points on the sun's visible disk. Then the angle between those two lines is the sun's angular diameter.

It is quite evident from this definition that the sun's angular diameter will be greater, in proportion as the sun is nearer to us. Figure 36 makes this quite clear. When the earth is near the sun, as shown at E_2, the angular diameter is greater than when the earth is farther from the sun, as at

E_1. Consequently, if we have measured the sun's angular diameter corresponding to each terrestrial position E in Fig. 35, we can mark off the relative lengths of the distances SE. Whenever the angular diameter was found to be large, we should make SE proportionately short, and *vice versa.* The first of the lines SE would be made of any convenient arbitrary length, according to the size chosen for the whole diagram.

When all this has been done, the points E will represent various positions of the earth in its orbit. A smooth curve can be drawn through them, and it will be found to be, not a circle, but a slightly flattened oval or ellipse. The point S, occupied by the sun, will not appear at the center of the ellipse, but at the point already mentioned as being situated a little to one side of the center, and called the focus.

But it is most important to notice that all this experimentation so far gives us only the true shape of the earth's annual orbit around the sun. It tells us nothing whatever about the actual size of the orbit in miles. This could not be otherwise, in the nature of things. For up to the present we have measured angles only; angular right-ascensions and declinations, and angular diameters. And it is a mathematical principle that angles alone can never make linear distances known.[1]

One more interesting fact might be verified experimentally by the methods we have just described. Referring again to Fig. 35, if the dates corresponding to the terrestrial positions E are taken into consideration, it will be found that the line joining the earth and the sun moves in a very peculiar manner. This line is called the Radius Vector. It is clear that it not only revolves around the sun as a sort

[1] Note 14, Appendix.

of pivotal point, but it also lengthens and shortens, according to the variations in the curvature of the terrestrial orbit.

It will be found that the radius vector, in the course of these motions and changes of length, always sweeps over equal areas in equal intervals of time. If we take three positions of the earth *E*, such that the time-interval between the first and second is equal to that between the second and third, then the space or area included inside the orbit between the first radius vector and the second is equal to the corresponding space between the second and third. Each of these areas is a kind of triangle, of which two sides are radii vectores, and the third side is a bit of the curved orbit. These facts were discovered by Kepler in 1609, using, as we have said, a method of investigation quite different from that here described.

Having now attained a notion as to the shape of the terrestrial orbit, it is possible to explain one of the astronomic phenomena most important to man,—the Seasons. What is the cause of summer heat and winter cold?

For the moment we shall consider the northern hemisphere only. At a first glance, one might suppose that the curved shape of the earth's orbit would cause the seasons. For the sun not being accurately at the center, it must happen that we are nearer the sun when at some particular point of the orbit than we are at any other time. When at this point nearest the sun, called Perihelion, the earth, as a whole, does actually receive a maximum of heat. But this is masked so completely by another phenomenon that it is largely without effect in determining the seasons. In fact, the date of perihelion occurs about January 1 each year, so that we are actually nearest the sun in winter.

The temperature at any given place on the earth depends,

not on our slightly varying proximity to the sun, but on the relative duration of day and night. When we have long "days" and short "nights"; when the sun is shining on us during more than half of each 24-hour day, — then is the time to expect hot summer weather.

We have already learned in Chapter II that half the ecliptic circle on the celestial sphere lies between the celestial equator and the north celestial pole; that the sun is seen in that northern half of the ecliptic circle during about half the year; and that during such half-year it is above the horizon daily for more than twelve hours. To be more precise, we found that at the times of the equinoxes, about March 21 and September 22, when the sun appears to cross the celestial equator, the days and nights are equal, and each is twelve hours long. But at the solstices (cf. p. 93), about June 21 and December 21, when the sun attains its greatest angular distance (or declination) north and south of the celestial equator, — at these solstices we have the longest and shortest days in the year, midsummer day and midwinter day.

But there is still another factor influencing this question of the seasons materially. As we have just seen, the earth's surface is heated more or less in proportion to the length of time the sun's rays fall upon it; but it is also heated in proportion to the directness with which it receives those rays. In summer, the sun is not only above the horizon each day longer than in winter, but it is also higher up in the sky when it is above the horizon. Its rays therefore fall upon the earth more nearly vertically; the sun not only acts during a larger number of hours, but it also acts more efficiently while the effect is being produced.

The next important question in connection with the

seasons is to inquire as to the date when we may expect the hottest day of summer. We might at first think it should occur at the time of the summer solstice, about June 21; and we do, in fact, on that date receive our maximum heat per hour and per day. But for a long time after that date the days continue longer than the nights; in each 24-hour period the earth is heated more in the daytime than it is cooled at night; it receives more heat than it radiates away into space, and is constantly becoming hotter.

But as this process of increased heating continues, the earth, being hotter, acquires an increased capacity to give up or radiate heat in the night, because a hot body radiates faster than a cool body. At the same time, the daylight receipt of heat by the earth diminishes constantly as we leave the solstice date in June. So the daily accretion of heat is diminishing, because of the shortening of daylight; the outgo is increasing, because of increased power of radiation; and so there must come a time when a balance occurs, after which the earth begins to become cooler again. In the temperate regions of the northern hemisphere this happens about August 1, instead of September 22, the approximate date of the autumnal equinox, which would be the date of balance if it were not for the hot earth's increasing capacity to radiate heat. After August 1 the night radiation begins to exceed the daily gain of heat, and the earth commences to cool, in anticipation of winter.

In the southern hemisphere all these effects are reversed. There the south celestial pole is elevated above the horizon instead of the north celestial pole; the southern half of the ecliptic circle corresponds to the long days and short nights, instead of the northern half; and midsummer comes in December instead of June.

And there is also another difference between the two hemispheres which is most interesting. We have already mentioned that the earth is nearest the sun about January 1, and that this causes a slight increase of heat, which we have so far neglected to take into consideration. In the southern hemisphere this little increase of heat occurs in summer, and so tends to make the southern summer somewhat hotter than the northern summer.

On the other hand, the fact that the radius vector sweeps over equal areas in equal time-intervals indicates that the earth must move faster in its orbit when near the sun than when farthest from the sun. Another reference to Fig. 35 (p. 117) will make this clear: when the earth is near the sun, the triangles have short sides, and therefore the earth must move through a large angle in a given time-interval so that the short sides of the triangle may be compensated by an increase in the curved base, and the area thus maintained unchanged. It is a principle of mechanics that the orbital speed of any planet must be greatest when it is nearest the sun.

The effect of this in the case of the earth is to make it traverse the perihelion half of its orbit seven days quicker than the other half. In other words, when the sun appears in the autumnal equinox point in September, we have to wait only about 179 days for it to reach the vernal equinox point in March. But the other half of the ecliptic circle, traversed apparently by the sun from March to September, requires about 186 days. These numbers may be verified by counting the days between these pairs of dates, taken from an almanac.

It follows that summer in the southern hemisphere is about seven days shorter than summer in the northern hemisphere;

and this just about balances the increased heat of the southern summer, which we have just seen is due to its occurring in the part of the year when the earth is nearest the sun. In the northern hemisphere, on the other hand, summer occurs when the earth is farthest from the sun; but it occurs in the long half-year of 186 days. So there is an equalization of the summers in the two hemispheres. Both are about equally hot. The southern has slightly warmer days because of the sun's proximity, but it has seven less summer days; the northern has slightly cooler summer days, but seven more of them.

The case is different with the winters, as shown in the following schedule:

	Northern Hemisphere	Southern Hemisphere
Summer	186 days (far from sun)	179 days (near sun)
Winter	179 days (near sun)	186 days (far from sun)

From this it appears that the southern winter is seven days longer than the northern, and also that the southern winter days are of the cooler kind on account of increased distance from the sun. So there is no equalization of winter between the two hemispheres, as there is in summer. The southern hemisphere has a somewhat colder winter than the northern hemisphere; and the summers are approximately the same in both hemispheres.

This interesting fact may be stated in a slightly different way: the difference between the average summer and winter temperatures must be greater in the southern than in the northern hemisphere. And this presents a much more important aspect of the whole question. If one hemisphere, taking the year as a whole, is somewhat colder than the other, can there not have been a remote age in the earth's past history when this difference was far greater than it now is? —

great enough, perhaps, to account for the vast glaciers of the geologic ice-age.

Of course there is but one way in which the difference could ever have been materially greater than at present: there must have been a time when the terrestrial orbit was flattened in a greater degree than now, and when the sun was consequently much farther from the center of the orbit. But was there ever such a time, and, if so, what was the cause?

It is an obvious fact that the motions of our earth will not only be influenced by the gravitational attraction existing between the earth and the sun, but also by that produced through the pull of the other planets. This latter effect is small compared with the solar effect; but it is powerful enough to bring about certain very slow and somewhat irregular changes in the earth's orbit around the sun.

But all these changes have one peculiarity: all are of the kind mathematicians call Periodic. That is to say, none can continue to act indefinitely in a single direction. Every part of the orbit that changes will change first one way and then the opposite way, so that after the lapse of sufficient ages of time, everything about the orbit must return again to its original form and condition.

There is thus a peculiarly impressive perfection about the operation of Newton's law of gravitation in the solar system. No matter what changes are destined to occur, these changes will never disrupt the system mechanically. So far as gravitational forces alone are concerned, the solar system may endure forever.

It may be of interest to give here the principal orbital changes of the above kind which have been brought to light by the labors of various mathematicians following Newton.

1. The orbital flattening undergoes slight changes with a period of 64,000 years.

2. The angle between the celestial equator and the ecliptic circle (cf. Fig. 6, p. 35) varies slightly, with a period of about 34,000 years.

3. The longest axis, or diameter, of the oval terrestrial orbit is slowly twisting around the sky with a period of 108,000 years.

While we are considering these peculiar variations of long period produced by the complicated action and interaction of gravitational forces, it will be of interest to describe briefly the famous phenomenon known as the Precession of the Equinoxes. To make this matter clear, it will perhaps be best to call attention to the methods probably used in ancient times to ascertain by observation the length of the year. In the first place, astronomers tried observations by means of shadows. For instance, setting up a vertical pole, it is easy to fix the date when the shadow at noon is shorter than it is on any other date. This must, of course, occur on the day of the summer solstice (p. 93), when the sun appears highest in the sky. And the sun will then appear at that point of the ecliptic circle which is farthest north of the celestial equator.

By counting the number of days until the same event occurs again, it is possible to obtain an approximate value for the length of the year. For the year is simply the period of time required by the sun to complete an entire circuit in its apparent motion around the ecliptic circle, due to the real circuit of the earth in its annual orbit around the sun. By counting in a similar way the number of days between two widely separated occurrences of the same observation, it is easy to find the length of a considerable number

of years joined together. In this way Hipparchus compared the date of the summer solstice fixed by Aristarchus of Samos in 280 B.C. with his own observation in 135 B.C., and thus found the number of days in 145 years. Dividing this by 145, he computed a very accurate value of the average length of the year. It was very nearly 365¼ days.

Another method of ascertaining the length of the year was used by the Egyptians long before the time of Hipparchus. They observed the phenomena called the Heliacal Risings of certain bright stars near the ecliptic circle. A star is said to have its heliacal rising where it rises above the horizon as near as may be at the time of sunrise. This can occur only on the date when the sun, in the course of its apparent motion around the ecliptic circle, happens to appear near the star in question. Star and sun will then rise together. By counting as before the number of days until the same event occurs again, it is possible to ascertain how many days the sun requires to complete an apparent circuit of the ecliptic circle from a given star back to the same star again.

But the length of the year obtained in these two ways is not quite the same. The shadow year is a little shorter than that deduced from the method of heliacal risings.

The sun, in its apparent motion, travels from a given point of the ecliptic back to that point again somewhat quicker than it proceeds from a given star back to the same star again.

It is a very singular thing that the sun should thus move along the ecliptic faster than it moves among the stars. There is but one way in which this could possibly occur. The entire ecliptic circle, or at least the equinox and solstice points, must have some kind of motion among the stars.

In other words, while the sun is apparently traveling along the ecliptic circle, that circle must itself be moving slightly in the opposite direction, so as to accelerate the sun's apparent motion. Or, to be more exact, if the sun starts from the vernal equinox in its annual apparent motion, and moves exactly one degree along the ecliptic circle, it will then be a very little more than one degree distant from the vernal equinox. While the sun was moving its one degree, the equinox also moved a tiny distance in the opposite direction; so that the distance from the vernal equinox to the sun is finally the sum of the two motions.

Astronomers call the kind of year whose length may thus be determined by shadows the Tropical Year. It is the interval between two successive apparent returns of the sun to that point of the ecliptic circle which is farthest north of the celestial equator. When the sun reaches that point in its apparent course, it turns, and begins to move southward again. The point is a turning-point; and the word "tropic" comes from a Greek word meaning "to turn."

Hipparchus was able to measure also with considerable precision the length of the other year,—the period of time required by the sun to move in its apparent course along the ecliptic from a given star back to that star again. This kind of year, from its relation to the stars, is called the Sidereal Year.

The difference between the two kinds of year is about twenty minutes, the tropical year being the shorter. Hipparchus explained the difference correctly as a consequence of the annual motion of the vernal and autumnal equinox points. Now these points are merely the intersections of the ecliptic circle and the celestial equator on the celestial sphere. If they are in motion, such motion may be caused

by a change in position of the ecliptic circle, or the celestial equator, or both. Hipparchus was able to show that the effect is produced by a slight motion of the celestial equator, the ecliptic remaining practically unchanged. The celestial equator is moving in such a way as to cause the equinoxes (its points of intersection with the ecliptic circle) to move along the ecliptic circle very slowly.

Hipparchus had no difficulty in satisfying himself that the ecliptic circle did not itself change, and that only the equator and the equinox points were in motion. For his star observations showed that all the fixed stars maintained constantly unchanged angular distances from the ecliptic circle on the sky; which could not have been the case if that circle was itself in motion. But the stars did change their angular distances (declinations) from the celestial equator.

In fact, Hipparchus discovered these phenomena first from his star observations, which he compared with those of Timocharis and Aristyllus, made about 150 years before his day. From this comparison he ascertained the quantity of motion of the equinoxes, and thence computed the difference in length between the tropical and sidereal years. The length of the tropical year he found, as we have seen, by means of shadow observations. The length of the sidereal year he then calculated by adding to the length of the tropical year the difference between the two as he had computed it. This was, and is, the best method of procedure, as the length of the sidereal year cannot be observed directly with high precision. It was Hipparchus who named the motion of the equinoxes Precession.

It is possible to explain the cause of precession by the aid of Newton's law of gravitation. We have already found

that the earth is not truly spherical, but that it is somewhat flattened at the poles. This amounts in effect to a spherical earth, with a girdle of protuberant material surrounding the equator. In other words, the earth has its biggest girth around the terrestrial equator. Figure 37 is intended to illustrate the existing state of affairs. It shows the spherical earth, with its north pole (N. P.), its equatorial protuberance, and the planes of the equator and ecliptic.

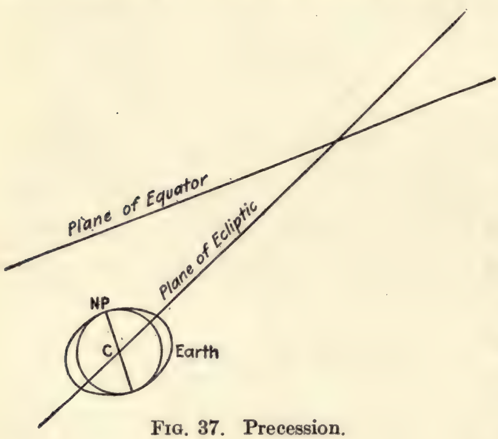

FIG. 37. Precession.

Now the sun and moon both exert a gravitational attraction upon the earth, and also upon its equatorial protuberance. And, as we have already seen, the sun is always in the plane of the ecliptic; we may add as a fact that the moon also happens to pursue an orbit that never goes very far from the same plane. But the lunar and solar attractions affect most strongly that part of the protuberant ring which is nearest to them. This tends to tip over the protuberant ring into the plane of the ecliptic. If no other forces were at work, the earth (Fig. 37) would simply revolve around an axis perpendicular to the paper and passing through the earth's center C, until the equatorial plane had been brought into coincidence with the ecliptic plane.

The force which prevents this rotation is due to the diurnal turning of the earth on its axis. The earth is

trying to turn around two axes at once, — its rotation axis through the north and south poles, and the other axis we have just mentioned. The result is to produce what is called in the science of mechanics a composition of rotations. This leaves the earth turning around its regular rotation axis once daily, but makes that axis itself move in space in such a way that the celestial pole on its extended end revolves slowly on the sky in a circle around a fixed center called the Pole of the Ecliptic. This is a point on the celestial sphere 90° distant from every point of the ecliptic circle. The celestial pole being merely the prolongation of the earth's rotation axis to the celestial sphere, and the rotation axis being set in motion by the composition of rotations, the celestial pole must evidently move on the sky. The pole of the ecliptic remains unmoved, because, as Hipparchus found, the ecliptic does not itself change. But the celestial pole, and consequently the celestial equator, are both subject to this precessional motion.

The angular radius of the circle in which the celestial pole revolves on the sky around the ecliptic pole is equal to the angle between the celestial equator and the ecliptic circle, which is about $23\frac{1}{2}°$. A complete revolution of the one pole around the other requires about 25,800 years; for the annual precession of the equinoxes upon the ecliptic circle is $50.2''$ and a complete revolution of the pole must, of course, correspond to a complete revolution of the equinoxes. And we have:

$$\frac{360°}{50.2''} = 25{,}800, \text{ approximately.}$$

It must not be supposed that this precessional motion proceeds with perfect uniformity, for there are various causes of inequality. When the sun appears at the equinoctial

points in March and September, it is for the moment also in the celestial equator, because the two circles, ecliptic and equator, cross at the equinoctial points. At such times the sun does not tend to tip the earth's equator. But at the time of the solstices, when the sun is far from the equatorial plane, it has its maximum tipping effect. The moon's effect is even more complicated, on account of certain periodic changes in the position of the moon's orbit. Thus the actual precessional circle marked out on the sky by the celestial pole really resembles a sort of wavy line, having about 1400 principal waves in an entire circuit of 25,800 years. These waves are called the Nutation, or nodding, of the terrestrial axis.

An interesting consequence of precession is its effect on the seasons in the northern and southern hemispheres. We have seen that the southern hemisphere is now on the whole colder than the northern. But after half a precessional cycle has elapsed, the northern will be the colder hemisphere. Thus the astronomical explanation of the geologic ice-age is made possible. For the ice-cap was in the northern hemisphere: it must have been formed at a time when precession made the northern hemisphere the colder one, and when, coincidently, the summer and winter halves of the year were unequal by much more than the present difference of seven days, on account of the periodic change of the earth's orbital flattening (p. 126).

Another important result of precession is the fact that the celestial pole is not always near our present pole star. This star is now about $1\frac{1}{4}°$ distant from the true celestial pole; in the time of Hipparchus it was 12° distant; 12,000 years hence Vega will be our nearest pole star; and 4000 years ago α Draconis was the pole star. This is well shown

PLATE 5. Precessional Motion of the Pole,

in the accompanying Plate 5, reproduced from Hevelius' *Prodromus Astronomiæ*, Gedani (Dantzig), 1690. It contains the constellation *Draco*, as drawn by Hevelius, inclosed in the precessional circle, having the pole of the ecliptic at its center. The pole star appears at the end of the long tail of *Ursa Minor*, the Little Bear. The circle is divided into degrees, and it indicates that Hevelius observed the pole star at an angular distance of 4° from the celestial pole, which is situated at the lowest point of the circle. This is in very close agreement with the theory of the precessional motion of the pole, as explained above.

Of peculiar interest, also, in this connection, are the theories held by Egyptologists as to the date of construction of the great pyramid. In that pyramid there is a long passage pointing due north, and elevated above the horizontal at exactly the right angle to view α Draconis when it was the pole star. There can be little doubt that this passage was purposely so built; and there is therefore little doubt left as to the approximate age of the pyramid.

There are still several other details in connection with the relation of earth and sun that we must consider here. For instance, we recall (p. 71) that astronomers use a mean solar day and a mean sun corresponding as accurately as possible to the actual performances of the real visible sun. It is now possible to make this relation between the mean sun and the real sun a little clearer.

Since the length of the mean solar day represents the average of all the actual solar days, it is evident that the mean sun must be sometimes in advance of the actual sun, and sometimes behind it. The difference between the two suns cannot continue to increase indefinitely; as a matter of fact, the extreme value of the difference is sixteen minutes.

In other words, mean solar time may be as much as sixteen minutes fast or slow of apparent solar time. The difference between the two kinds of solar time is called the Equation of Time. The equation of time at any moment is defined, then, as the quantity of time we must add to the apparent solar time at that moment, to make it equal to the mean solar time.

There are two principal causes producing the equation of time. The first has already been mentioned repeatedly. The earth does not move in its orbit with uniform velocity, but travels most rapidly near perihelion (pp. 70, 123). Consequently the sun, projected on the sky at its various apparent positions in the ecliptic circle, also appears to move other than uniformly in that circle. This, of course, puts the real sun in advance of the mean sun at times, and behind it at other times.

But even if the real sun were projected upon the ecliptic circle with uniform motion, still there would not result an equality of the actual solar days. A reference to a celestial globe, or to Fig. 6, p. 35, shows that there is a variable angle on the celestial sphere between the ecliptic circle and the celestial equator. At the equinox points, where the two circles cross, there is an angle of $23\frac{1}{2}°$ between them; but at the solstices, where the distance between the two circles is greatest, they are practically parallel for a short distance.

Therefore even uniform motion in the ecliptic would not give uniform motion when projected on the equator. But it would require uniform apparent motion of the sun on the celestial equator to produce equality of the actual solar days. For we have seen (cf. p. 69) that the sun would have to move exactly the same distance on the equator each day to make all the apparent solar days exceed the unvarying sidereal day by exactly the same amount. To repeat, then,

the two causes of the equation of time are: first, variable motion of the earth in its orbit, producing variable apparent motion of the sun in the ecliptic circle; second, variable angle between the ecliptic circle and the celestial equator.

We have already given (p. 82) a table of the equation of time for various dates in the year. It there appears as a table of errors of the sundial, because the dial keeps apparent solar time by the shadow of the actual visible sun, and must be corrected by the amount of the equation of time to make it conform to mean time.

An interesting and frequently misunderstood consequence of the equation of time is the inequality of the mornings and afternoons at certain dates in the year. Morning begins at sunrise and ends at noon. Afternoon begins at noon and ends at sunset. Now sunrise and sunset occur when the actual visible sun appears or disappears at the horizon; by convention, noon occurs when the mean sun is on the meridian. Thus the morning will be shortened and the afternoon lengthened by the amount of the equation of time, or *vice versa*. The difference on any date will be twice the equation of time on that date.

In February the afternoons are about half-an-hour longer than the mornings; in November, they are half-an-hour shorter; on account of this effect of the equation of time. Furthermore, we have seen (p. 74), when considering standard time, that the times in actual use in certain places may differ from their proper mean solar times by as much as half-an-hour. This again affects the difference between the morning and afternoon by twice its amount, or a full hour. On the dates mentioned above, it may happen that this hour is added to the half-hour arising from the equation of time; so that on certain dates and in certain places

morning and afternoon may differ as much as an hour and a half. It is easy to find an example in the ordinary almanacs. Thus for November 20, 1913, the almanac gives the standard times of sunrise and sunset at Detroit as 6.28 A.M. and 4.7 P.M. This makes the forenoon $5^h\ 32^m$ long, while the afternoon lasts only $4^h\ 7^m$. The difference is nearly an hour and a half.

There now remains but one more phenomenon of importance requiring attention in connection with our earth's annual motion around the sun. It is called the Aberration of Light, and was discovered by James Bradley, astronomer royal of England, in 1728. Bradley had been making some very precise observations of the declinations (p. 34) of certain stars, and had found that observations made six months apart could not be brought into agreement. There was a slight displacement of the stars on the sky at the end of six months; after the lapse of a whole year they were back again in their old places.

This matter puzzled Bradley greatly; for a long time he was quite unable to find any satisfactory explanation. Finally he came upon the solution of the problem while he was sailing in a small boat on the Thames. At the mast-head of the boat was a pennant; and Bradley noticed that whenever the boat changed its course in tacking, the pennant changed its direction a little with respect to the river bank. There seemed no reason for this, because the wind was quite steady in direction.

Then it occurred to him that the boat's own motion influenced the pennant. Its position would be determined by a combination of the wind's velocity and direction, together with the boat's speed and direction of motion. And he saw at once that light coming to us from a star would

seem to come in a direction similarly depending on the true direction of the star, and the light's velocity combined with the direction and velocity of our terrestrial orbital motion. The earth is here the boat; and the aberration of light was explained.

There is another familiar explanation which may make this phenomenon clearer. Imagine a person standing perfectly still in a rain storm on a windless day. The drops will seem to fall perpendicularly downward; but if the person runs rapidly, they will strike him in the face, precisely as if they were coming down in a slanting direction. The drops will seem to come towards the runner; more exactly stated, the direction from which the drops seem to come will be thrown forward in the direction of the running observer's motion.

In a similar way, the direction from which starlight seems to come is thrown toward that point on the sky toward which the terrestrial motion is for the moment aimed. We see the star a little too near that point. But the earth moves in a nearly circular orbit, and so is constantly changing the direction of its motion. Therefore the aberrational change in the stars' positions is also constantly and similarly changing its direction. The final result is to make each star seem to describe a little closed curve on the sky, which is a sort of miniature copy of the terrestrial orbit around the sun. This little aberrational curve is, of course, different for different stars, depending on their positions in the sky with respect to the earth's orbit. And the reason why these aberrational curves are so small is that the velocity of light is very large compared with the earth's linear velocity in its orbit. For if light moved instantaneously, or if the earth had no motion, there would be no aberration.

CHAPTER VIII

THE CALENDAR

PERHAPS the chief duty of astronomers has always been the orderly measurement of time; not merely short intervals such as the hour and minute, but also the much longer periods represented by months and years. For the latter purpose various calendars have been devised. The most ancient were doubtless based on the motions of the moon, and were consequently very irregular and complicated. It will not be of interest to trace their development beyond the year 45 B.C., when Julius Cæsar put in force at Rome the form of calendar which bears his name, and which had been arranged for him by the Greek astronomer Sosigenes of Alexandria.

The first thing to understand about a calendar in the modern sense is that every date, such as Wednesday, August 27, 1913, is composed of four different constituent parts: the day of the week, the day of the month, the name of the month, and the number of the year. We may then define the fundamental problem of the calendar thus: having given any three of these constituent parts of a date, to find the fourth. This is the problem we shall solve in the present chapter, both for the Julian calendar of Cæsar, and the modern Gregorian calendar, now in general use. This calendar was named after Pope Gregory XIII, by whose orders it was introduced in 1582, though it did not receive recognition in England until 1752, and is not yet used in Russia.

Our fundamental problem may present itself in several different forms. For instance, an important event in American history happened on March 4, 1865; on what day of the week did it occur? This event was the second inauguration of Abraham Lincoln as President.

This same event suggests a good illustration of another form in which our problem may present itself. Presidents of the United States are always inaugurated on March 4, at intervals of four years; and, with rare exceptions, in years following a "leap-year." In what years during the twentieth century will these inauguration dates fall on Sunday?

A third form of the problem might be as follows: an old letter, of great historic interest, happens to have its date blurred so as to be partly illegible. Suppose we can read, however, that it was written in a certain year, and on the 17th day of the month. It also appears from some remark in the letter itself that it was written on a Thursday. In what month in the year of the letter was the 17th a Thursday? Such are the problems we can solve through a proper understanding of the calendar.

The first difficulty that arises in devising a calendar comes from the odd lengths of the week and the year. We all know that there are seven days in the week, and we have learned that the year contains about $365\frac{1}{4}$ days. And it is impossible to divide $365\frac{1}{4}$ by 7 exactly, without a "remainder." Therefore the number of weeks in a year cannot be expressed as a whole number; this fact makes the year and the week "incommensurable," as it is called. The difficulty could not be avoided by changing the number of days in the week, because no whole number of days, such as 6 or 9, can be an exact divisor of $365\frac{1}{4}$.

To bring about an exact division, it would be necessary

to change either the length of the day or the length of the year. But neither of these can possibly be altered, because both are natural units of time. The day (p. 66) is the quantity of time required by the earth to make one complete rotation on its axis. This quantity of time is fixed by nature, and is therefore called a natural unit. We have also artificial or conventional time-units, such as the hour and minute. For instance, the hour is defined conventionally as one twenty-fourth part of the time required by the earth to complete one axial rotation. Being an artificial unit, it would be within our power to make the hour one twenty-fifth or one twenty-third part, and to have twenty-five or twenty-three hours in the day. This makes clear the difference between an artificial and a natural unit of measurement: one is man's creation, and subject to change by him at will; the other is fixed and unchangeable by nature.

But chronology does not concern itself with minor subdivisions of time, such as hours and minutes; and the year of chronology, like the day, is a natural unit quite beyond our control. So we must perforce deal with the year and day as we find them; our artificial chronological units are the week and month. We have just seen that nothing would be gained by changing the number of days in the week; we may add that it would be impossible practically to make such a change, even if it were desirable. Both the week and the month have acquired, from their antiquity, a species of historic changelessness which lends them a kind of permanence almost as great as that possessed by the natural units themselves.

We must next explain what is meant by the year in chronology. We have already had definitions (p. 128) of two different kinds of astronomic years. In chronology,

we use one only of these two time-periods, the tropical year. This is the interval of time between two dates when the sun, in its apparent motion around the ecliptic circle, attains its greatest angular distance from the celestial equator. When this occurs at the summer solstice (p. 93) we have the date when the sun climbs highest in the sky at noon, when shadows are shortest, when midsummer day occurs.

These facts make plain at once the reason for using the tropical year in calendar making. Suppose we have become accustomed to midsummer day occurring on June 21. It is obvious that midsummer must necessarily happen when the noon shadows are shortest, etc. Now suppose (to exaggerate) that the calendar year differed by a day from the tropical year. If one midsummer day then fell on the calendar date June 21, the next midsummer day would fall on June 22. And each midsummer day would come a day later in its turn, until, after the lapse of a century or so, we should have midsummer in December, and our calendar would be completely reversed. The one absolutely essential thing is to have the calendar year as nearly equal to the tropical year as it is possible to make it.

We have seen that the length of the tropical year can be determined easily by astronomical observations. It has been found to contain 365.2422 days; or, approximately, $365\frac{1}{4}$. Now the calendar year must of course contain a round number of days, without fractions; the most obvious way to bring this about is to use a year of 365 days, and put in a leap-year of 366 days every fourth year. This is the Julian calendar already mentioned as having been put in force under Julius Cæsar.

The error of this calendar is found easily as follows:

Julian Calendar

	1st year	365 days
	2d year	365 days
	3d year	365 days
	4th year	366 days
	Total, 4 years,	1461 days
Actual length of 4 tropical years (365.2422 × 4)		1460.9688 days
Error of Julian calendar		.0312 day in 4 years
	or	.0078 day in 1 year

The above simple calculation shows that the Julian calendar runs into error at the rate of 0.0078 day per annum. This amounts to one day in 128 years, and the Julian calendar will therefore pass out of accord with the true tropical motion of the sun at that rate.

Another simple calculation shows how to correct this error almost exactly, and this leads to our present Gregorian calendar. It is clear that the Julian method of introducing a leap-year every four years somewhat over-corrects the error that would be caused by the use of a uniform year of 365 days. We need to omit one of those leap-years every 128 years. To do this most simply, it was decided under Pope Gregory to omit a leap-year once every century for three centuries; and in every fourth century to omit no leap-year. This omits three leap-years in 400 years, or one in 133 years, instead of one in 128 years, as required. And the Gregorian rule for leap-year then becomes the following:

The year is a leap-year if the year number is divisible exactly by 4, without a remainder; except that in the case of century years like 1500, 1600, etc., the divisor must be 400 instead of 4.

Under this rule 1912 was a leap-year; 1900 was not; but 2000 will be.

Let us now calculate the error of the Gregorian calendar. In 400 years there are 100 leap-years in the Julian calendar. The exception in the Gregorian rule reduces this number of leap-years to 97. We therefore have the following calculation:

GREGORIAN CALENDAR

Number of days in 400 years (400 × 365) is	146,000
and 97 leap-year days	97
Total number of days in 400 calendar years	146,097
Number of days in 400 tropical years (365.2422 × 400)	146,096.88
Error of Gregorian calendar in 400 years	.12

This makes the Gregorian error in one year only .0003 day; so that 3333 years will pass before there is an accumulated total error of a single day. This is an entirely negligible quantity, and so the Gregorian calendar may be regarded as perfectly satisfactory for all practical purposes.

Having thus explained the construction of the calendar, the next step is to show how to calculate the week-day corresponding to any date in the past or future. Let us, for convenience, attach to the seven days of the week seven numbers, thus:

Sunday, 1,
Monday, 2,
Tuesday, 3,
Wednesday, 4,
Thursday, 5,
Friday, 6,
Saturday, 7.

Let us also designate as the "century number" the first two digits of the year number. Thus, in 1913, 19 is the

century number and 1913 is the year number. Then we have the following:[1]

Rule for Finding the Week-day

1. Divide the century number by 4 and 7, and call the remainders resulting from the division the first and second remainders.

2. Divide the year number by 4 and 7, and call the remainders the third and fourth remainders.

3. Add five times the first remainder to the second remainder, and call the sum the "constant."

4. Add the following five numbers; viz.: the constant; five times the third remainder; three times the fourth remainder; the day of the month; and the following number depending on the name of the month:

in Jan.;	6, in ordinary years; 5, in leap-years;
" Feb.;	2, in ordinary years; 1, in leap-years;
" March;	2, in all years;
" April;	5, " " "
" May;	0, " " "
" June;	3, " " "
" July;	5, " " "
" Aug.;	1, " " "
" Sept.;	4, " " "
" Oct.;	6, " " "
" Nov.;	2, " " "
" Dec.;	4, " " "

And call the sum of the five numbers thus added the "sum."

5. Divide this sum by 7, and call the remainder the fifth remainder.

Then this fifth remainder, when increased by unity, will be the week-day number required.

[1] For a demonstration of this rule, see Note 15, Appendix.

As an example, let us find the week-day corresponding to July 4, 1913. We have:

1. 19 divided by 4 gives first remainder, 3;
   19 divided by 7 gives second remainder, 5.
2. 1913 divided by 4 gives third remainder, 1;
   1913 divided by 7 gives fourth remainder, 2.

3. Five times first remainder,	15,
second remainder,	5,
The constant is	20.
4. The constant,	20,
Five times the third remainder,	5,
Three times the fourth remainder,	6,
Day of month, July 4,	4,
The month number for July,	5,
The sum is	40.
5. 40 divided by 7 gives fifth remainder,	5.

The fifth remainder increased by unity gives the week-day number as 6, corresponding to Friday. Therefore July 4, 1913, is a Friday.

The above rule applies to the Gregorian calendar; but we may use it in the Julian calendar also if we simply omit the first and second remainders, and for the constant always use 0.

The foregoing method of calculation may be replaced by a device called a Perpetual Calendar, by means of which all calendar problems may be solved with ease. The accompanying form of perpetual calendar was arranged by Captain John Herschel; it is convenient in use, and may be extended easily, indefinitely in either direction, backwards from 1860, or forwards from 1995, for which limiting dates it is here given. The months *January* and *February* appear twice in it; the *italicized January* and *February* to be used in leap-years, which are also italicized in the columns of year numbers. In ordinary years the unitalicized January

and February are to be used. The calendar is Gregorian. The following examples will illustrate the use of this perpetual calendar in finding the fourth constituent part of a date for which three parts are given.

1. What day of the week is July 4, 1913? Opposite 4, under Day of the Month, and in the column headed July, we find the letter *F*. We then find 1913 in the third vertical column of year numbers. Running up this column to the letter *F*, and thence turning to the right, we find Friday for the day of the week. This agrees with our former calculation by the rule.

2. In what years following leap-years does inauguration day, March 4, fall on a Sunday? Opposite 4, and under March, we find the letter *B*. Opposite Sunday, *B* occurs in the first column. Consequently, March 4 is Sunday in 1860, 1866, 1877, 1883, etc. But the only years in this column that follow leap-years are 1877, 1917, 1945, and 1973. In these years, therefore, inauguration day falls on Sunday.

Having thus explained the civil calendar in ordinary use, we shall next, to complete the subject, describe the Ecclesiastical Calendar as briefly as possible. The fundamental problem of this calendar is to find the date of Easter Sunday in any given year. Following Gauss,[1] we shall divest the subject of all non-essential details; and especially exclude the ancient terminology, which tends to involve this somewhat complicated problem in unnecessary obscurity.

Fundamental data are to be found in regulations adopted by the famous Ecclesiastical Council of Nice, which met in the year 325 A.D. According to decree of that council, Easter Sunday is the first Sunday that follows the first

[1] Gauss, *Berechnung des Osterfestes; v.* Zach's *Monatliche Correspondenz*, August 1800.

Perpetual Calendar

Day of the Month	Jan. Oct.	Apr. July <u>*Jan.*</u>	Sept. Dec.	June	Feb. Mar. Nov.	Aug. <u>*Feb.*</u>	May	
1 8 15 22 29	A	B	C	D	E	F	G	Monday
2 9 16 23 30	G	A	B	C	D	E	F	Tuesday
3 10 17 24 31	F	G	A	B	C	D	E	Wednesday
4 11 18 25	E	F	G	A	B	C	D	Thursday
5 12 19 26	D	E	F	G	A	B	C	Friday
6 13 20 27	C	D	E	F	G	A	B	Saturday
7 14 21 28	B	C	D	E	F	G	A	Sunday
	1860	1861	1862	1863		*1864*	1865	
	1866	1867		*1868*	1869	1870	1871	
		1872	1873	1874	1875		*1876*	
	1877	1878	1879		*1880*	1881	1882	
	1883		*1884*	1885	1886	1887		
	1888	1889	1890	1891		*1892*	1893	
	1894	1895		*1896*	1897	1898	1899	
	1900	1901	1902	1903		*1904*	1905	
	1906	1907		*1908*	1909	1910	1911	
		1912	1913	1914	1915		*1916*	
	1917	1918	1919		*1920*	1921	1922	
	1923		*1924*	1925	1926	1927		
	1928	1929	1930	1931		*1932*	1933	
	1934	1935		*1936*	1937	1938	1939	
		1940	1941	1942	1943		*1944*	
	1945	1946	1947		*1948*	1949	1950	
	1951		*1952*	1953	1954	1955		
	1956	1957	1958	1959		*1960*	1961	
	1962	1963		*1964*	1965	1966	1967	
		1968	1969	1970	1971		*1972*	
	1973	1974	1975		*1976*	1977	1978	
	1979		*1980*	1981	1982	1983		
	1984	1985	1986	1987		*1988*	1989	
	1990	1991		*1992*	1993	1994	1995	

What day of the week was March 4, 1865?

Under March, opposite 4 is the letter *B*. In the 1865 column, opposite B, is Saturday.

Note. In leap-years, use *Jan.* and *Feb.* that are *underlined.*

full moon occurring on or after March 21, the date of the vernal equinox (p. 72). And it was further ordered that the day of the ecclesiastical full moon shall be the fourteenth day after new moon, or the day when the so-called "moon's age" is 14 days. Our problem is to calculate the date of Easter Sunday in accordance with this regulation.

Gauss' rule[1] is approximately as follows, some of the successive operations being identical with those already used for the ordinary calendar:

1. Divide the century number by 4 and 7, and call the remainders resulting from the division the first and second remainders.

2. Divide the year number by 4 and 7, and call the remainders the third and fourth remainders.

3. Divide the year number by 19, and call the remainder the sixth remainder.

4. Add five times the first remainder to the second remainder, and call the sum the "constant."

5. Add eight times the century number to the number 13; divide the sum by 25, and call the remainder the eighth remainder.

6. Subtract the first remainder from the century number, divide the difference by 4, and call the quotient (which will be a whole number) the result of operation 6.

7. Add eight times the century number to the number 13, deduct from the sum the eighth remainder, divide what is left by 25, and call the quotient (which will be a whole number) the result of operation 7.

8. Add the century number to the number 15, deduct from the sum the results of operations 6 and 7, and call what is left the result of operation 8.

[1] For a demonstration, see Note 16, Appendix.

9. Add the result of operation 8 to 19 times the sixth remainder, divide by 30, and call the remainder resulting from the division the seventh remainder.

10. Add five times the third remainder, three times the fourth remainder, the constant, the number 2, and the seventh remainder; divide by 7, and call the resulting remainder the ninth remainder.

11. Subtract the ninth remainder from the seventh remainder, and increase the difference by 28. The result will be the date of Easter Sunday in March. But if this date is greater than 31, Easter Sunday will fall in April, and its date in April will be found by subtracting the ninth remainder from the seventh, as before, and diminishing the difference by 3.

The above rule applies to the Gregorian calendar. In the Julian calendar the same rule may be used; but the constant is then always 0, and the result of operation 8 always 15. Furthermore, two exceptions to the above rule exist in the Gregorian calendar:

1. When Easter Sunday comes on April 26 by the rule, April 19 must be substituted for April 26.

2. Take eleven times the result of operation 8, and increase the product by the number 11. Divide the sum by 30. If the remainder resulting from this division is less than 19, and if at the same time the seventh remainder was 28 and the ninth remainder 0, the rule will give April 25 for the date of Easter Sunday. When these conditions all occur, substitute April 18 for April 25.

As an example, let us calculate the date of Easter Sunday in 1913. We have:

1. Century number 19 divided by 4, gives first remainder 3,
   Century number 19 divided by 7, gives second remainder 5.
2. Year number 1913 divided by 4, gives third remainder 1,
   Year number 1913 divided by 7, gives fourth remainder 2.
3. Year number 1913 divided by 19, gives sixth remainder 13.
4. The constant is 20.
5. The eighth remainder is 15.
6. Result of operation 6 is 4.
7. Result of operation 7 is 6.
8. Result of operation 8 is 24.
9. The seventh remainder is 1.
10. The ninth remainder is 6.
11. Easter Sunday is March (1–6 + 28), or March 23.

CHAPTER IX

NAVIGATION

The guiding of a ship across the unmarked trackless ocean is strictly a problem of astronomy: among the many problems of the science it is the most important commercially; certainly there is no other so astonishing to those who do not understand the simple methods employed for its solution. Briefly stated, the astronomic problem of navigation consists in ascertaining a ship's latitude and longitude by observing the heavenly bodies. If this can be done, we know the exact position of the ship on the earth's surface; and knowing also the latitude and longitude of the port to which the ship is bound, it becomes an easy matter to calculate, or to measure on a chart, whether the ship must be steered north, east, south, or west, in order to reach its destination.

Inasmuch as ocean currents, leeway, or other causes may produce unperceived deflections as a ship moves through the water, it is necessary and customary for navigators to determine their position astronomically at frequent intervals; once each day, if possible. These successive astronomical observations furnish a continuous check upon the running of the ship. Each new observation gives a new "departure," as it is called; and helps to assure a correct "land-fall" at the end of the voyage, as sailors say. When clouds prevent satisfactory observations of the sky, the ship must be run by "dead reckoning"; a very expressive

term which indicates how little confidence mariners have in their compass, as compared with observations of the unerring heavens.

Our concern is with the purely astronomic question of navigation; with seamanship, knotting and splicing, charts and compass, leadline and sounding machine, we have nothing to do. And the astronomic problem is a very simple one, for both the latitude and longitude of the ship may be calculated if we measure with a suitable instrument, and in a suitable way, the angular elevation or altitude (p. 36) of the sun above the visible sea-horizon.

The instrument used for this purpose at sea is called a Sextant; it may be defined as a portable instrument for measuring the angular altitude of the sun above the horizon. Figure 38 will enable the reader to form an idea of its appearance, and to understand its principle. The essential parts are two small silvered mirrors, *M* and *m*; a telescope, *EK*; and a circle, *AA*, engraved with "graduations" as they are called, by means of which angles may be measured upon it in degrees, minutes, and seconds. The mirror *m* and the telescope *EK* are firmly attached to the sextant; but the mirror *M* is pivoted in such a way that it can be turned, and the angle through which it is turned measured on the circle by means of the index *CB*. When the mirror *M* is turned back until it is parallel to the mirror *m*, the circle reads 0°, because the angle between the two mirrors is then 0°. In all other positions the circle measures the angle between the two mirrors. *P* and *Q* are sets of colored glasses, which can be interposed temporarily, when the sun's rays are so brilliant as to be hurtful to the observer's eye. *R* is a small magnifying glass, pivoted at *S*, intended to facilitate the examination of the index *CB*. At *C* and *B*

are shown the "clamp" by means of which the index can be fastened to the circle, and the "tangent-screw," which will adjust it delicately, after it has been "clamped." *I* and *F* are accessories for the telescope.

The mirror *m* has an important peculiarity. The silvering is scraped away at the back of the mirror from one-half its surface. Thus only one-half reflects; the other half is

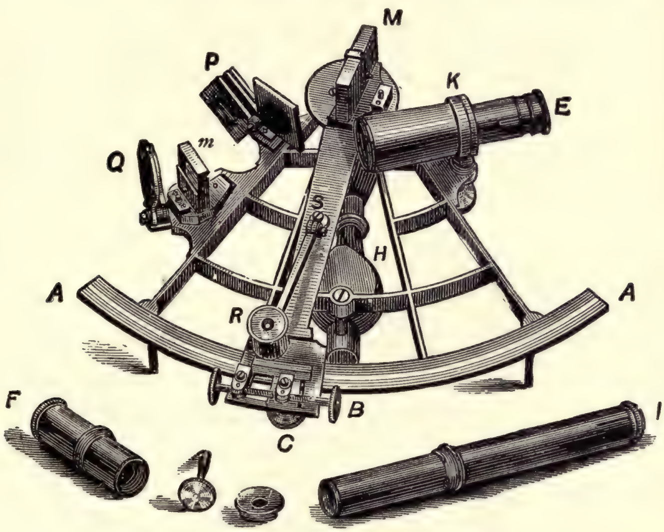

Fig. 38. The Sextant.
(From Bowditch's *Navigator*, 1912 ed., p. 66, Bureau of Navigation, U. S. Navy.)

simply transparent glass. A navigator looking into the telescope at *E* will therefore look *through* the mirror with half his telescope, and with the other half he will look *into* the mirror.

Now it is a fact that half a telescope acts just like a whole one, always. If a person using an ordinary "spy-glass" covers half of the big end with his hand, he will see the same view he saw with the whole glass. Only, as half the light-

gathering power is cut off, this view will be fainter or dimmer, — less luminous. Applying this fact to the sextant telescope, it is clear that the observer will see two things at once with the telescope: he will see what is visible *through* the mirror *m* with half the telescope; and with the other half he will see what is visible by reflection *from* the mirror *m*.

If he holds the sextant in such a position that the telescope is horizontal, he will see the visible sea-horizon with half the telescope through the mirror. If the other mirror *M* is then turned to the proper position, and the sextant held in the hand with its telescope still horizontal, and its circle vertical, it is possible to see the sun at the same time with the other half of the telescope, the solar rays having been reflected from *both* mirrors. To make this possible, the horizontal telescope must, of course, be aimed at that point of the sea-horizon which is directly under the sun. The solar rays will then strike the mirror *M* first; be thence reflected to the silvered part of the mirror *m*; and finally into the telescope. So the observation consists in so adjusting or turning the mirror *M*, that the sun and the horizon can be seen coincidently in the telescope.

The angle between the mirrors can then be measured on the circle; and it is easy to prove[1] that the angular altitude of the sun will be twice the angle between the two mirrors. Thus the sextant becomes an instrument for measuring the sun's altitude; it remains to explain how a knowledge of that altitude will furnish us with the ship's latitude and longitude.

To obtain the ship's latitude, it is best to measure the solar altitude when the sun is on the meridian, at apparent

[1] Note 17, Appendix.

solar noon. Omitting certain very small corrections, the sun will then have its greatest altitude for the day; so that the navigator need only begin measuring altitudes a few minutes before noon, and continue as long as the altitude is increasing. The moment it begins to diminish, he stops and "reads" his sextant circle; thus obtaining the meridian altitude of the sun. The accompanying Fig. 39 shows how the latitude is obtained from such an observation. O is the observer on the ship. The semicircle $H'PSEH$ is that half of the celestial meridian (p. 36) which is above the horizon. H and H' are the south and north points of the horizon, where it is intersected by the celestial meridian. P is the north celestial pole (p. 31), and S the sun as observed on the celestial meridian. E, 90° from the pole, is a point on the celestial equator, where it crosses the meridian. The angle SOH, or the arc SH, is then the observed altitude of the sun; and the arc SE is the declination (p. 34) of the sun. This declination is always known; it can be calculated in advance, because we know the annual orbit of the earth around the sun and the point of the ecliptic circle (p. 27) at which the sun appears on the date when the observation was made. In fact, the navigator always has at hand a copy of the *Nautical Almanac,* which is a book published annually by the United States government, in which the sun's declination is printed for every day in the year.

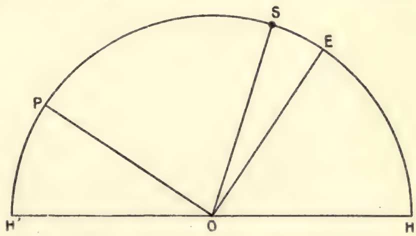

FIG. 39. Latitude from Observation.

The navigator then simply subtracts the known declination SE from the observed altitude SH, and thus obtains

the arc EH, or the altitude of the celestial equator above the south point of the horizon. As soon as EH becomes known, it is easy to obtain the latitude. For the arc PE is also known; it is always 90°, because it is the angular distance from the equator to the pole. Therefore we need merely subtract PE and EH from 180°, to get PH', the angular altitude of the celestial pole above the horizon. But this (p. 40) is always equal to the latitude; and so the latitude of the ship becomes known from the sextant measurement of altitude.

In making observations of this kind, it is necessary to apply certain corrections to the observed altitude, of which the two most important are the correction for refraction, and the correction for semi-diameter. The former is due to the bending of the sun's light as it comes down to us through the terrestrial atmosphere (p. 114). The amount of this bending can be found in refraction tables which are printed in all books on navigation; the navigator merely subtracts it from the altitude as actually observed.

The other correction for semi-diameter is due to the fact that we cannot measure the altitude of the sun's center because the sun appears in the sextant telescope as a round disk, and it is impossible to estimate the position of its center accurately. Therefore navigators always measure from the lowest or highest point of the disk: in either case, the angular semi-diameter must be added to or subtracted from the observed altitude to get the altitude of the sun's center. This semi-diameter varies a little through the year because, as we know, the flattening of the earth's orbit around the sun (p. 118) alternately increases and diminishes the distance of the earth from the sun. But the exact value of the semi-diameter is printed for each day in the

nautical almanac, whence the navigator obtains it, together with the sun's declination.

To ascertain the ship's longitude, a somewhat different process is employed. In principle it depends upon the time-differences which, as we have seen, exist between different places on the earth (p. 72). Strictly speaking, longitudes are longitude differences. The longitude of New York, for instance, is really the longitude difference of New York from Greenwich. And the time difference between New York and Greenwich corresponds exactly to the longitude difference, one hour of time corresponding to each fifteen degrees of longitude.

The navigator takes advantage of these facts in a very simple way. He carries in the ship one or more marine chronometers. These are merely very large watches, accurately made, and mounted in boxes with swinging supports, so as to prevent the ship's rolling from influencing the exact running of the instrument. Before leaving port, these ship's chronometers are "rated" accurately, by comparing them on successive days with standard telegraphic time signals from some astronomical observatory.

By rating a chronometer we do not mean merely ascertaining its error, or the number of minutes and seconds it may be fast or slow on a given date. Rating includes also the determination of the fraction of a second by which the chronometer increases or diminishes its error on each succeeding day. For instance, if a chronometer is found to have the following error and rate:

April 15, 1913, chronometer fast 28.0 seconds, and gaining 0.3 daily;

then on April 30, 1913, fifteen days later, the chronometer would be fast $28.0 + 15 \times 0.3$ seconds, or 32.5 seconds.

Knowing the error and rate, the navigator can always obtain correct time from his chronometers, within the limits of accuracy with which they can be made to maintain a constant or unvarying rate.

Marine chronometers are always set to Greenwich time; so that when a navigator takes the time from the chronometer, allowing for its rate, it is always Greenwich time. Now suitable sextant observations enable him to determine also the correct local mean solar time of the ship; this having been done, a simple comparison with the Greenwich time of the chronometer furnishes the time difference between the ship and Greenwich, and therefore also the longitude difference, or "longitude of the ship."

It remains to explain how the ship's local time may be ascertained by observation with the sextant. This is accomplished by observing the altitude of the sun, just as we have explained in the case of latitude determinations; only, while latitude observations are made at noon, time or longitude observations must be made rather early in the morning, or late in the afternoon.

It is quite obvious that a measurement of the altitude or angular elevation of the sun in the sky must make the time of day known. For the altitude is zero at sunrise, and greatest at noon; consequently, if we know the altitude, we must be able to calculate[1] how far the sun has proceeded from sunrise toward noon in its apparent diurnal rotation across the sky. The calculation involves the use of spherical trigonometry, and cannot be explained in detail here; but enough has been said to show that such a calculation is possible.

The methods given here for navigating a ship are the

[1] Note 18, Appendix. p 394.

simplest and most easily understood. Many other methods, or modifications of the above methods, have been devised, and may be found in any standard book on navigation.

Older methods are perhaps of minor interest, but the reader will surely wish to know how ships were navigated before the days of chronometers. The first chronometer capable of keeping reasonably accurate time at sea was not made until 1736, although it was in 1675 that Charles II issued his royal warrant establishing the office of astronomer royal, and making it the duty of that official to "apply himself with the most exact care and diligence to the rectifying of the tables of the motions of the heavens and the places of the fixed stars, in order to find out the so much desired longitude at sea, for the perfecting the art of navigation."

Without the chronometer the navigator could still obtain his local time, but he had no Greenwich time with which to compare it. But his latitude from a noon observation was always available, since comparatively rude instruments for measuring altitudes existed for centuries before the invention of the sextant. Thus, in the early days, the navigator was forced to find his way with latitudes only. For instance, in a voyage from England to Rio, the ship would be steered southward and westward, more or less, until the "noon-sights" showed that the latitude of Rio had been reached. It was then merely necessary to steer due west, along the latitude parallel of Rio, and checking the latitude of the ship by daily noon-sights; the lookout man forward would notify the navigator when he "raised the land." But with no knowledge of longitude, and in a sailing ship, the navigator might be uncertain by many weeks as to the date when he would reach the port of Rio.

CHAPTER X

MOONSHINE

The moon, more than any other celestial body, is in a very peculiar sense our own, for it is a satellite of the earth, revolving around us, and accompanying our annual orbital journey about the sun. Earth and moon together follow the same path, completing each year a full circuit around the sun. And the moon is important, too, in the history of astronomy: upon its peculiarly intricate motions; its connection with eclipses; its lifting of the great waters of ocean in tidal ebb and flow, — upon a due explanation of all these things men have exercised their highest powers from the very beginning of the science.

The moon is not self-luminous like the sun, but shines only by receiving light from the sun, and reflecting it to the earth (cf. p. 16). Its orbit around the earth, like most orbits, is a slightly flattened oval or ellipse, with the earth situated at the focus (cf. p. 116), a little to one side of the center. The orbital plane can be imagined extended outward indefinitely, so as to cut out a great circle on the celestial sphere, similar to the ecliptic circle (p. 27). Somewhere in this great circle the moon will always be seen projected on the sky. The plane of the lunar orbit is inclined to the ecliptic plane by a small angle, about 5°; so that this is also the angle between the ecliptic circle on the celestial sphere and the great circle belonging to the lunar orbit.[1]

[1] We learn in Spherical Geometry that the angle between any two great circles drawn upon a sphere is equal to the angle between the two planes in which the circles are situated.

As the moon travels around the earth in its orbit, we see it projected on the sky, and apparently progressing around its orbital great circle, just as the sun appears to travel around the ecliptic circle. No wonder the ancients were puzzled when they saw both sun and moon alike moving around the sky in their respective great circles. Of course they thought both bodies were alike revolving around the earth, and concluded that the earth must be the immobile center of all things. But we now know that the moon appears to progress around the sky because it is really moving around the earth, and we see this real motion projected on the sky. The sun, on the other hand, only seems to travel around the sky; it is really stationary, and its motion is an apparent one, due to the real motion of the earth in its own annual orbit (p. 116). But to the eye both sun and moon alike seem to circle the sky.

The angular velocity of lunar motion in the moon's projected great circle is far greater than that of the sun in its ecliptic circle. Both bodies appear to move in the same direction, from west to east; but while the solar apparent revolution takes about a year, and therefore averages about 1° daily, the moon completes a circuit from any fixed star back to the same star again in about 27¼ days, corresponding to an average daily angular motion of about 13°. This period of 27¼ days, from star to star, is called the lunar Sidereal Period, and corresponds to the sidereal year (p. 128) of terrestrial orbital motion around the sun. But the moon has also another period, called the Synodic Period. To understand it, we must remember that as the moon's angular motion among the stars is about thirteen times as rapid as the sun's apparent angular motion, the moon must be constantly overtaking and passing the sun, much as the

minute hand of a watch is constantly overtaking the hour hand. The synodic period is defined as the interval of time between two such successive overtakings of the sun by the moon. It is about $29\frac{1}{2}$ days long, or about $2\frac{1}{4}$ days longer than the sidereal period.

To explain this in a different way, suppose the moon and the sun are to-day both projected on the sky near a certain fixed star. Then, $27\frac{1}{4}$ days later, the moon will have circled the sky completely, and will be back near the same star. During the $27\frac{1}{4}$ days, however, the sun will have moved apparently some $27\frac{1}{4}°$ eastward, because of its apparent motion in the ecliptic circle. Therefore, to rejoin the sun, the moon will still need to travel those $27\frac{1}{4}°$; and this, at the rate of about 13° daily, will require approximately $2\frac{1}{4}$ days. So the synodic period is again seen to be $29\frac{1}{2}$ days long.

Probably the first astronomical phenomenon ever observed by man was the "waxing and waning" of the moon; its change in shape from a thin crescent, gradual, night after night, to the "half-moon" of Plate 3, p. 17; and finally its increase to the brilliant circular orb we call the full-moon. The accompanying Plate 6 is a photograph of the moon, nearly full; and the small additional picture is the crescent moon. The dim visibility of the remaining lunar surface within the crescent is explained on p. 164.

What are these "phases" of the moon, and what is their cause? We have just seen that the moon is not self-luminous, but shines by reflected sunlight. If the moon were incandescent, like the sun, we should see it always as a full-moon, or complete luminous circle. But it is a globe, and so only one-half its surface can be illuminated by the sun at any given moment. Now if the earth happens to be so

Photo by Slocum.

Photo by Barnard.

PLATE 6. Full Moon and Crescent Moon.

placed that we can see the entire illuminated hemisphere, full-moon occurs. If the earth is so situated that we see only the unlighted hemisphere, the moon is wholly invisible, and we say it is "new-moon."

Evidently, we shall see the illuminated hemisphere when we are on that side of the moon which faces the sun and receives light; when we are on the side of the moon opposite the sun, we see the dark part. And as the moon goes completely around the earth with respect to the sun in $29\frac{1}{2}$ days, it must happen once in each such period that we are suitably placed for each of these phenomena. And, of course, at intermediate dates, we must be so placed as to see larger or smaller portions of the illuminated part, giving rise to the other visible phases. This is the simple explanation first found by Aristotle.

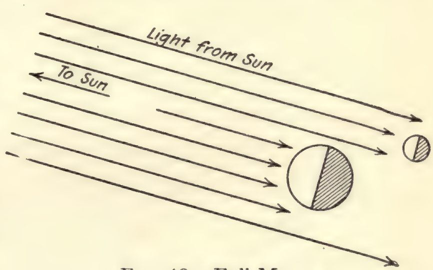

Fig. 40. Full Moon.

It follows from the above, as shown in Fig. 40, that full-moon must always occur when the sun and moon are seen projected at nearly opposite parts of the celestial sphere. The figure shows how light from the sun illumines half of both earth and moon. To the inhabitants of the dark side of the earth, the sun is not visible, and it is night. But those same inhabitants evidently see the bright half of the moon, in the full-moon phase. Under these circumstances, the figure shows that the directions of the sun and full-moon, as seen from the earth, point toward opposite sides of the sky, approximately.

It may be remarked, also, that if the sun, earth, and moon

were always in a single plane, the earth, at the time of full-moon, would be exactly in line between the moon and sun. It would then cut off the solar light from the moon and give rise to the phenomenon called an Eclipse of the Moon. But we have already seen that the two orbit planes are not identical; that there is an angle of 5° between them. It is this angle between the planes that prevents the occurrence of an eclipse during every $29\frac{1}{2}$-day period of lunar orbital motion, as will be more fully explained in a later chapter. Finally, Fig. 40 shows that the interval between two successive full-moons or two successive new-moons is the synodic period of $29\frac{1}{2}$ days, not the sidereal period of $27\frac{1}{4}$ days. For these phases must recur when the moon has made a complete revolution around the earth, measured by the sun, not by a star.

Closely connected with lunar phases is the phenomenon called the "earth-shine," or "the old moon in the new-moon's arms," shown in Plate 6, p. 162, small photograph. It often happens that when the first slender lunar crescent is seen, a few days after the date of new-moon, the dark part of the moon, within the horns of the crescent, will be illuminated faintly. This illumination of the dark part cannot come directly from the sun, under our accepted theory of lunar phases; nor can it be light from the moon itself, for we know the moon to be non-luminous. But it is explained easily if we once more examine Fig. 40, p. 163. This figure makes clear that when we see the moon in the full-moon phase, the earth turns its dark side toward the moon. As seen from the moon, the earth is in the "new-earth" phase.

All the earth phases, as seen from the moon, are opposite to the lunar phases, as seen from the earth. Thus, when

we see the moon nearly new, as a slender crescent, the earth is nearly a full-earth to the moon. And the slight illumination of the dark part of the moon, as we see it, is then due to the strong light thrown upon it by the brilliant full-earth, doubtless several times more luminous than the full-moon seems to us.

The small photograph of Plate 6, p. 162, also gives a good opportunity to notice that the "horns" of the moon always appear to be turned directly away from the sun, as they are seen by us projected on the sky. This follows from the explanation of phases: we can understand it easily, if we paint a ball half black and half white, to represent the moon, with half its surface illuminated by the sun. If we now hold this ball so as to see only a narrow sickle of the white half, we shall always find the horns of that sickle turned to the right, if the white half of the ball, which faces the sun, is turned to the left.

Now the small photograph of Plate 6 was made by Barnard, at the Yerkes Observatory near Chicago, Feb. 14, 1907, about one hour and twenty minutes after sunset, when the moon was very near the western horizon, where the sun had set. So, in Plate 6, if we imagine a line drawn between the two ends of the moon's horns, and a second line perpendicular to it, and passing downward in the Plate, this second line, if drawn far enough below the horizon, would pass through the sun on the sky.

The small photograph, therefore, appears on Plate 6 just as it appeared to the eye when Barnard photographed it. The large photograph was taken with a different instrument at a different time, but it has been purposely turned around in Plate 6 to agree with the small photograph. This agreement may be verified readily by comparing the configuration

of markings on the two pictures. The photograph of Plate 3, p. 17, shows the moon as it would be seen on the meridian with an astronomical telescope; to make the large photograph of Plate 6 agree with it, it would be necessary to turn Plate 6 around through more than a right angle in the direction in which the hands of a watch move. The configuration of markings would then again be in agreement.

Having now explained briefly some of the lunar phenomena of phases and motions, let us next consider a peculiarity in which the moon differs absolutely from the earth. Astronomers have ascertained quite definitely that lunar air or atmosphere is altogether absent; or, if present, exists only in an extremely attenuated form. The principal observational proof of the non-existence of atmosphere is derived from phenomena known as "occultations" of stars. We have seen that the moon, as it moves in its orbit around the earth, travels among the stars about 13° daily (p. 161). But the stars are very much more distant than the moon, though we see both stars and moon alike projected on the background of the celestial sphere.

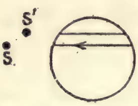

Fig. 41. Occultations.

Therefore it must happen occasionally that the moon passes between us and some individual star. In such a case that star is, of course, concealed from our view temporarily. Usually such "occultations" last about an hour, the duration varying according to the part of the moon the stars happen to meet. In Fig. 41, the moon moves across the sky in the direction of the arrow. The star S will therefore be occulted longer than the star S', because it meets a wider part of the disk of the moon.[1]

[1] The two lines shown in the figure, along which the two stars are about to be occulted, are called "chords" of the moon's disk.

Now we find from telescopic observation that no matter where the occultation takes place, the disappearance of the star is always perfectly instantaneous; there is no gradual fading away; it is blotted out with very striking suddenness while still in full brilliancy. If there were a lunar atmosphere, we would surely see a progressive dimming of the star, particularly as it passed from the outer less dense layers of lunar air into the denser layers near the surface.

Knowing, then, that the moon has no atmosphere, we must inquire what has become of it. For we now accept the plausible theory that the moon was once part of the earth, and that it was separated from the parent planet as a result of a continued and peculiar action of gravitational forces. In that case the moon must have taken some atmosphere with it when it left the earth. What has become of that atmosphere?

The most plausible explanation of its loss is derived from the kinetic theory of gases. According to that theory, the molecules of a gas are in constant violent motion, and continually colliding with each other. If this was true on the outer confines of the moon's original gaseous atmosphere, it must have frequently happened that an outer molecule, after collision, bounced off in a direction away from the moon. It then encountered no other molecule, and was prevented from escaping into space by nothing but the moon's gravitational attraction. That is the only force to hold it.

But the moon's gravitational attraction is comparatively slight, as compared with the earth's; for, as we shall see later, the mass of the moon is only about $\frac{1}{81}$ part of the earth's mass; and gravitational attraction varies proportionally to the mass of the attracting body. Therefore it

is quite conceivable that the moon may have lost its atmosphere by the kinetic method, while the earth, by reason of superior gravitational attraction, is able to retain it. However this may be, physicists are now agreed that there is ample molecular velocity to carry gases gradually away from the moon.

Absence of atmosphere means also absence of water; for water, if present, would evaporate and form an atmosphere. And without air and water, there can be no lunar inhabitants similar to ourselves.

The shape of the lunar orbit around the earth, to which we have already referred, might be ascertained observationally in the manner already explained for the earth's orbit around the sun (p. 117). It would merely be necessary to measure frequently the lunar angular diameter, and the moon's exact place as projected on the sky with reference to the celestial equator; in other words, the moon's declination and right-ascension. This would enable us again to draw the outline of an orbit similar geometrically to the moon's actual orbit. But as in the case of the earth's path, observations of this kind give us no notion as to the actual size of the orbit in miles. To know this we must measure a linear distance somewhere, just as we found when describing the similar state of affairs in connection with the earth's path around the sun.

We shall therefore next outline the method by which this may be done in the case of the moon. The easiest way is to observe the moon's position, as projected on the sky, simultaneously from two observatories widely separated on the earth. We can then use the known distance between the two observatories as a "base-line" for calculating the moon's distance. Nor is it difficult to show that

such calculations will make this distance known. It is found to be about 240,000 miles.[1]

It is interesting to note that we have now for the first time outlined a method of finding by observation the actual distance separating a heavenly body from the earth. We now see that astronomy can make measurements other than mere angular diameters and angular distances. Its grasp extends outward into space; by indirect methods, but methods perfectly valid, man has learned the distance of the moon just as though he could go there and measure it with a surveyor's tape-line.

Closely related to the method of ascertaining the moon's distance is the mysterious word "parallax." The moon's parallax is defined as the angular semi-diameter or radius of the earth, as seen from the moon. Thus, in Fig. 42, AC is the earth's radius; M is the moon; and the small angle at M is the lunar parallax.[2]

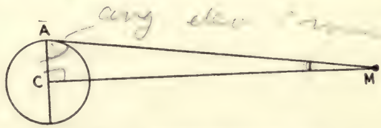

FIG. 42. Parallax of the Moon.

The moon's distance (240,000 miles, in round numbers) is about 60 times the earth's radius. But of course the flattening of the lunar orbit makes the distance vary, just as we found was the case with the earth and sun, when we discussed the terrestrial seasons. Just as the earth has a perihelion, or nearest approach to the sun (p. 120), so the moon has a "perigee," or nearest approach to the earth. And the lunar orbit is more flattened than that of the earth; the actual distance of the moon may vary all the way from 222,000 to 253,000 miles.

The axial rotation of the moon is a subject often found

[1] Note 19, Appendix. [2] Note 20, Appendix.

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puzzling, though really very simple. Here the crucial fact is derived from the most elementary telescopic observation of the moon. We find that the moon always turns approximately the same hemisphere toward the earth. Whenever we look at the moon, we see the same configuration of surface details, lunar mountain ranges, etc.; we never see the mountain ranges on the moon's opposite side.

There can be but one reasonable explanation of this. The moon must have an axial rotation just rapid enough to produce this peculiar result. And here is the puzzle: many persons ask how the moon can have any axial rotation at all, if it constantly turns the same face toward us. The matter will be understood most easily by means of a simple experiment. Let the reader face a table in the middle of a room. Let him imagine himself to be the moon, and the table to be the earth. Let him now walk around the table in such a way that he faces it constantly. When he has gone halfway around the table, always facing it, he will find that he is looking at that wall of the room toward which his back was turned when he began the experiment.

Thus he must have turned himself halfway around, while constantly facing the table. If his face was turned toward the north when he began, it is now turned toward the south. And if he completes a circuit of the table in the same way, returning finally to his original position, he will find that he has faced successively every point of the compass. This proves that he has turned himself around, or rotated once on his vertical axis; yet, representing the moon, he has at all times turned his face toward the table, representing the earth.

Accurately stated, the case stands thus: the moon makes an axial rotation in exactly the same time it takes to make an

orbital revolution around the earth. We have seen that it revolves in its orbit in $27\frac{1}{4}$ days; it also finishes a rotation on its axis in $27\frac{1}{4}$ days. This is the whole explanation of the mystery.

To complete this matter of the moon's rotation, we must now point out that the explanation, so far given, is not quite exact, though it is very nearly so, and quite sufficiently so for a first approximation. There is a phenomenon called Libration of the moon, which makes it possible for us to see a somewhat different part of the lunar surface at certain times. The lunar rotation axis is slightly inclined from perpendicularity to the plane in which is situated the orbit of the moon around the earth. The inclination is small, about $6\frac{1}{2}°$; but it has the effect of tilting the moon, as it were, $6\frac{1}{2}°$, first one way and then the opposite way, according to its position in its orbit around the earth. For the lunar rotation axis remains constantly parallel to its original direction in space during the entire orbital revolution. On account of this tilting effect, we see a slightly different hemisphere of the moon at different dates in each lunar period.

Still another libration of the moon exists. It is true that the moon rotates on its axis in the same period of time it requires for its orbital revolution around the earth; but while the axial rotation is uniform, the orbital motion, of course, is variable. As in all orbital motion, the velocity is greatest when the moon is in that part of its orbit which lies nearest the earth. Consequently, the axial rotation and the orbital revolution do not increase at precisely the same rate; and from this cause, also, we see a slightly different hemisphere of the moon at different dates in the month.

These are the two principal librations; there remain certain other very slight ones which we may here omit

as unimportant. But the combined effect of them all is as follows:

> $\frac{41}{100}$ of the lunar surface is always visible,
> $\frac{41}{100}$ of the lunar surface is never seen,
> $\frac{18}{100}$ of the lunar surface is sometimes visible.

It is of interest to note in passing that this agreement of the lunar axial rotation period with the period of orbital revolution is not due to chance. It must have resulted from some physical cause; the theory at present accepted by astronomers considers it to be a result of forces, interacting between the moon and the earth, and analogous to those producing ocean tides. These forces probably brought about the existing state of things long ago, in early cosmic ages, when the moon may be considered to have not yet become perfectly solid, and to have therefore been subject to enormous tidal distortions.

The next thing we have to do is to explain how astronomers measure and weigh the moon. Of course this cannot be done by the methods used in the case of the earth, because it is impossible to visit the moon to make surveys and perform the Cavendish experiment (p. 107) for determining mass. But the moon's size can be derived easily from measures of its angular diameter, combined with the knowledge we have already obtained as to its distance from the earth.

In Fig. 43, the angle *AEB* is the moon's angular diameter (p. 118), as seen from the earth *E*. *BE* and *AE* are each equal to the known distance of the moon from the earth. The triangle *ABE* is therefore fully known,[1] and we can calculate

[1] All parts of a triangle can be calculated by trigonometric methods, if we know two sides and the angle between them.

the number of miles in the lunar diameter *AB*. The average angular diameter is measured easily with astronomical instruments; it is found to be about 31′ of arc. This, combined with the known distance (about 240,000 miles), makes the moon's diameter about 2200 miles, or a little more than one-quarter of the earth's diameter. And as we know from geometry that the volumes of spheres are proportional to the cubes of their diameters, it follows that the volume of the earth is somewhat more than 64 times that of the moon ($64 = 4 \times 4 \times 4$). More accurately stated, the earth's volume is about 50 times that of the moon.

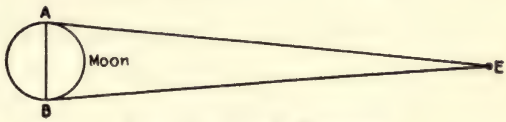


FIG. 43. Moon's Diameter.

A somewhat more difficult problem is the "weighing" of the moon, which, as we have already seen in the case of the earth (p. 103), really means a determination of the moon's mass. Curiously enough, the mass of the moon is most simply determined by observations of the sun. To understand how this is done, we must begin by correcting an approximately accurate theory which we have so far found sufficient for our explanations. It is frequently convenient, for the sake of lucidity, to begin the explanation of some phenomenon by assuming a state of affairs resembling closely that which actually exists in nature, and afterwards substituting new explanations successively, each more closely approximating to the truth, until we can finally consider a tolerably complete theory in its full complexity.

In the present instance, we must now correct a previous statement concerning the earth's orbit around the sun. The earth has so far been said to pursue an oval or elliptic orbit,

with the sun at one focus. And by the earth we mean, of course, the earth's center. But we now know that the earth and moon together are traveling in that orbit around the sun. Therefore, speaking accurately, it is not the earth's center that is exactly in the orbit, but rather the combined "center of gravity" of the earth and moon. And by center of gravity we mean a point so situated on the line joining earth and moon that their weights, as it were, would just balance about the center of gravity. Figure 44 shows this position of the center of gravity at *c*. If we imagine the earth and moon attached to the ends of a rigid bar 240,000 miles long, their weights would balance if the bar were supported at the point *c*. And owing to the great mass of the earth as compared with the moon, this center of gravity is much nearer the earth's center than the moon's. It is, in fact, inside the earth's surface.

FIG. 44. Center of Gravity of Earth and Moon.

Now this center of gravity has another peculiarity of the utmost importance. Not only is it the point that is really following out the terrestrial orbit around the sun, but it is also the real focus about which the moon pursues its monthly orbit around the earth. The moon, accurately speaking, does not revolve about the earth, but about the point *c*, the center of gravity of the earth and moon combined. Furthermore, while the moon is going around this center, the earth is doing the same thing, though in a much smaller orbit. Again imagining both bodies attached to the ends of a rigid rod, it is a little as though this rod were pivoted at the center of gravity, and turning around it. Thus the force of gravitation causes both bodies to revolve

about their common center of gravity, but little moon cannot make big earth travel in as large an orbit as big earth imposes on little moon.

The final result is to swing the earth each half month a short distance either forward or backward with respect to the position it would occupy in its annual orbit around the sun, if there were no moon. Sometimes the earth is in advance of the center of gravity; sometimes behind it. But we always see the sun projected on the celestial sphere at a point on the ecliptic circle directly opposite the earth's actual position in its orbit; therefore this center of gravity effect must show itself by slightly advancing or retarding the sun's apparent motion in the ecliptic circle.

The whole phenomenon is very slight, amounting to a total change in the sun's apparent place on the ecliptic circle of only 12″ of arc. Yet this can be measured with accurate instruments; and a simple calculation then shows that the common center of gravity of earth and moon is distant only 2880 miles from the earth's center. This is about $\frac{1}{82}$ part of the total distance between the centers of these two bodies; therefore the lunar mass must be about $\frac{1}{81}$ part of the earth's mass.[1]

Having thus found the moon's volume to be about $\frac{1}{50}$ that of the earth, and its mass only $\frac{1}{81}$, it follows that the moon must on the average be composed of materials less dense than those of which the earth is made. If the moon were equally dense with the earth, a cubic foot of average lunar material would weigh as much as a cubic foot of average terrestrial material; and these ratios of masses and volumes between the two bodies would be equal. The figures we have

[1] Note 21, Appendix. p. 396

obtained make the moon's density about six-tenths that of the earth.

The interval of time between two successive returns of the moon to the meridian of any place on the earth may be called the Lunar Day. Its length depends on the diurnal axial rotation of the earth, in a manner analogous to the relation between sidereal and solar days (p. 65). We have seen that the sidereal day is equal to the period of the earth's axial rotation, and is therefore the interval of time between two successive returns of the vernal equinox to the meridian. We have also seen that the solar day is about four minutes longer than the sidereal day, because the sun's apparent daily motion of one degree along the ecliptic circle makes the sun lag a little behind the equinox point, so that the apparent rotation of the heavens must continue about four minutes after each complete axial rotation of the earth, to enable the sun to reach the meridian again (p. 69). The case of the moon is precisely similar; only, as its daily motion averages about 13° instead of 1°, the excess length of the lunar day is about 52 minutes, instead of 4 minutes. This makes the lunar day average about $24^h\ 52^m$ of sidereal time.

The fact that the moon thus reaches the meridian about 52 minutes later each night means that it will also rise and set about 52 minutes later each night. But this is only an average figure; in the latitude of New York, for instance, the daily retardation of moonrise may vary all the way from 23 minutes, to 1 hour 17 minutes.

When this retardation of the time of moonrise is at the minimum of 23 minutes, the moon will rise at nearly the same time on two or three successive nights. If the moon also happens to be almost a full-moon on such an

occasion, we have the phenomenon known as the Harvest Moon. This is defined, then, as the rising of the moon, nearly full, on two or three successive nights at nearly the same hour.

To ascertain when this will occur, we must discuss the principal cause of these large variations in the daily retardation of the time of moonrise. For this purpose we may, with sufficiently close approximation, consider the moon as appearing always in the ecliptic circle on the sky; as we already know, it is actually never very far from that circle. This being premised, it is clear that the time-interval between the moonrises on two successive nights will depend on the angle between the ecliptic circle and the horizon, as shown in Fig. 45. *HH* is part of the horizon; *VV* part of the ecliptic circle. Let us suppose the moon was at the intersection *I* when it rose on a certain night. Exactly twenty-four hours later the point *I* will be again rising above the horizon *HH*. But in those twenty-four hours the moon will have moved along the ecliptic to the point *I′*, about 13° from *I*.

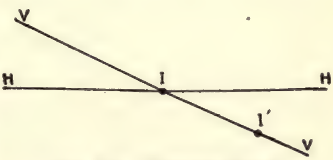


FIG. 45. Harvest Moon.

How much later will the moon rise on the second night? Clearly, by a time-interval exactly equal to the time in which the apparent rotation of the celestial sphere will move the point *I′* up to the horizon *HH*. This interval will be short, if the angle *HIV* between the horizon and ecliptic is small.

But the angle *HIV* is not always the same. It is easy to demonstrate, by the aid of a celestial globe, that it is a minimum when the point of intersection *I* is at the vernal equinox (p. 35). This is well illustrated by the small photo-

graph of Plate 6, p. 162. The line joining the moon's horns being nearly horizontal, the ecliptic must be nearly perpendicular to the horizon if the horns are to point directly away from the sun (p. 165). And the date of the photograph being near the vernal equinox, about the time of sunset, it follows that the angle *HIV*, being nearly a right angle, is at a maximum at the western horizon on or about March 21. Moreover, near the eastern horizon, it will be at a minimum on the same date. That the ecliptic rises very high from the western horizon at sunset on March 21 is also shown by the table on p. 49.

It results from these considerations that if the full-moon occurs when the moon appears near the vernal equinox point, the daily retardation of moonrise will be a minimum. But we have already found (p. 163) that the full-moon always necessarily appears opposite the sun in the sky. Therefore, on the occasion of a harvest moon, the sun must be at the autumnal equinox (p. 35), which is directly opposite the moon's position at the vernal equinox. But the sun appears in the autumnal equinox about September 22 in each year. Consequently, the harvest moon is always the full-moon which happens nearest to September 22.

And this explains the name "harvest." For certain harvests are gathered in September; and it is of consequence to farmers to have plenty of moonlight, so that their work may be completed before rain falls. The full-moon, being opposite the sun, will rise when the sun sets, which occurs at six o'clock on the day of the equinox. Thus the harvest full-moon will rise on two or three consecutive dates at about six in the evening, and will remain visible until sunrise the next morning.

Still another phenomenon of interest arises from the fact

that the full-moon always appears opposite the sun in the sky. Near the time of the winter solstice (p. 121) in December the full-moon must be near the summer solstice point of the ecliptic circle, in order that it may be opposite the sun. It follows from this that the winter full-moons appear far north of the celestial equator, like the sun in summer. Consequently, the full-moon in winter "rides high," as the saying is; when on the meridian it will appear near the zenith, while the summer full-moons are low down in the sky, like the sun in winter.

These variations in the time of moonrise are always set forth in ordinary almanacs; but a certain peculiarity of this part of the almanacs requires explanation. In the case of the sun, the almanacs give both the time of sunrise and sunset, all of which is understood without difficulty. But for the moon, the almanacs give only the time of rising or the time of setting, — never both. And both are not needed. If the moon, for instance, rises shortly after sunset, it will set shortly after the next sunrise. It will therefore be in the sky when the sun rises, and will set during daylight, — a phenomenon not usually observable. In other words, only one of the two phenomena, moonrise or moonset, can be observed on any given date, and the almanac always gives the time of the observable phenomenon.

But this introduces another complication. As the lunar "day" is 24^h 52^m long, it may happen now and then that a given solar day of 24 hours contains no moonrise at all. The moon might have risen just before the beginning of the solar day, and might rise again just after the ending of it. In fact, this must occur once each month. If the moon-column in the almanac contains the word "rises," the following numbers in the column are the successive times of moon-

rise. On the date when the moon does not rise, the abbreviated word "morn" is then substituted in the moon column for the usual time of moonrise. The following numbers in the column then indicate that the moon rises after midnight, — in the morning.

In the lunar orbit there exists still one more peculiarity that illustrates the tendency of astronomy to deceive us by entangling the seeming and the true, — a tendency that has much to do with the peculiar fascination of the science. To an observer on the earth the moon's orbit seems to be an ellipse or oval curve; but the true orbit is not really an ellipse at all. For while the moon is traveling around the earth, the earth is itself speeding through space in its annual orbit around the sun, dragging with it the moon and the lunar orbit around the earth.

Consequently, though the moon's orbit is an ellipse, so far as we dwellers on the earth are concerned, its real path in space is compounded of the two motions involved: first, the lunar motion around the earth; and second, the terrestrial motion around the sun. Now the earth's linear velocity of motion around the sun is much more rapid than the lunar motion around the earth, and is therefore of greater influence in fixing the true shape of the orbit in space. And it is known that the sun is itself also moving through space, carrying with it the earth and the whole solar system, including the moon. This motion would also affect the shape of the lunar orbit; but we shall here consider only the two principal causes already mentioned, — the moon's motion around the earth, and the earth's motion around the sun.

It is a very singular thing, and one not altogether easy to understand, that the combination of these motions makes the true path of the moon always concave toward the sun,

as shown in Fig. 46. The arrow indicates the direction of the sun; E_1, E_2, E_3, etc., are five successive positions of the earth in its annual orbit around the sun, separated by an interval of about $7\frac{1}{2}$ days, or one-quarter of a lunar synodic period (p. 161). The points M_1, M_2, M_3, etc., are five corresponding positions of the moon. M_1 and M_5 are new-moon positions; M_3 a full-moon position; M_2 and M_4 represent quartered phases. The whole line $M_1M_2M_3M_4M_5$ represents a part of the moon's actual orbit in space with respect to the sun; and we can prove without difficulty that it is everywhere concave toward the sun.[1]

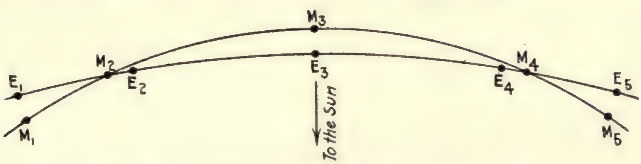


FIG. 46. Moon's Path with Respect to the Sun.

When considering the lunar atmosphere we found the moon quite unlike the earth. But there exists also a very conspicuous similarity between the two bodies, — the mountainous character of their surfaces. There are a number of mountain ranges on the moon, and numerous craters apparently of volcanic origin; but there are no active volcanoes. These lunar mountains are from 1000 to 2000 feet high, and some of the craters are 50 miles in diameter. In the center of the crater there is often a conical mountain peak; it is as though the crater wall was formed by a shower of volcanic material ejected from a center, and falling in a circle around it. The central peak may then have resulted from a final outburst of the volcanic discharge, after the explosive force of the volcano

[1] Note 22, Appendix.

had become too feeble to throw its lava far from the eruptive center. The moon's surface also shows many "rills" or crooked valleys radiating from certain craters. These surface features are well seen in Plate 7. At the bottom of this photograph is the great crater Theophilus, with its rugged central mountain peak.

The height of lunar mountains and crater walls may be measured with the telescope. In certain lunar phases, when sunlight falls obliquely on the moon's surface, the mountains cast long black shadows, seen conspicuously in Plate 7. It is possible to measure in the telescope the angular length of such shadows; and knowing the moon's distance, we can then calculate the shadow lengths in miles from the measured angular lengths. (Cf. p. 172.) Then, from the calculable angle at which sunlight falls on the lunar surface at the moment when the shadows were measured in the telescope, and the known shadow lengths in miles, we can compute the mountain heights, also in miles, by methods well known to surveyors.

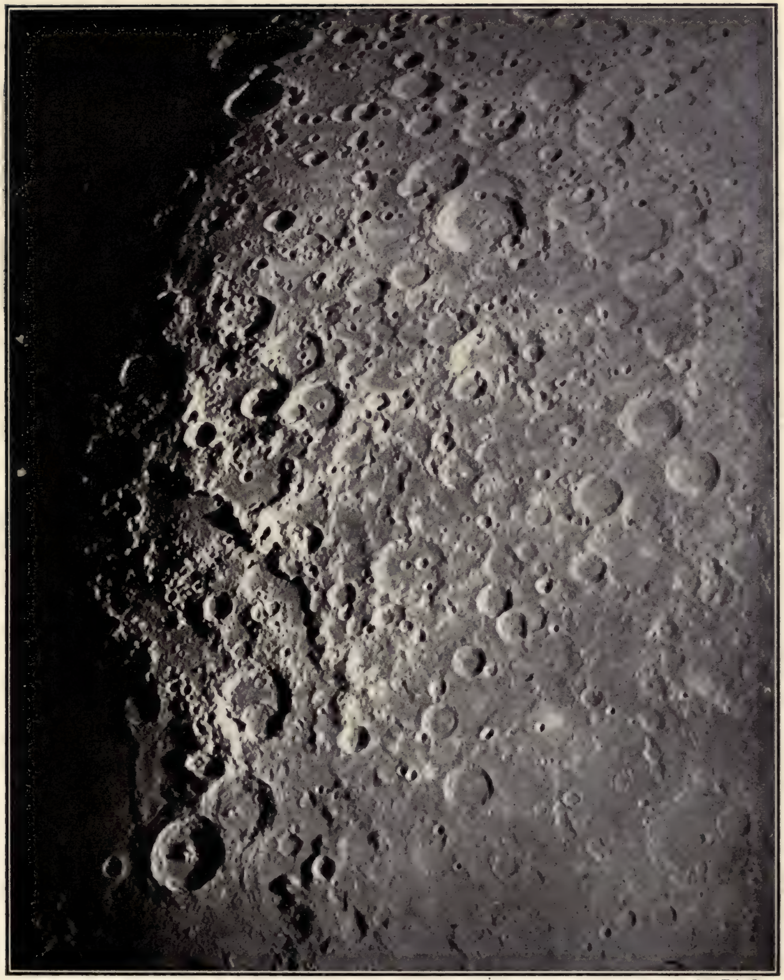


Photo by Ritchey.

PLATE 7. Lunar Enlargement.

CHAPTER XI

THE PLANETS

In discussing the celestial sphere (p. 23) and the appearance of the stars projected upon it, we found that the great mass of these luminous points retain practically unchanging relative positions on the sphere, and are subject only to such apparent motions as result from the earth's daily rotation on its axis and annual orbital revolution around the sun. At the same time, a certain small number of stars move about among their fellows (p. 10). These are the "wanderers," — the Planets. Five are easily visible to the unaided eye, — Mercury, Venus, Mars, Jupiter, and Saturn. Uranus may also be seen without a telescope under favorable conditions; Neptune, and the great body of tiny telescopic objects of the planetary class, called Planetoids, require optical help to be seen.

The distinguishing thing about these planets is that they all belong to our solar system. The earth is merely one of the planets in that system; the others, like the earth, revolve around the sun in orbits analogous to the earth's own annual orbit. These planetary orbits are all oval or elliptic, and have the sun at a point near the center of the orbit, — the focus (p. 116).

When explaining the earth's annual orbital revolution around the sun, we described a simple method of observation by which the form of the terrestrial orbit might be determined experimentally. These simple observations were also

found capable of establishing for the earth a law of planetary orbital motion first discovered by Kepler; viz. that the "radius vector" (p. 119), or line joining the planet and the sun, moves over equal areas in equal times. Thus, in Fig. 47, S represents the sun, P_1, P_2, P_3, P_4, four positions of a planet in its orbit, such that the motion from P_1 to P_2 is accomplished in the same interval of time required for motion from P_3 to P_4. Then the triangular area SP_1P_2, included between the two radii vectores SP_1, SP_2, and the arc of the curved orbit P_1P_2, is equal to the other triangular area SP_3P_4, similarly included between two radii vectores and an arc of the curved orbit.

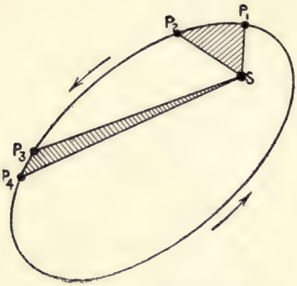


FIG. 47. Law of Areas.

We must now prove that this law applies universally to all planets, and that it is a necessary consequence of Newton's law of gravitation. This latter law, as we have already seen (p. 103), declares that an attraction exists between the sun and planet, directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

Let us consider Fig. 48, and suppose that at a certain instant of time the sun is situated at S, with the planet at P_1; and let us first examine what the planet's motion would be if there were no such thing as an attraction toward the sun. We may suppose the planet to be traveling with a certain velocity, and in a certain direction, such as would carry it to P_2 at the end of

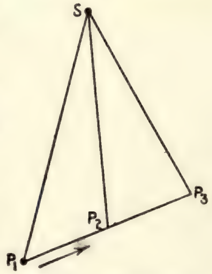


FIG. 48. Planetary Motion.

one second of time. This original velocity and motion may be regarded, if we choose, as a result of the original cataclysm whereby the planet was first brought into separate existence.

Now if there were no attraction toward the sun, and as there can be no friction or resistance to motion in empty space, the planet will arrive at P_2 endowed with the same velocity and direction of motion which it originally possessed at P_1. Therefore it will, under these circumstances, travel an equal distance along the same straight line in the next second, and thus arrive at P_3. The line $P_1P_2P_3$ is the planet's orbit, if there be no attraction toward the sun, and the lines SP_1, SP_2, SP_3, are three positions of the planet's radius vector.

The area traveled over by the radius vector in the first second is the triangle SP_1P_2; and in the second second it is the triangle SP_2P_3. But these two triangles have equal areas;[1] and this constitutes a proof that the radius vector moves over equal areas in equal times, if there exists no attraction whatever toward the sun.

Next suppose that the solar attraction exists, but that instead of being continuous in action it is applied suddenly in the form of an impulse toward the sun at the end of each second of time. Suppose the first impulse is applied at the end of the first second of time, when the planet has reached P_2, and that it is applied toward the sun along the radius vector P_2S. Now consider Fig. 49, and imagine the impulse toward the sun strong enough to have carried the planet to P_2' in one second of time, supposing the said im-

[1] Readers familiar with geometry will recognize that these triangles are equal because they have a common vertex at S, and equal bases P_1P_2 and P_2P_3 situated upon a single straight line.

pulse toward the sun to have acted alone during one second. But the planet at P_2 is also subject to the original force, which, acting alone, would have moved it to P_3 in the second second of time. Thus the planet at P_2 is subject to two forces, one of which, acting alone, would have carried it to P_2' in the next second; and the other, likewise acting alone, would have carried it to P_3 in that next second.

Where will the planet go under the combined action of these two forces in the second second of time? It must evidently move along a line intermediate in direction between P_2P_3 and P_2P_2'. That line will in fact be P_2P_3', and at the end of the second second the planet will arrive at the point P_3'.[1] Its radius vector will then be the line SP_3'; and the areas traversed by the radius vector in the two consecutive seconds of time here under consideration will be the triangles SP_1P_2 and SP_2P_3'. It is not difficult to show that the areas of these two triangles are also equal.[2] Consequently, under our present supposition as to the nature of the attraction toward the sun, the planetary orbit $P_1P_2P_3'$ still satisfies the law of areas.

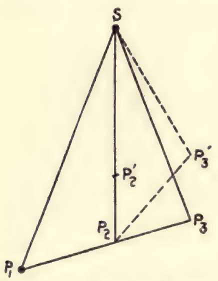


FIG. 49. Law of Areas.

It is evident that any number of impulses toward the sun at the ends of other successive seconds of time would produce similar results. And the same reasoning would hold true if we suppose the impulses to occur more frequently; say ten or a hundred times in a second of time. It follows that if we increase sufficiently the number of supposed impulses per second, we can at last transform our orbit from a series of very short straight lines into an actual curve; for

[1] Note 23, Appendix. [2] Note 24, Appendix.

every curve may be regarded as made up of an infinite number of excessively short straight line elements. And at the same time, the supposed series of impulses toward the sun, coming infinitely close together, are transformed into the continuous action of gravitational attraction. The above reasoning therefore constitutes a proof that a planet moving under the influence of an original impulse in any direction, *plus* a gravitational attraction toward the sun, will pursue an orbit satisfying the law of equal areas for the radius vector.

One of the most interesting things in the above proof is the absence of any special requirements as to the nature of solar gravitational attraction. Nothing in the proof demands an attraction acting accurately in accordance with Newton's law (p. 103). To satisfy the law of areas, it is merely necessary that the attracting force be what is called a "central" force, directed always toward a definite point occupied by the sun within the orbit. And conversely, the fact that the planets can be observed to travel in orbits that satisfy the law of areas, proves merely that they are moving under the influence of a central force, but not necessarily that particular variety of central force which we know under the name of Newtonian gravitation.

But in addition to this law of areas, which can be deduced as a fact directly from observation (p. 120), two other similar laws are known, — also obtainable directly from observation. All three laws were first found by Kepler; they are called, to the present day, Kepler's three laws of planetary motion; and they may be formulated as follows:

1. The orbit of each planet is an ellipse, with the sun at the focus of the curve.

2. The radius vector of each planet passes over equal areas in equal times.

3. If the time required by any planet to complete a revolution in its orbit is called its "period," then the squares of the planetary periods are proportional to the cubes of their average distances from the sun. This third law is called the "harmonic law."[1]

We have just proved that the second law, or law of areas, is a necessary consequence of the existence of a central force pulling always toward the sun. It is similarly possible to prove, by the aid of mathematics, that all three laws follow as a necessary consequence of a central attracting force, provided that force acts in accordance with the Newtonian law. Thus the three laws of Kepler are merely corollaries or consequences of Newton's more general law; Newton's great service consisted in bringing everything under the sway of a single law, instead of three separate ones, apparently unrelated.[2]

In the light of the above explanation of Kepler's and Newton's work, it will now be of interest to give a brief account of the two best known explanations of planetary motion within the solar system, — the Copernican theory, which, with some modifications, is the one now accepted, and the older Ptolemaic theory. It may possibly seem out of place to give any attention to the abandoned Ptolemaic hypothesis; it is like studying something we know to be untrue. But there are many references to that theory in

[1] The harmonic law may be represented mathematically by a simple proportion:

Let t_1, t_2, be the periods of two planets, a_1, a_2, their mean distances from the sun.

Then: $$t_1^2 : t_2^2 = a_1^3 : a_2^3.$$

[2] Note 25, Appendix.

literature: a few pages may well be devoted to a description of it; enough, at least, to form some idea of its peculiarities. It is also of interest that the Ptolemaic theory was actually taught in early days at Harvard and Yale colleges, as being a possible alternative theory to the Copernican.[1]

Ptolemy (140 A.D.), following Hipparchus, supposed the earth to be immobile, near the center of the universe. For each planet a circular orbit was provided (Fig. 50), which circle was called the planet's "deferent." Upon the deferent moved, not the planet itself, but an imaginary planet, represented by a point. The actual planet moved in another circle called the "epicycle," whose moving center was the imaginary planet. The sun and moon had deferents, but no epicycles. Each deferent was supposed to be traced on the surface of a perfectly transparent separate crystal sphere;[2] and all these crystal spheres rotated once a day around an axis passing through the poles of the heavens. The outermost crystal sphere had no deferent or attached epicycle; but to it were fastened all the fixed stars. This

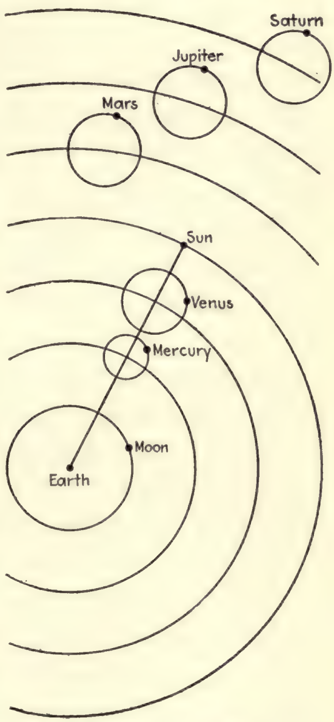


FIG. 50. Ptolemaic Theory.

[1] Young, *Manual of Astronomy*, p. 323.

[2] These spheres, by their motion, produced the famous "music of the spheres."

star-sphere also rotated around the polar axis of the heavens.

The spheres being all of crystal, and perfectly transparent, did not interfere with a view of what was going on outside of each in connection with the exterior deferents and epicycles. The daily axial rotation of the spheres produced all the diurnal phenomena we now believe to result from the axial rotation of our earth. And the spheres, of course, revolved from east to west, not as our earth does, from west to east.

The deferents of Mercury and Venus were inside the solar deferent. The imaginary planets Mercury and Venus revolved in their deferents once a year, keeping pace with the solar motion in its own deferent circle. The sun and the two imaginary planets Mercury and Venus were always in line, as shown in the figure. The revolution of the actual planets Mercury and Venus in their epicycles thus made them swing back and forth, east and west of the sun, in a manner quite similar to their actual observable apparent motions to be described later in the present chapter.

Mars, Jupiter, and Saturn were connected by Ptolemy to deferent circles exterior to the sun. The periods of revolution of the imaginary planets were not here assumed equal to that of the sun, as was the case for the inferior planets Mercury and Venus; and in this way the observable phenomena were also reproduced for these superior planets. Later investigators, following the Ptolemaic theory, added further secondary imaginary planets, revolving in Ptolemy's epicyclic circles; with the actual planets attached to additional corresponding epicycles. In this way they were able to reproduce all irregularities of motion, as improving methods of observing brought them to light.

In contradistinction to the above, the Copernican theory, as we have already seen, supposes the sun immobile, and the planets moving in flattened oval orbits with the sun at one focus. The great objection to this system, an objection that long prevented its adoption by men of science, is this: if the earth really revolves in an orbit around the sun, the fixed stars should change their apparent positions, as seen projected on the sky, while the earth progresses around its orbit. Figure 51 makes this clear. Let S be the sun; E' and E'' two positions of the earth at opposite points of its orbit. Suppose a star to be situated in space at P, fixed and immobile. Then from E' we should see the star projected on the celestial sphere at P', and from E'' we should see it at P''. As a matter of fact we see each fixed star constantly projected in the same place on the celestial sphere; and this seemed an insuperable objection to many early astronomers, including the famous Tycho Brahé. On the other hand, if the earth does not move, there would of course be no change in the direction of the lines $E'P'$ and $E''P''$. There would be but one such line if the earth were constantly in the center at S.

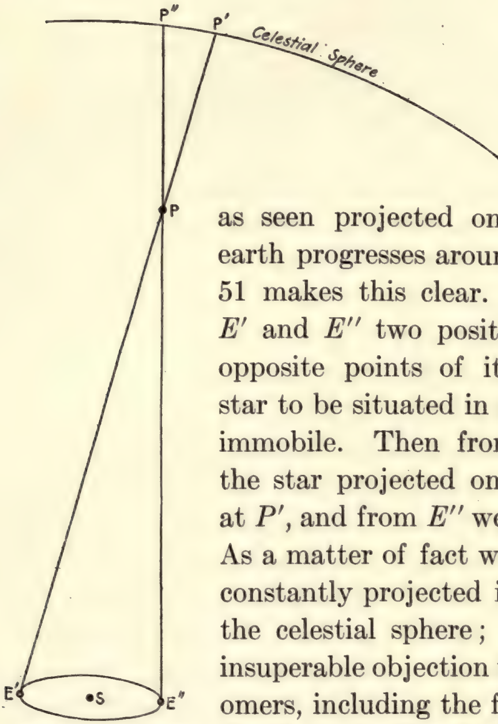


FIG. 51. Copernican System.

This objection to the Copernican system was not removed until the middle of the nineteenth century, when, for the

first time, Bessel was able to measure with certainty a slight difference between the two sight lines from the earth to a certain star in the constellation Cygnus. It then appeared that the trouble arises from the extreme minuteness of the angle $E'PE''$, caused by the fact that the fixed stars are all so excessively distant, in comparison with the diameter of the earth's orbit. And, of course, the angle $E'PE''$ will diminish with an increasing distance of the stars. Up to the present time, no star has been found for which this angle exceeds 1.5 seconds of arc; and in the case of but very few stars has the angle been found large enough to be measured, even with the powerful astronomical instruments of to-day. The angle subtended at P by the radius of the earth's orbit is of course half the angle $E'PE''$. This half-angle is called the star's "parallax." And the measurement of even a single stellar parallax removes the fundamental difficulty of the Copernican theory.

Of historic importance even greater than the above theory of Ptolemy are certain very old and very simple methods of determining observationally a planet's period of revolution around the sun and distance from the sun in terms of the earth's distance. It is evident that before Kepler discovered his harmonic law, no relation was known to exist between distance and period; but there were always simple methods for determining the period by direct observation. When we were discussing the earth in its relation to the sun (Chapter VII), we found that the great ecliptic circle on the sky is cut out by the plane of the terrestrial orbit produced outward to infinity. It must also be a fact that any planetary orbit plane cuts the ecliptic plane in a straight line, because any two planes in space must intersect in a straight line. This intersection line is called

the line of nodes of the orbit. Twice in the course of its revolution around the sun the planet must reach this line of nodes. When this occurs, the planet is for a moment in the ecliptic plane as well as in the plane of its own orbit; and as the earth is always in the ecliptic plane too, it follows that, at this critical moment, the straight line joining the earth and planet will lie entirely in the ecliptic plane.

But we see the planet along that line, observing from the earth toward the planet. Consequently, if we observe the planet at the critical moment, we shall see it projected on the sky somewhere in the great circle cut out on the sky by the ecliptic plane. So we can ascertain by observation when the planet is in the node, by noting the instant of time when it crosses the ecliptic circle, as seen projected on the sky. The interval between two successive passages of the planet through the same node is then its period of revolution around the sun.

Kepler made certain important improvements in the above method of determining planetary periods; and, of course, he also gave much time to the study of planetary distances from the sun, in the work preparatory to his discovery of the three great laws. As an example of Kepler's ingenious methods, we shall give here his investigation of the variations in the distance between the earth and the sun in different parts of the terrestrial orbit.[1] Kepler had at his

[1] Kepler's works are in Latin, and are difficult to read. The original book from which we quote in modernized form is called "*Astronomia Nova seu physica coelistis tradita commentariis de motibus stellae Martis ex observationibus Tychonis Brahé.*" It was published in 1609, but there is a reprint by Dr. Charles Frisch, published in 1860 "Frankofurti et Erlangae."

A most excellent commentary on Kepler was also published in London in 1804 by the Reverend Dr. Robert Small, and dedicated to the Earl of Lauderdale.

disposal a long series of observations of the planet Mars, accumulated by his master Tycho Brahé. These observations recorded the positions of Mars as seen projected on the sky on a very large number of different dates. He selected certain of these observations dated as follows:[1]

1590, March 5, $7^h\ 10^m$,
1592, Jan. 21, $6^h\ 41^m$,
1593, Dec. 8, $6^h\ 12^m$,
1595, Oct. 26, $5^h\ 44^m$.

It will be seen that he had been able to choose four observations separated by exactly the same interval of time; viz.: $686^d\ 23^h\ 31^m$, which interval corresponds very nearly with the known average period in which Mars completes a revolution around the sun. In the accompanying Fig. 52, therefore, Mars must occupy the same position M on each of the above dates, while the earth will occupy the successive positions E, F, G, H. These terrestrial positions will be equidistant points on a circle with its center at B, if we suppose that the earth moves uniformly in a circular orbit. Under this supposition, these points must be equidistant, since they are separated by a series of equal time-intervals, each equal to the Martian period. And it is important to notice that the successive returns of Mars to the same point M are independent of any assumption as to the form or

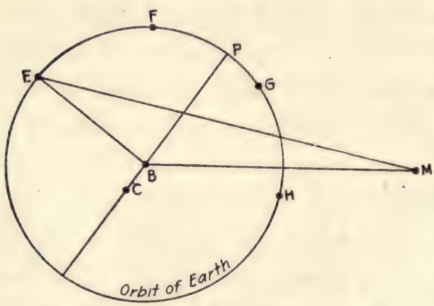


Fig. 52. Kepler's Mars Observations.

[1] Frisch, *Kepler*, Vol. 3, p. 275; Small, *Kepler*, p. 202.

position of the Martian orbit. Whatever and wherever this orbit may be, Mars must return to the same point after each complete orbital revolution has been terminated.

For the date 1590, Mar. 5, when the earth was at E, Kepler had Tycho's observation of the position of Mars as projected on the ecliptic circle, or rather the position of that point on the ecliptic circle which was nearest to Mars. This gave the direction of the sight-line EM from the earth to Mars. The directions of the lines from the center B to the earth, and from the center to Mars, were furnished by the tables of planetary motion in Tycho's possession. Thus the directions of the three sides of the triangle EBM were known, and from these the three angles of the triangle were obtained by subtraction.

But when the three angles of a triangle are known, it is possible to calculate the relative lengths of the triangle's sides.[1] By successive applications of this process, Kepler computed that [2] —

$$BE = .66774 \times BM.$$
$$BF = .67467 \times BM.$$
$$BG = .67794 \times BM.$$
$$BH = .67478 \times BM.$$

These numbers should all be equal if the point around which the earth describes equal angles in equal times were at B, the center of the circle in which the earth is supposed to move. So Kepler's numbers show that the point about which the earth's angular motion is uniform is not at the center B of the earth's orbit, supposed circular; but that it is at some point C, outside the center of the orbit. Kepler was

[1] In the language of trigonometry, the sides are proportional to the sines of the opposite angles.

[2] Frisch, *Kepler*, Vol. 3, p. 275.

able to compute the position of this point C; and the corresponding changing distance between the earth and the sun. These results were of course obtained long before he perfected his three laws; they are regarded justly as marking one of the most difficult and important advances ever made in human knowledge.

There is still another remarkable peculiarity about the planetary distances from the sun; like the foregoing, of historic interest only. When we compare these distances, we find an accidental relation between them. Let us number the planets consecutively, from the sun outward, calling Mercury, 1; Venus, 2; Earth, 3; Mars, 4; the Planetoids, 5; Jupiter, 6; Saturn, 7; Uranus, 8; Neptune, 9. Let us then multiply the number 2 by itself four times, say for Mars, which is planet number 4. This gives 16. Then take three-quarters of this number, giving 12. Increase this result by 4, giving 16. Divide this by 10, giving 1.6. The result is an approximate value for the distance of Mars from the sun, counting the earth's distance from the sun as 1.

This curious arbitrary rule is known as Bode's law; astronomers have been acquainted with it for more than a century; but we know of no physical reason why it should have a real existence. The following little table contains a comparison of the known planetary distances with their values calculated as above. In the case of the planetoids an average value is given.

The table shows that the law is quite accurate until we reach Neptune; then the error increases suddenly; and we must conclude that the whole thing is one of those rare and remarkable coincidences that nature sometimes provides, apparently to mislead scientific investigators.

Planet	No.	Known Distance	"Bode" Distance	Error "Bode"
Mercury	1	0.4	0.5	0.1
Venus	2	0.7	0.7	0.0
Earth	3	1.0	1.0	0.0
Mars	4	1.5	1.6	0.1
Planetoids	5	2.6	2.8	0.2
Jupiter	6	5.2	5.2	0.0
Saturn	7	9.5	10.0	0.5
Uranus	8	19.2	19.6	0.4
Neptune	9	30.0	38.8	8.8

Having thus described certain famous historic methods of studying the planetary distances, etc., we shall next give a somewhat more detailed description of the planetary orbits, and the exact nature of the observations by means of which we study them in modern times. When we determine the position of a planet by observation, we really determine only the direction in which we see it projected on the celestial sphere. We point a telescope at the planet, and, by moving the telescope, bring the center of the planetary disk very accurately into the middle point of the field of view, which, for this purpose, may be supposed to be fitted with a very fine pair of cross threads to mark the center. Then, if the telescope mounting be provided with suitable "graduated" [1] circles, we can read the angles measured by those circles, and thus ascertain the direction of the planet in space, referred to certain points and lines, such as the celestial poles and equator. In other words, we measure the planet's right-ascension and declination (p. 34), as it is seen projected on the celestial sphere.

We can also note the exact time when this observation

[1] Brass circles divided into degrees, minutes, and seconds of arc.

was made, thus fixing the moment when the planet's direction from the earth was measured. There are other methods of making these observations in addition to direct measurement with graduated circles attached to the telescope; but all are alike in this: they furnish us with the direction in space of the sight-line joining the earth with the planet, and the instant of time when that line had the direction in question. Direct observation gives no information whatever as to the planet's distance from the earth. It tells us nothing about the length of the line joining earth and planet; only its direction in space.

If several observations of this kind have been made at different times, separated perhaps by a number of days, or even months, the earth will itself have moved considerably in its own orbit in the interval between the observations. The planet will also have moved in its own orbit. Consequently, both ends of the line will have moved in different orbits and with different velocities; so that the changes in direction of the line will have been of an extremely complicated nature.

But the changes in space of one end of the line are well known to us, — the earth end. For we know the orbit of the earth around the sun, and can calculate the terrestrial position in space accurately for each moment of time when an observation was made. Knowing thus, from calculation, the position of one end of the line, and, from observation the direction of the line, the line itself becomes fully known, all but its length. Thus, in Fig. 53, if at a certain time t_1 the earth was at a known point of its orbit E_1, and the planet was seen in the observed direction P_1, we know the line E_1P_1, all but its length. If, at a second observation, made at the time t_2, the earth was at E_2, and the planet

was seen in the direction P_2, we again know the line E_2P_2, excepting its length. And the same is true of a third line E_3P_3. But it is to be remarked that these three lines will not lie in a single plane, unless the terrestrial and planetary orbits around the sun should happen to lie in the same plane, which is not accurately the case in our solar system.

The problem now is to determine the planetary orbit from observations of this kind. But we know certain additional things about this planetary orbit. We know that it is an ellipse or oval; that the sun is in the focus; and we know the position of the sun with respect to the earth from our knowledge of the terrestrial orbit, since the sun is also in the focus of that orbit. Both orbits have the same point for a focus, and the sun is in that point. Furthermore, we know the planet's orbital motion must be such as to satisfy Kepler's laws of planetary motion (p. 187), and so we know the planet must have moved in such a way as to cause its radius vector to sweep over areas proportional to the known time-intervals between the observations.

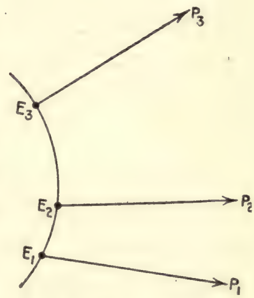


FIG. 53. Planet Observations.

It is a fact that an unknown planetary orbit can be thus determined from three observations such as have been described. Our geometric problem may therefore be stated thus:

Given three observed straight lines in space; it is required to find an ellipse, cutting these three lines in three points, such that the radii vectores to the sun or focus from these three points will satisfy Kepler's laws.

It would carry us too far afield in mathematical astronomy

to deduce here the methods by which this problem can be solved; but several interesting things about it can be enumerated. In the first place, the problem always has two solutions: there are always two ellipses in space that satisfy the problem. One of these is the planetary orbit; the other is the earth's own orbit. For the latter is also an ellipse; it cuts the three lines because they are sight lines from the earth to the planet; and the earth's motions in its own elliptic orbit, of course, satisfy Kepler's law.

But suppose the problem to have been solved for the planet, also; let us see what we need to know about the orbit in order to say that we know the orbit completely. Six different things must become known, and six only; these are called the six "elements" of a planetary orbit.

First we must know the length of the largest diameter of the ellipse, and the degree of flattening, — the eccentricity, as it is called. These two elements being known, we know the size and shape of the orbit. We could draw it to scale.

Next we must know two more things, to define where the orbit is located in space. These two elements fix or determine the position in space of the plane in which the orbit lies. To fix this plane, we must know the angle it makes with the plane of the earth's orbit, the ecliptic plane; and we must know the position in the ecliptic plane of the line along which the planetary orbit plane cuts the ecliptic plane. This, as we have seen, is called the "line of nodes"; and the angle between the two planes is called the "inclination" of the planet's orbit.

Having thus fixed the size and shape of the orbit in its plane, and the position of the plane itself in space, we must still know two more elements. We must know where the planet was in its orbit at some definite time; and we must

know the position of the orbit in its own plane. As we have already seen (p. 120) the planet is said to be in perihelion when it is so placed in its orbit as to be at its nearest possible approach to the sun. The perihelion point is that point of the orbit which is nearest the sun. Therefore we use for one of the orbital elements the exact time of perihelion passage. This element fixes the position in the orbit occupied by the planet at a definite moment of time. Finally, to locate the orbit in its own plane, we must know the direction in that plane of the "major axis," or longest diameter of the oval orbit.

The six elements of a planetary orbit are therefore the following:

<table>
<tr><td>1. Longest diameter of oval
2. Eccentricity, or degree of flattening</td><td>These fix the size and shape of the orbit.</td></tr>
<tr><td>3. Inclination of orbit plane
4. Position of lines of nodes</td><td>These fix the position in space of the orbit plane.</td></tr>
<tr><td colspan="2">5. Time of perihelion passage.</td></tr>
<tr><td colspan="2">6. Direction of orbital major axis in its own plane.</td></tr>
</table>

A seventh orbital element is usually added: the Period, or time required for a complete orbital revolution of the planet. But this element is not really an independent one; for the planetary periods and the diameters of the orbits are connected by Kepler's harmonic law (p. 188), by means of which either may be calculated from the other.

The elements of an orbit once computed from three complete observations of the planet's apparent position, as projected on the celestial sphere, and seen from the earth, the problem can be inverted, and the subsequent apparent projected positions of the planet calculated from the elements. Thus is it possible to predict exactly where each planet may be seen in the sky. If a series of such calculated predictions

are tabulated for every day in the year, the tabulation is called a planetary Ephemeris. The United States government publishes such a tabulation annually, under the title "The American Ephemeris and Nautical Almanac." In it the planetary positions are printed for each day in the form of right-ascensions and declinations; and by means of these printed numbers it is easy to find the planets in the sky.

The measurement of a planet's axial rotation period, corresponding to the terrestrial sidereal day, is not a very easy matter. The best method of doing it is to observe with the telescope any spot or mark that may be distinctly visible on the planet's surface. As the planet turns on its axis, this spot will alternately appear and disappear; for it will of course be invisible when the planet's rotation carries it around to the side which is turned away from our earth. If we note the exact time elapsing between two successive returns of the spot to the apparent center of the planet's disk, this interval will be the planet's rotation period, or day.

Such an observation must, of course, be corrected for any effects produced by variations in the relative positions of the planet and the earth, due to their respective orbital motions. And the result can also be much improved by allowing a considerable number of rotations to elapse between the two observations. If this can be done, the effect of any error in noting the exact time when the spot arrives at the center of the disk will be greatly diminished. But none of the planets, with the exception of Mars, have spots sufficiently perfect to admit of precise observation. Our knowledge as to the duration of the planetary days is therefore still very defective.

If the paths of the spots, as they move across the visible

planetary disk, can be mapped with sufficient accuracy, we can further ascertain from them the location of the planetary rotation poles, the inclination of the planetary equator to the plane of the orbit, and other related matters. Unfortunately, information of this kind is still very meager, on account of the lack of suitable spots on the surfaces of most planets.

We shall next consider the measurement of a planet's size, its diameter, surface area, and volume. We have seen that ordinary astronomical instruments enable us to measure

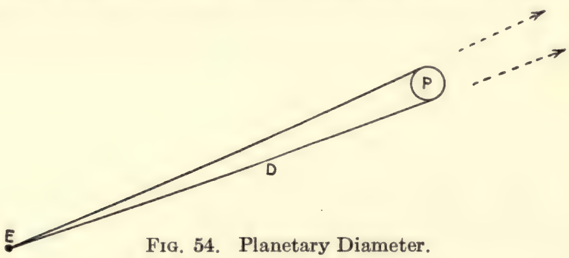


FIG. 54. Planetary Diameter.

only the directions in which we observe the heavenly bodies projected on the celestial sphere. Thus, for instance, we can determine whether a star lies in the direction of the celestial equator, or whether its direction makes an angle of 10° with the direction of the celestial equator. If the former, the declination (p. 34) of the star would be 0°; if the latter, 10°.

Now if we thus observe the difference in direction of the two sides of a planetary disk (pp. 13, 52); we have at once the "angular diameter," or the angle subtended by the planet to an observer on the earth. Figure 54 explains this matter. E is the observer on the earth, P the disk of the planet. The two arrows show the directions in which the observer sees the two sides of the planetary disk projected

on the celestial sphere. The small angle at E is the difference of these two directions, and it is the angular diameter of the planet, which is measured by observation.

A knowledge of this angular diameter tells us nothing about the actual diameter of the planet in miles, unless we know also the distance D between the earth and the planet. For it is obvious that it would require twice as big a planet to subtend the angular diameter observed at E, if the planet were removed to double the distance D. But the distance D, at the moment of observation, can always be calculated, if we know the dimensions and other particulars of the orbits pursued by the earth and the planet around the sun. And with the distance D available, it is easy to calculate the planet's diameter in miles from the observed angular diameter.[1]

Having thus found the planet's diameter in miles, it is frequently convenient to represent it in terms of the earth's diameter as a unit. We can then find the surface area of the planet, as compared with that of the earth, by simply squaring the planet's diameter expressed in terms of the earth's diameter as unity. And the same number cubed will give us the planet's volume, as compared with the earth's. For it is a well-known mathematical principle that the areas of spherical bodies are proportional to the squares of their diameters; and their volumes are proportional to the cubes of the diameters.

A somewhat more difficult problem is the determination of a planet's mass. If there happens to be a satellite revolving around the planet, the problem is comparatively

[1] This involves merely a trigonometric solution of the long, narrow triangle shown in Fig. 54, using the angle at E, which has been measured, and the two including sides, which are both equal to D in length.

easy. We can then determine by observation the period of the satellite's revolution in its orbit around the planet; and its distance from the planet in miles can also be observed by precisely the same process just used to ascertain the planet's own diameter in miles. From these data the planet's mass can be computed.[1]

With regard to the planet's satellites in general, there is not much more to be said. Their distances from the planets are determined, as we have just seen, by means of angular measures. Their periods of revolution around the planets are best found by noting the time elapsing between successive "elongations," or occasions when the satellite's orbital motion around its planet carries it to its greatest apparent angular distance from the planet.

Most satellite orbits are almost exact circles: our own moon has an exceptionally flattened or elliptic one. And the planes of the satellite orbits are mostly very near the planes of the planets' equators; indeed, the equatorial bulging of the planet itself should suffice to pull the orbit plane of a close satellite into the planetary equatorial plane, from gravitational causes alone. That the planets have an increased diameter at the equator, and a corresponding polar flattening, has been verified by direct measurements in the case of our earth (p. 97). For the other planets its existence is proved by comparing separate determinations of polar and equatorial angular diameters, if the position of the poles has become known. When the satellites are unusually far from their planets, as in the case of our moon, their orbits lie nearly in the planes of the planets' own orbits around the sun.

Before leaving this subject of orbits in the solar system,

[1] Note 26, Appendix.

we shall discuss briefly the permanence or "stability" of those orbits. Will they endure forever? Will the solar system change materially in the course of time?

The planets move primarily under the influence of solar attraction as if they were themselves mere particles devoid of more than an infinitesimal mass. They are, in fact, all extremely small in comparison with the great sun. Nevertheless, they do possess mass in a certain degree; and consequently there is an interaction between them, which shows itself in slight perturbative effects upon the planetary orbits. In other words, if, by any method, we determine the elements of a planetary orbit in any given year, we shall not find these elements remaining unchanged forever. After the lapse of sufficient centuries, the planetary interactions and perturbations effect changes in the orbital elements of the solar system.

These changes are of two kinds:

1. The Periodic perturbations.
2. The Secular perturbations.

The periodic perturbations increase and diminish in comparatively brief intervals of time, comparable in length to the orbital periods of the planets themselves. But the secular changes, produced, as it were, in each orbit by all the other orbits acting upon it, are extremely slow in period, requiring many thousands of years to complete a cycle.

The periodic perturbations never displace the position in which we see a planet projected on the celestial sphere more than about one or two minutes of arc, except in the case of Jupiter and Saturn, which are at times displaced from their proper or unperturbed orbital positions as much as half a degree, more or less.

The most interesting facts about the secular perturbations,

known from the researches of Laplace and Lagrange, are as follows:

1. The major diameters and periods of the orbits do not change.

2. The inclinations and eccentricities vary in an oscillatory manner.

3. The nodal points and perihelion points move around the ecliptic and orbital planes, respectively.

4. All changes of whatever kind are probably oscillatory; so that the solar system is stable and permanent. After the lapse of sufficient ages, it will always return again to its original condition, no matter what changes it may have undergone. Of this, however, there exists a slight doubt, due to a possible imperfection discovered recently in Laplace's mathematical demonstrations.

5. There is in the solar system an "invariable plane," not subject to change, and containing the center of gravity of all the bodies composing the system.

Throughout the foregoing explanations, the word "period" has been used to indicate the interval of time required by a planet to complete an orbital revolution around the sun. But there exists more than one kind of planetary period. When we were discussing the planet earth, the sidereal year (p. 128) was defined as the time required by the earth to complete one orbital revolution around the sun. Thus, if we imagine an observer situated on the sun, the sidereal year will be the time elapsing between two successive apparent returns of the earth to the same fixed star, if both star and earth are supposed to be seen from the sun, and projected on the celestial sphere. In the same way, the sidereal period of any planet is the time required for a complete orbital revolution, from any fixed star back to the

same star, and seen from the sun. So far as the sidereal period is concerned, then, the earth is in precisely the same condition as all the other planets.

We also found (p. 128) that the earth has a tropical year, used especially in calendar making. Of course no other planet has a tropical year, so far as dwellers on the earth are concerned. But the other planets all have another important kind of year, which the earth does not have. It is called the Synodic year and corresponds to the synodic period (p. 161) in the case of the moon. To define it, suppose, in Fig. 55, we have the orbits of the earth and Jupiter. For both planets the sidereal year is the time required to complete revolutions from any two points such as E and J back again to the same points. But for Jupiter, which has a synodic year, this synodic year is defined as beginning when a straight line drawn from the earth to the sun at S passes through Jupiter at J. And, similarly, the synodic year ends when the revolutions of both bodies make it again possible to draw a straight line from the earth to Jupiter through the sun.

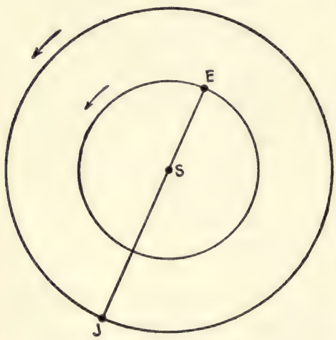


Fig. 55. Synodic Year.

We have here supposed the orbits of both earth and Jupiter to lie in a single plane. This may be done as a first approximation for all the planets, since none of their orbits lie in planes very greatly inclined to the ecliptic plane, in which the terrestrial orbit is situated.

Both the sun and Jupiter are seen from the earth projected on the background of the celestial sphere; conse-

quently, when they are in this straight-line position, they should appear to us at the same point on the sky. Owing to the existing small angle between the orbit planes, it will happen only rarely that they will appear to occupy the same point quite exactly. So the synodic year is considered to commence when they are as nearly as possible in a straight-line position, and therefore in the closest possible apparent proximity, as seen by us projected on the sky.

At such a time, we say that Jupiter is in Conjunction with the sun. In general, the term "conjunction" is thus used whenever two celestial bodies are in very close proximity, as seen from the earth, projected on the celestial sphere.

A very simple mathematical relation exists between the synodic and sidereal periods of any planet. It is based on the fact that the synodic period depends on a line passing through the earth as well as the planet, and must therefore be affected by the terrestrial as well as the planetary rate of orbital motion; while the sidereal period depends on the planetary motion alone.[1]

The foregoing reasoning applies strictly to those planets only whose distances from the sun are greater than that of the earth from the sun. These are called Superior planets to distinguish them from Mercury and Venus, which are accordingly called Inferior planets, because their orbits lie within that of the earth.

These inferior planets, of course, have sidereal and synodic periods defined in the same way as the corresponding periods of the superior planets. The accompanying Fig. 56 represents the case of an inferior planet such as Venus. The sidereal period of Venus, like that of Jupiter, is the

[1] Note 27, Appendix.

time required by Venus to complete an orbital revolution around the sun, from any fixed star back to the same star again, supposed seen from the sun. But when we draw our straight line passing through the sun, the earth, and Venus, Fig. 56 shows that such a line can be drawn when Venus is in the position V, or in the position V'. In either case, Venus and the sun will be seen from the earth close together, as projected on the celestial sphere; and will therefore be in conjunction. When Venus is thus in conjunction through being situated between the sun and the earth, we call the phenomenon Inferior Conjunction; and when the sun is between Venus and the earth, we call it Superior Conjunction.

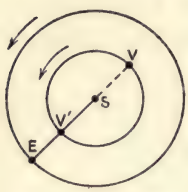


FIG. 56. Inferior Planet.

Of course a superior planet, like Jupiter, whose orbit is entirely outside that of the earth, can never be placed between the earth and the sun, and can therefore never have an inferior conjunction. Superior planets have superior conjunctions only; inferior planets have both inferior and superior conjunctions.

The synodic period of Venus is, then, the time in days elapsing between two successive inferior conjunctions, or two successive superior conjunctions. But the mathematical relation connecting the synodic and sidereal periods is slightly different from that which holds in the case of a superior planet.[1]

The following little table contains approximate planetary periods; and exhibits the interesting fact that both kinds of periods increase from Mercury to Mars, inclusive. Also, for this part of the table, the synodic periods are always the greater periods. But for all the other

[1] Note 28, Appendix.

	Sidereal Period	Synodic Period[1]
Mercury	88 days	116 days
Venus	225 "	584 "
Earth	365 "	——
Mars	687 "	780 "
Jupiter	12 years	399 "
Saturn	30 "	378 "
Uranus	84 "	370 "
Neptune	165 "	368 "

planets the synodic periods are far smaller than the sidereal periods; and they are all nearly equal in duration.[2]

It is plain that when any planet is in conjunction with the sun, we shall be unable to see it. Sun and planet being then projected on the sky at nearly the same point, the bright solar light will, of course, overcome the faint planet, and make it invisible. In other words, the planet, appearing near the sun, will be above the horizon in daytime. To make the planet visible, it must be far from the sun, as seen projected on the sky; *i.e.* there must have been considerable synodic motion since the time of conjunction. Visibility from the earth depends on synodic motion, not actual motion in the orbit.

It is customary to use the term "elongation" to designate a planet's angular distance from the sun, as we see it projected on the sky. At the time of conjunction, the planet's elongation is very small; it may even be zero. We have seen in Figs. 55 and 56 the state of affairs when a conjunction with the sun occurs in the case of a superior and inferior

[1] These periods have been used on pp. 50 and 51.

[2] Note 29, Appendix.

planet. As the synodic motion advances after conjunction, the planets increase their elongation from the sun. Figures 57 and 58 show the maximum elongations the two kinds of planets can attain. For the superior planets, like J, representing Jupiter (Fig. 57), the elongation may reach 180°. For the inferior planets, like V, representing Venus (Fig. 58), there is a certain definite maximum angle of elongation SEV, which occurs when there is a right angle at V; *i.e.* when there is a right angle between the directions of earth and sun, as seen from the planet.

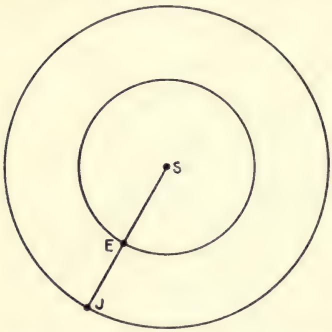


FIG. 57. Superior Planet. Greatest Elongation.

When the elongation is 180° in the case of a superior planet (Fig. 57), the sun is directly opposite the planet, as seen from the earth, projected on the sky. Thus, in the figure, the sun would be seen from the earth E projected toward the upper part of the page, and Jupiter directly opposite, projected toward the lower part of the page. The planet is then said to be in Opposition. The greatest possible elongations (Fig. 58) for the inferior planets Mercury and Venus, which can never be in opposition, are 47° [1] for Venus, and 28° for Mercury. These numbers may be verified by means of a simple mathematical calculation.[2]

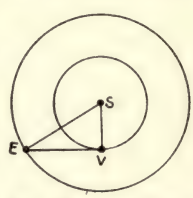


FIG. 58. Inferior Planet. Greatest Elongation.

Let us still remember that for the purpose of a first approximation we may consider all the planetary orbits to lie in a single plane, the plane of the ecliptic. It follows that

[1] Cf. p. 51.

[2] Note 30, Appendix.

we must always see the planets projected on the sky near the great circle cut out by that plane, — the ecliptic circle, in which we also see the sun projected. Now since Mercury is thus always near the ecliptic circle, and always within 28° of the sun, it must appear to us to oscillate back and forth near the ecliptic circle, appearing now on one side of the sun, now on the other. This is also true of Venus, the other inferior planet, though here the arc of oscillation is much greater, as we have seen. When Mercury is at either extreme of its oscillation, it is in greatest elongation. When it is an eastern elongation, Mercury being east of the sun, the planet is visible for a short time after sunset. When it is a western elongation, the planet is west of the sun, and is visible a short time before sunrise. But owing to the apparent proximity of the sun, Mercury is always projected against the rather bright background of the sky near the point where the sun rises or sets at the horizon. Thus Mercury is not very easy to see. Venus, with its much greater possible elongation angle, is a very easy object to the unaided eye.

In general, we thus find that the visibility of an inferior planet depends on the production of these maxima of elongation by the synodic motion (cf. p. 50).

In the case of a superior planet the state of affairs is very different. Visibility still depends on synodic motion; as before, the planets cannot be seen near the time of conjunction. But as their synodic motion advances, these planets do not approach a moderate maximum elongation, and appear to oscillate back and forth across the sun. For, as we have already seen, the superior planets have their oppositions when their elongation from the sun is 180°; then they are directly opposite the sun; and are therefore observ-

able on the visible part of the celestial meridian at midnight, when the sun is on the lower and invisible part (cf. p. 51).

But, nevertheless, the superior planets do have certain oscillations in their apparent motions among the stars, as seen from the earth. These oscillations cause them to perform at times so-called "retrograde" motions, traveling apparently among the stars from east to west instead of west to east, which is their usual direction of apparent motion. Sometimes, too, they have temporary "stationary points," appearing immobile for a short time, like fixed stars.

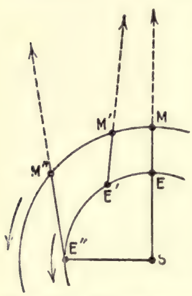


FIG. 59. Retrograde Motion of Mars.

To understand this state of affairs, let us consider for a moment the orbits of the earth and a superior planet like Mars. The accompanying Fig. 59 shows these orbits, not drawn to scale, but again supposed to be in a single plane, and circular. Beginning at the time of opposition, Mars, earth, and sun are shown on the line *MES*. At the end of one month, Mars will be at M' and the earth at E'. After three months, Mars will be at M'' and the earth at E''. At these three dates, therefore, terrestrial observers will see Mars projected on the sky along the three successive directions EM, $E'M'$, and $E''M''$. Both planets have been constantly and uniformly moving in the direction of the curved arrows, yet from E' we see Mars along $E'M'$, apparently retrograded back of the direction EM, or contrary to the direction of orbital motion for both planets. At $E''M''$, Mars has again begun to move forward in its apparent motion among the stars, and that forward motion will evidently become more rapid a little later. It is also

clear that about the time the apparent motion changes from retrograde to direct, Mars will for a short time appear quite stationary among the stars. And it is further evident from the figure that the middle of the arc of apparent retrogression must occur about the time of opposition, when the planet is nearest the earth.

There is but one more peculiarity of importance in connection with this apparent motion of the planets as seen from the earth, and projected on the sky. It arises from the fact that the orbital planes do not coincide accurately with the ecliptic plane, and therefore the planets do not always appear to us on the sky projected accurately on the ecliptic circle. They have certain small apparent motions toward the ecliptic circle, and again away from it. It follows that a planet's arc of retrograde motion does not simply return along the same line over which it traveled in its direct motion, as would be the case if all planetary motions were accurately in the ecliptic plane. The actual retrograde apparent motions usually involve peculiar curves, both for the superior and inferior planets.

We shall close this chapter with another reference to the Keplerian method of determining the planetary periods. The matter could not be explained fully until the synodic period had been made clear. By the aid of that period, astronomers of old possessed still another simple way of ascertaining the sidereal period by observation. They could observe the date when the planet was in opposition to the sun, when it comes to the meridian at midnight. Then the interval between two successive oppositions is the synodic period (p. 208); and from the synodic period they could calculate the sidereal period, which is the true period of orbital revolution, by means of a simple mathematical

equation.[1] This method cannot be used for the inferior planets, as they do not have oppositions.

The accuracy of this measurement of period could be increased greatly by comparing two oppositions between which the planet had made many revolutions around the sun. Thus, by a comparison of two oppositions separated by five hundred synodic periods, the error of observation affecting the exact times of opposition would, of course, be divided by 500. This was actually possible in the case of the principal planets, by utilizing existing ancient records of opposition observations.

Furthermore, it was necessary to compare distant oppositions, to eliminate the effects of orbital flattening in the case of both the planet and the earth. For it is clear that successive synodic periods will not be accurately equal: they would be so, if the orbits were truly circular; but from the average of a large number of successive revolutions this source of error is practically removed.

[1] Note 27, Appendix.

CHAPTER XII

THE PLANETS ONE BY ONE

IT will now be of interest to consider separately the many details in which the planets differ amongst themselves; and we shall begin with Mercury, the one nearest the sun. As we know, it always appears projected on the sky in the vicinity of the sun (p. 50); sometimes on one side of it, sometimes on the other. The ancients did not perceive that this planet, seen alternately on opposite sides of the sun, was a single body. They had two names for it, — Apollo and Mercury.

The seasons on Mercury must present a rather curious problem. We have no means of ascertaining with any degree of certainty the angle between this planet's rotation axis and the plane of its orbit (p. 203). On the earth this angle is $66\frac{1}{2}°$; and it is owing to the existence of such an angle that we have the regular terrestrial seasons (p. 120). Therefore we know very little about the seasons of Mercury, so far as they may be analogous to terrestrial seasons. But we know that the distance of this planet from the sun has so large a variation between perihelion and aphelion that a very variable quantity of solar heat must reach it at different times. There must exist a variability from this cause, great enough to make very appreciable temperature changes. The interaction of this with possible seasons of the terrestrial kind may give rise to hot summers and cold summers, etc., in different years.

In the telescope, Mercury exhibits phases like our moon, and due to the same cause. It has little or no atmosphere in all probability; and most astronomers can see but the faintest surface markings. Lowell, however, has published drawings of Mercury showing many geometric lines and angles; and he thinks they change their apparent positions on the planet very slowly. This would indicate that there is no rapid axial rotation like the earth's. If the planet turned quickly on its axis, the rotation would soon carry some of the markings out of sight around the edge of the planet (p. 202). Possibly, therefore, Mercury, like the moon, rotates on its axis but once, while making an orbital circuit around the sun in 88 days. If this be so, there must be a very hot hemisphere, always facing the sun, and a very cold opposite hemisphere. But this would be modified somewhat by the very large librations (p. 171), which would result from Mercury having an unusually flattened orbit around the sun.

The surface of Mercury is not very brilliant. It has been calculated that it reflects only 13 per cent of the solar light falling upon it. This percentage of light-reflection is called the planet's Albedo; and Mercury has the lowest albedo in the solar system.

The planet Venus, the other inferior planet, is also seen alternately on opposite sides of the sun, appearing as morning and evening star. But it attains a much greater angular distance from the sun than does Mercury, and is also more brilliant. It is at times the brightest of all the planets, and can even be seen by the unaided eye in full daylight near the occasions of its greatest elongation from the sun.

The telescopic phases of Venus range all the way from a

complete circle down to a narrow crescent.[1] According to old Ptolemy's theory (p. 189), we should never see the phase of Venus larger than the half-moon shape. For Ptolemy supposed Venus moving on a circle whose center was always near a line joining the earth and the sun. It is clear, from Fig. 60, that the angle at Venus between the earth and the sun could never be as small as a right angle; and so Venus could never show a phase bigger than the half-moon, according to the accepted Aristotelian theory of phase phenomena (p. 163).

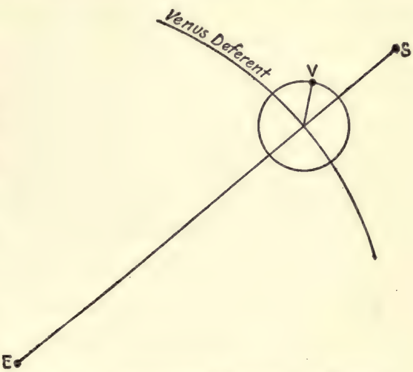


FIG. 60. Ptolemy's Theory of Venus.

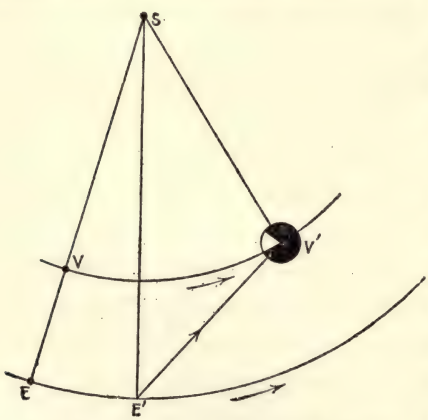


FIG. 61. Greatest Luminosity of Venus.

This matter is most interesting; the moment Galileo turned the first astronomic telescope upon Venus, about the year 1610, and saw a phase larger than the half-moon, he had at once a strong proof that something was wrong with this particular detail of the sacrosanct Ptolemaic theory. To remove this difficulty, however, it would only have been necessary for Ptolemy to lengthen the radius of the Venus epicycle.

[1] Well shown in Plate 8, p. 225, a reproduced photograph.

Venus gives a good example to demonstrate that a planet does not attain its greatest luminosity when nearest the earth, nor when exhibiting the largest possible phase. Figure 61 illustrates this problem. The points S, V, E represent positions of the sun, Venus, and the earth at inferior conjunction (p. 210). Venus has then no perceptible disk; we see its dark side. As time goes on, the phase of Venus grows; and the light reflected toward us increases in proportion to the increasing area of the visible disk. But at the same time the distance from Venus to the earth is increasing; and the intensity of the planet's light, as received by the earth, of course diminishes rapidly with the increase of our distance from Venus. Thus, in the figure, at the moment when Venus has reached the point V', the earth is at E'. The area or phase of the visible disk has grown from zero at V to the segment shown unshaded at V', but the distance between the two planets has increased from VE to $V'E'$.

Thus we see that, beginning with inferior conjunction, the disk area grows much more rapidly than the distance; consequently, Venus grows more brilliant to our eyes. But, later on, this is reversed; so there must be a certain point where Venus suddenly begins to substitute a decrease of visible brilliancy for the previous increase. This is the moment of maximum luminosity, as seen from the earth; it is a nice problem, requiring the infinitesimal calculus for its solution, to determine this moment exactly. It will suffice here to say that it occurs about 36 days from inferior conjunction, when Venus has a phase like the crescent moon.

Venus is believed to possess an atmosphere, for it has a very high albedo, or light-reflecting power, which indicates a reflecting surface containing clouds. Moreover, Venus is

occasionally seen to pass between the earth and the sun, — a phenomenon called a Transit of Venus. When these transits are about to occur, and just as the planet is beginning to encroach upon the solar disk, as seen from the earth, a ring of light becomes visible around the part of Venus not yet projected upon the sun. This cannot be explained otherwise than as a refraction or reflection of solar light by the planetary atmosphere.

Certain ill-defined shadings have been seen at times on the planet's surface: Lowell goes so far as to give a map of Venus showing very clear geometrical structures of straight lines. These are of interest because of their bearing on Lowell's observations and theories as to the Mars "canals." From observations of the markings he concludes that Venus (as he also found in the case of Mercury) has a very slow axial rotation; that it probably turns on its axis in 225 days, which is also its sidereal period. If this be correct, Venus must always turn the same face toward the sun.

The planet Mars, which we shall next consider, differs greatly from Mercury and Venus. Its orbit is exterior to that of the earth and varies quite considerably from an exact circular form, so that the planet's distance from the sun, and its distance from the earth, undergo very wide variations, corresponding to the planet's motion in its orbit. Furthermore, unlike Mercury and Venus, Mars has certain very well-defined and constantly visible surface markings. These have enabled astronomers to ascertain with precision the length of the Martian day, or period of axial rotation. It is found to be about $24\frac{2}{3}$ of our terrestrial hours, or nearly the same as the day of our earth. The diameter of Mars is about half that of the earth; and the inclination of its axis to the plane of its orbit around the sun is 65°. Since the

corresponding angle of inclination in the case of the earth is $66\frac{1}{2}°$, it is clear that the Martian seasons will resemble closely those experienced by ourselves. Thus there are many points of resemblance between the two planets, Mars and the earth; and therefore is Mars the best hunting ground for those who seek a planet with intelligent inhabitants.

In the telescope, Mars shows no crescent phases like the moon or the inferior planets, because its orbit is outside that of the earth; and so the angle at Mars between the earth and the sun can never be as big as a right angle. Its atmosphere should be less dense than that of the earth; for the absence of clouds is indicated by our seeing constantly permanent markings on the planet's own surface, and by the observable fact that Mars has an unusually low albedo, or light-reflecting power. Moreover, owing to its small size and small mass, the attractive force of gravity on Mars is less than half that existing on the earth. Consequently, it is not improbable that Mars has been deprived of its atmosphere in the same way that the moon is believed to have lost its own air (p. 167).

Mars has two satellites or moons; and they are in some respects the most peculiar bodies in the solar system. Their special oddity arises from their close proximity to the planet, and the consequent shortness of their periods of orbital revolution about it. Deimos, the outer satellite, has an orbital period of 30^h 18^m. Phobos, the inner one, revolves in its orbit in 7^h 39^m. These brief intervals are the "lunar sidereal periods" (p. 161) for Mars. Now the planet itself takes about $24\frac{2}{3}$ hours to complete an axial rotation. Therefore the orbital motion of Phobos, as seen from Mars, makes it move among the stars from west to east much faster than the apparent diurnal motion of the Martian celestial sphere

makes the satellite seem to move from east to west. In other words, Phobos rises in the west and sets in the east!

Deimos, however, with its period of $30^h\ 18^m$, travels diurnally from east to west. We can investigate easily its apparent diurnal motion. Its period being 30.3^h, in one hour it moves among the stars $\frac{360°}{30.3}$. In the same time the diurnal rotation of the Martian celestial sphere is $\frac{360°}{24.7}$. The apparent motion of Deimos is therefore:

$$\frac{360°}{24.7} - \frac{360°}{30.3} \text{ in an hour, east to west.}$$

It will therefore make an apparent rotation of 360° around the sky in a number of hours found by dividing 360°, the circumference of an entire diurnal circle, by the above hourly motion. The result of this division is 128 hours; and this is the "lunar day" (p. 176) of Deimos. And so we have the unusual condition that the lunar day is far longer than the lunar "month," or sidereal period.

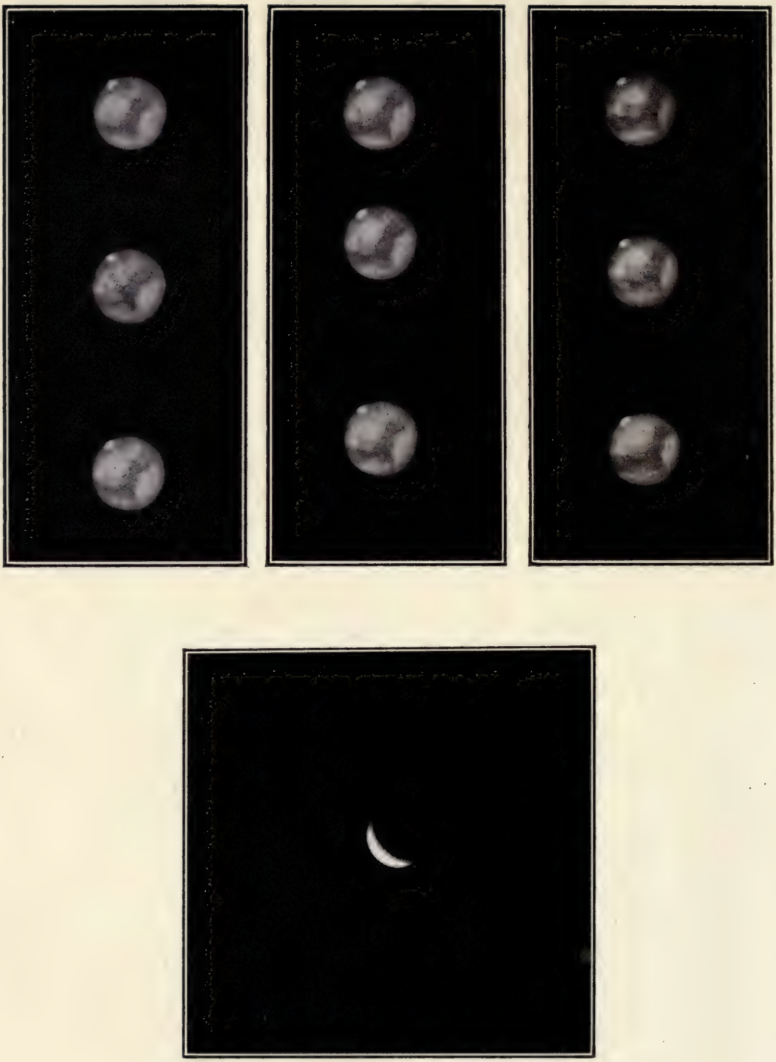
Approaching now the question of Martian "inhabitants," and their canals, we must first inquire as to the existence of water vapor in the atmosphere of the planet. Is there any? This is a matter of prime importance in connection with the famous supposed canals. If there exists on the planet a network of geometric markings, their explanation as waterways must stand or fall by the water vapor in the planetary air. A flow of water in canals can be imagined only if we suppose also evaporation of that water into an atmosphere, and subsequent precipitation of it as snow or rain. If this precipitation occurs for some reason principally near the planet's poles, while the evaporation takes place all along the canals, we might imagine the latter to have

been constructed artificially to carry the water away from the poles, so as to fructify and irrigate the entire planetary surface.

When these markings were first seen by Schiaparelli the weight of observational evidence favored the presence of this water vapor. Such observational evidence is all obtained by means of an instrument called the spectroscope (p. 282). If the solar light reflected from Mars passes through a planetary atmosphere containing water vapor, its "spectrum," as seen in the spectroscope, will show certain bands called water vapor bands. Unfortunately, we cannot observe the Martian light until it has passed through the terrestrial atmosphere, which always contains some water vapor. The difficulty is to determine whether any observed vapor bands are due to the Martian atmosphere, or to that of the earth.

There is but one way to distinguish between the two: we must compare the Martian spectrum with that of our moon. The lunar spectrum will show no water vapor effects except such as are due to the earth's air, for the moon itself has no atmosphere. Consequently, a lunar observation gives us only terrestrial water vapor bands; a Mars observation gives us the terrestrial plus the Martian effects. Any observable difference between the two is due to Martian vapor alone.

It is clear that this method of observation cannot be successful if there is very much water vapor in our own air. If there is, the slight difference between the moon and Mars will be masked completely. As existing observations have been found discordant, Campbell made an expedition to the summit of Mt. Whitney (15,000 ft.) in 1909. He took the necessary instruments with him and photographed the spectra of Mars and our moon at the exact time when Mars was most favorably situated in proximity to the earth.



Photos by Barnard.

PLATE 8. Mars and the Crescent Venus.

At the great elevation of Mt. Whitney there was but little terrestrial atmosphere above the observer. The lunar spectrum, which exhibits the effects caused by our own air, showed so little water vapor that Campbell concludes it would never have been detected at all by a person previously ignorant of its existence. And the Martian spectrum was equally destitute of water vapor bands, even the faintest. On account of this elimination of the earth's air, these observations must be considered by far the most reliable in our possession. They seem to settle the water vapor question in the negative; with the Martian vapor the Martian water goes; and, without water, the canals are impossible as artificial waterways.

The markings on Mars of which we are certain consist of various permanent patches, lines, and areas of different shades; and there are also two bright spots at the poles known certainly since the time of Herschel. The accompanying Plate 8 contains a series of photographs of Mars, showing these markings and white spots very plainly. There is also distinct evidence of axial rotation, the exposures having been made in sets of three, with an interval of $1^h\ 22^m$ between the first and last set. In that interval the markings have moved perceptibly across the disk (p. 202). The lower part of the plate shows a photograph of Venus, in the crescent phase (p. 219). The polar spots seem to increase in the Martian winter season, and to diminish in the summer. If so, they may be ice-caps; and it is this notion that gives color to the canal theory. For the melting ice-caps, in this theory, are used as the source of water to be pumped through the canals. Later, the water evaporated out of the canals is supposed to be returned to the poles by atmospheric movements, and there again precipitated as snow.

But if the planet, in comparison with the earth, is as cold as it should be according to its distance from the sun, there is quite a possibility that the caps are not ice at all, but perhaps some other substance, such as solidified carbon dioxide.

It is not difficult to calculate the theoretic temperature on the surface of Mars, and it is found to be $-33°$ on the Fahrenheit scale.[1] Since this low theoretic temperature would negative the existence of water in an uncongealed state, the advocates of canals are compelled to assume a heavy blanket of atmosphere, charged with much water vapor, to keep Mars warm, as it were, and cause the actual surface temperature to be far above its theoretic value. Such an atmosphere might act in this way; but here again we find the absolute necessity of assuming the presence of water vapor in spite of observational evidence to the contrary; and we are asked to imagine Mars to be an arid desert requiring irrigation, and yet above this arid desert a wet, foggy atmosphere, highly charged with water vapor.

In view of the great public interest in this Mars matter, we shall venture to quote briefly from an article published by the writer a few years ago. First, as to the question: Do the geometric markings really exist? The evidence here is almost all positive. Most astronomers who have observed Mars under favorable conditions and with powerful telescopes have seen markings, but the number of lines reported varies from several hundreds down to two or three. Finally, a very few prominent markings have been photographed.

Let us first consider the visual evidence. Let us examine the witnesses, for that is what these astronomers really are, eye-witnesses of the lines on Mars. Some years ago, Lowell observed on the planets Venus and Mercury certain

[1] Note 31, Appendix.

systems of geometric markings. As it is impossible to suppose that all planets possess intelligent engineers, it is essential to the Martian theory to show that these Venus markings are quite unlike those now seen on Mars. Accordingly, in his book entitled *Mars and its Canals*, published in 1906, Lowell refers to these older Venus observations in the following words: "The Venusian lines are hazy, ill-defined, and non-uniform." But in the original article in which he described what he saw on Venus,[1] we find the following: "They (the markings on Venus) are not shadings more or less definite, but perfectly distinct markings. I have seen them when their contours had the look of a steel engraving."

The only way in which these two statements concerning Venus can be brought into accord is to suppose that in the interval of nine years the observer has, for some reason, changed his opinion.

Additional important testimony is furnished by Mr. A. E. Douglass, who was chief assistant at the Lowell observatory in Flagstaff for seven years, from 1894 to 1901, and since then held the position of astronomer there for a considerable period. In May, 1907, he published an article in the *Popular Science Monthly*, entitled "Illusions of Vision, and the Canals of Mars." This title alone shows that he had changed his views; and his actual words in the 1907 article are:

"The ray illusion (*sic*) is to me a very satisfactory explanation of many faint canals . . . the only objective reality is the spot from which they start."

Again, speaking of what he calls the "halo illusion," he says:

[1] Monthly Notices of the Royal Astronomical Society, London, 1897.

"The double canals of Schiaparelli in 1881–2 and of Perrotin and Thallon in 1886 . . . are . . . due to this cause."

And again:

"Thus, in conclusion, we see that there are fundamental defects in the human eye producing faint canal illusions."

Having thus outlined briefly the apparent contradiction in Lowell's testimony and the reversal of Douglass', it will be of interest to explain how it may be possible for observers to be in error to such an extent. For this purpose, we must mention some of the possible causes that may impair the correctness of an observation. At least five imperfections come into play: imperfections of the earth's atmosphere, the telescope, the eye, the optic nerve, and the imagination. The process of seeing a thing is not at all simple. Light waves coming from the object under examination, after passing through the atmosphere and telescope, fall upon the outer surface of the eye. They are concentrated, or focused, by the lens in the eye, and produce an effect, which we do not quite understand, upon the retina at the back of the eye. This, in some unexplained way, results in an impression being received by the brain through the optic nerve. Then, the brain in its turn does an unexplained something with that impression; what we think we see is equal to that which came through the eye and optic nerve, *plus* what the brain does with it later. The mind cannot distinguish between an impression caused by the eye and optic nerve and one produced by action of the brain itself.

Now it is important to remember that imperfections of the atmosphere, such as clouds, and all imperfections of the telescope, generally tend to diminish or destroy the

possibility of vision; but those of the eye and imagination, if they act, are just as likely to increase the number of details we think we see. Especially when the object is faint and indistinct, — trembling, as it were, on the very limit of visibility, — then especially can a very slight activity of the imagination either prevent our seeing it, or bring it seemingly into view. And this extreme faintness admittedly exists in the case of almost all the Martian markings.

This theory explains why highly experienced observers see so much more than beginners. They think they are training the eye, so as to increase its powers, while in reality they may only be training that slight imperfection of the imagination which tends to increase details thought to be visible. The theory also furnishes an explanation of the fact that so considerable a number of observers think they have seen the faint canals. Nothing more strongly increases the powers of imaginary seeing — of seeing the unseen — than the knowledge that others have already made the observation. We are very prone to "see" what we are told by others is visible: we think we see what we desire and hope to see; do what we will, we cannot prevent this.

Coming now to a consideration of the photographic observations, we must mention one or two matters that are not well known to the general public. In the first place the size of a Mars picture made by direct exposure of a photographic plate at the focus of the Lowell telescopes is not larger than the head of an ordinary pin. From so small a picture we could not even hope to discover any details. Therefore we must enlarge it as much as possible; and there are two ways of doing this. The first is to place an enlarging lens in the telescope itself. Two disadvantages limit this

method. First, it complicates the optical system of the telescope, with consequent loss of distinctness in the image; and secondly, it makes the image on the plate less brilliant. The cause of this loss of brilliancy is simple. The total quantity of light received from the planet is constant; if, therefore, we spread it over an enlarged surface, each part of that surface will receive less light. For instance, with an enlargement of five diameters, the surface of the image is 25 times as large. The resulting diminution of light makes necessary a longer exposure of the photograph, and a consequent increased difficulty in making the clock mechanism attached to the telescope follow with exactness the motion of Mars in the sky. Experiment has shown the greatest photographic enlargements that can be made in this way with the Lowell telescopes; and the negatives of Mars, including the most recent ones made in the Andes, never exceed three-sixteenths of an inch in diameter.

The other method of increasing the size of photographs is to use an ordinary enlarging camera, after the telescopic negative has been finished. There is here no difficulty in securing sufficient light, as in the case of enlargements made in the telescope itself. For we can use artificial illumination of the original negative, and make this illumination as strong as may be necessary. But there is another serious difficulty. Every photographic negative is developed by placing the plate in a chemical bath, after it has been exposed to light. This results in the precipitation of silver particles upon the plate, wherever its sensitive surface has been exposed to light. The picture is thus built up of separate particles of silver. These particles are so small that the eye cannot distinguish the separate ones; they run together, as it were, to form the picture. But the case

is very different if we magnify the negative. We then see the separate grains of silver, scattered here and there about the surface, and the picture itself is lost altogether. The same difficulty occurs if we attempt to examine any photograph with an ordinary microscope of considerable power. The separate silver grains at once appear, and the picture effect is lost.

All this photographic experimentation, therefore, has not yet resulted in good pictures more than three-sixteenths of an inch in diameter and produced by purely photographic processes, though somewhat larger negatives may possibly be made in the future.[1] All larger published pictures have been reproduced from hand drawings, and are therefore simply visual observations. The alleged photographic verifications have been made by the same observers who have studied Mars in the visual telescope; again the eye, optic nerve, and brain were brought into play, and exactly the same causes as before impair the reliability of these visual observations of photographs.

We conclude that neither by visual nor by photographic evidence has the existence of an artificial network of markings been proven, or even rendered highly probable. Therefore the time has not yet come when we shall have to inquire whether geometric lines indicate the presence of intelligent inhabitants; that time will arrive if the lines themselves are ever shown to possess a real or even a highly probable existence.

We shall next consider the Planetoids, or Minor Planets (pp. 183, 196). A large number of these tiny bodies travel in

[1] Those shown in Plate 8, p. 225, made with the 40-inch Yerkes' telescope, are about twice as large as the Lowell photographs; and they show no signs of geometric canal networks.

orbits situated between Mars and Jupiter; up to the present time several hundreds have been discovered and their orbits and motions computed.

Much interest attaches to the history of the first one ever observed, — Ceres. It had long been noted that the space between Mars and Jupiter was an exceptionally large empty space in the solar system; and it seemed strange that no planet should exist there. The matter appeared still more peculiar after Bode's empirical law was published (p. 196) in the latter part of the eighteenth century; for this law indicated that there should be a planet between Mars and Jupiter. And in 1781 this indication received stronger confirmation, when the older Herschel found Uranus, one of the modern exterior planets, in or near the position predicted by the law. An astronomical society was accordingly organized to make a systematic search for the expected unknown planet. But not until the first day of the nineteenth century did the long-sought object reveal itself; and to an independent observer in the Italian city of Palermo. There Piazzi was making an accurate catalogue of the fixed stars. Every night he made telescopic observations from which he could compute stellar right-ascensions and declinations (p. 34); and he planned to enter in a catalogue these two coördinates for every star in the sky, bright enough to come within the range of his telescopic power.

But he did not confine himself to single observations. Each night's work was checked by careful repetition on several other nights. Sometimes he found an error, which usually consisted in the discovery of a star that had escaped his notice at some previous observation. But on this historic occasion, he found that a star was absent, although he had observed it on another night. And, strange to say, there

was also an additional star close by, one that had apparently remained unobserved on the previous night. The conclusion was irresistible that the new star was the same one he had observed before, and that it must have moved among the other stars in the interval. This motion among the stars (p. 10) is the distinguishing characteristic of planets. A third observation made the matter sure: the second star was again absent, and a third new star once more appeared in a place previously vacant. The apparent motion between the second and third observations was proportional, both in magnitude and direction, to that between the first and second observations. So it must surely be a wandering star, — a true planet. Discovery and fame were his.

But Piazzi was able to observe the new planet during a few weeks only, on account of illness. When news reached the astronomers of northern Europe, Ceres had already passed so near conjunction (p. 209) with the sun that further observations were impossible. There was well-grounded fear that the planet would not be found again; for astronomers at that time had no good method of determining a planet's orbit from observations extending through such a short time. The older planets had been observed through many complete revolutions, and there was never any danger of their being lost, because they are bright enough to be visible easily by the unaided eye.

But there was a young astronomer at Göttingen, Gauss by name, who succeeded in solving this difficult problem; and from his published orbit and ephemeris it was easy to find the planet again as soon as the apparent motion of the sun in the ecliptic had brought the planet to a position where it could again be sought in darkness.

A year later, in 1802, the second minor planet Pallas was

found. In 1804 Juno was added; and in 1807, Vesta. It was not until 1845 that another appeared; and three more in 1847. From that time on discovery proceeded but slowly, because the method in use was still the tiresome and arduous process employed by Piazzi. But in 1891, Wolf attacked the problem photographically, the photographic method having just commenced to be widely applied to astronomic purposes. His procedure was perfectly simple. A photographic plate was exposed in the telescope for several hours; and care was taken to make sure that clockwork attached to the telescope moved it accurately during those hours, so as to keep pace exactly with the diurnal rotation of the celestial sphere.

The photograph, when developed, would of course show a round dot corresponding to each fixed star within the field of view of the telescope. But if there was a wandering planet in range, it would move slightly with respect to the stars during the period of photographic exposure; and consequently its image in the picture would be drawn out into a short line, instead of appearing as a round dot like the stellar images. Thus the presence of a line would infallibly betray the existence of a planet. As many as seven planetoids have been thus found on a single plate; so the method is enormously effective. To it we owe an immense increase in our minor planet knowledge during the past twenty years.

Plate 9 is a photographed field of stars, with two planetoid lines, or "trails." They will be found near the middle of the picture, as indicated by the marginal arrows. The trails are not quite parallel, showing that the orbit planes of these two planetoids are inclined at slightly different angles to the ecliptic plane. The difference in thickness of the trails indicates a difference of luminosity in the two planetoids.

Photo by Barnard.

PLATE 9. Discovery of Planetoids.

The orbits of the small planets present some interesting peculiarities. There are several open spaces where practically no orbits appear. Curiously enough, these open spaces occur at points where the minor planet periods of orbital revolution in accordance with Kepler's harmonic law (p. 188) bear a simple relation to the period of Jupiter. It was long ago explained by Lagrange that if two planets have periods connected by a simple proportion, such as $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{3}$, etc., then persistent perturbations (p. 206) must result, which will gradually change the orbits until the simple relation is destroyed. It is in accord with this principle that Jupiter has forced the minor planet orbits out of these critical positions in space, and made them congregate at intermediate positions.

As to the size of the planetoids, it has been computed that the mass of the entire group can be but a small fraction of the earth's mass. The individual planetoids are probably not more than one ten-thousandth as massive as the earth, and their diameter will not average more than twenty miles.

As to the evolution of these minor planets, there is not much doubt. If we accept the hypothesis of Laplace, usually known as the Nebular Hypothesis, the planets were formed by the concentration of matter thrown off from the sun in early ages, while it was still in a gaseous or nebulous condition. This matter is supposed to have been detached from the central mass in the form of a ring: we have only to imagine the minor planets an exceptional case, in which the ring, after breaking up, was prevented from concentrating into a single body by the perturbative action of the big planet Jupiter. Any other hypothesis as to the early history of the planets must, of course, also explain the planetoids as an exceptional case in cosmic evolution.

Among the minor planets is one very remarkable one, discovered by Witt in 1898 and by him named Eros. Its orbit comes well within that of Mars, and it approaches the earth at times nearer than any other planetary body except our own moon. It can pass within about $13\frac{1}{2}$ million miles of the earth ; and this makes it an especially valuable planet to observe for the purpose of ascertaining by certain indirect methods the distance from the earth to the sun. It is altogether probable that observations of Eros will give us ultimately the most accurate value of the sun's distance yet attained. There will be a very favorable opportunity to attempt the necessary observations in 1931 (cf. p. 263).

Proceeding outward from the sun, we now reach the planet Jupiter, the largest in the solar system, and the most brilliant object in the sky at night, with the possible occasional exception of Venus. In the telescope Jupiter is a magnificent object, second only to Saturn in interest. It surpasses Saturn in size, but it lacks the splendid, calm mysterious ring. Markings of a more or less permanent character exist ; they look like cloud-belts running along the planet's equator. And clouds they doubtless are ; for Jupiter must have retained, and must possess, a deep layer of atmosphere, on account of his very high gravitational attraction (see pp. 167, 222) ; and since there is also a high albedo, or reflecting power, we should expect the outer surface to be made of clouds, which have this power in a high degree.

Jupiter's rotation period, or day, can of course be determined from the markings. It is $9^h\ 55^m$; but there is some uncertainty in this period, because the cloud-markings probably have a drift of their own on the planet's surface, and thus do not determine the rotation with precision. The axis is only 3° out of perpendicularity with the orbit

plane; so that there should be no considerable seasonal differences of temperature. But the average Jovian temperature must be very high, for the constant visibility of clouds indicates a hot surface temperature. If this be correct, Jupiter must be in a condition slightly resembling that of the sun. It must furnish its own heat, for it is too far from the sun to receive much thermal assistance from that body.

Jupiter has eight moons, of which four can be seen with a small telescope. They are very interesting historically, for their discovery in 1610 by Galileo gave its death-blow to the old Ptolemaic theory of the universe. The following is a brief account of this great discovery, partly quoted from an article by the writer, first printed in the *New York Evening Post.*

What must have been Galileo's feelings when he first found with his "new" telescope the satellites of Jupiter? They were seen on the night of Jan. 7, 1610. He had already viewed the planet through his earlier and less powerful glass, and was aware that it possessed a round disk like the moon, only smaller. Now he saw also three objects that he took to be little stars near the planet. But on the following night, the three small stars had changed their positions, and were now all situated to the west of Jupiter, whereas on the previous night two had been on the eastern side. He could not explain this phenomenon, but he recognized that there was something peculiar at work. Long afterwards, in one of his later works, translated into quaint old English by Salusbury, he declared that "one sole experiment sufficeth to batter to the ground a thousand probable Arguments."

The 9th was cloudy, but on the 10th he again saw his little stars, their number now reduced to two. He guessed

that the third was behind the planet's disk. The positions of the two visible ones were altogether different from either of the previous observations. On the 11th he became sure that what he saw was really a series of satellites accompanying Jupiter on his journey through space, and at the same time revolving around him. In the 12th, at 3 A.M., he actually saw one of the small objects emerge from behind the planet; on the 13th he finally saw four satellites. Two hundred and eighty-two years were destined to pass away before any human eye should see a fifth. It was Barnard, in 1892, who followed Galileo.

To understand the effect of this discovery upon Galileo requires a person who has himself watched the stars, not as a dilettante, seeking recreation or amusement, but with that deep reverence that comes only to him who feels — nay, knows — that in the moment of observation just passed he, too, has added his mite to the great fund of human knowledge. Galileo knew, on that 11th of January, 1610, that the memory of him would never fade; that the very music of the spheres would thenceforward be attuned to a truer note, if any would but hearken to the Jovian harmony. For he recognized that the visible revolution of these moons around Jupiter, while that planet was itself visibly traveling through space, must end the old Ptolemaic theory of the universe. Here was a great planet, the center of a system of satellites, and yet not the center of the universe. Surely, then, the earth, too, might be a mere planet like Jupiter, and not the supposed motionless center of all things.

The most interesting phenomena about the Jovian satellites are their frequent eclipses and transits (p. 13). Any satellite may be eclipsed to us, either through passing behind the ball of the planet, or by moving into such a

position that the planet interposes between the satellite and the sun. In the latter case, the satellite receives no sunlight to reflect in our direction, and so becomes invisible.

At other times a satellite will "transit" between us and Jupiter. Then it generally becomes invisible, too, unless it happens to be projected against a dark part of the Jovian surface, such as one of the cloud-belts. Finally, a satellite may pass between Jupiter and the sun, when its shadow is thrown on the planet's surface, and is plainly visible as a round black dot slowly crossing the bright planetary disk. None of the transits or eclipses occur suddenly: the satellites all have disks of sensible magnitude, and thus encroach upon the planet's edge very gradually.

Observations of the exact time of these satellite eclipses are useful as an easy approximate method of determining terrestrial longitudes. If we note the instant of local time when an eclipse occurs, and compare it with the calculated Greenwich time, as given in the astronomical almanac, we ascertain at once the time difference of the observer's position on the earth, measured from Greenwich. And this, multiplied by 15, gives his longitude at once in degrees (p. 74). This method requires no instruments beyond an ordinary small telescope; but it is not very precise on account of the impossibility of observing the exact instant when the eclipse happens. Somewhat higher precision, with equal simplicity in method, may be secured from observations of star occultations (p. 166).

It was from observations of Jupiter's satellite eclipses that Roemer, in 1675, first ascertained that light is not propagated through space instantaneously, but requires an appreciable time for its transmission. He used a long series of satellite eclipses, and found they did not succeed each other at equal

intervals. During half the year they came too soon, by gradually increasing amounts; during the other half-year they came too late, by similar quantities of time. Roemer soon found that they came too soon when the earth was approaching Jupiter; too late when it was increasing its distance from Jupiter. He concluded correctly that they came too soon because the earth's approach to Jupiter diminished the distance the light traveled before reaching the earth, after the eclipse actually occurred. Consequently, the light arrived at the earth sooner, and observers saw the event sooner, too. It is clear from Fig. 62 that the extreme difference between accelerated and retarded eclipses must be the quantity of time required by light to cross a diameter E_1E_2 of the earth's orbit. This is found, by observation of the eclipses, to be 998 seconds. It follows that when the earth is at E_1, an eclipse will be observed 998 seconds sooner than it would have been observed if the earth had remained at E_2 instead of traveling around its orbit. Bradley knew of Roemer's observations when he explained aberration (p. 136), nearly a century later, as a result depending on the velocity of light and the earth's velocity in its orbit.

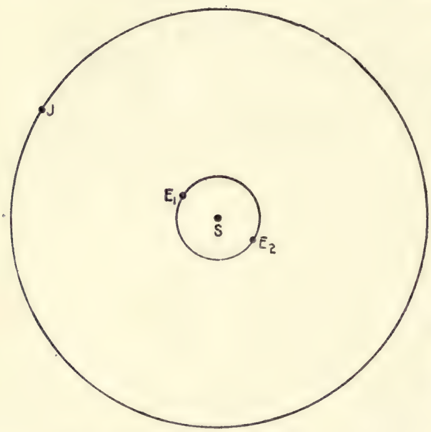


FIG. 62. Roemer's Discovery.

Since modern laboratory experiments have made known that light moves at the rate of 186,000 miles per second, we

have only to multiply this number by 998 to obtain the diameter of the earth's orbit in miles. This gives 186,000,000 miles, approximately; and this number is correct.

The planet Saturn is the last of the large planets observed by the ancients. The most interesting thing about it is its magnificent ring system,—a series of three disk-like rings, situated nearly in the plane of the planet's equator. The history of their discovery is worth noting. We hear of them first from Galileo, who saw a couple of "handles" or *ansæ* attached to the planet in 1610. He was unable to explain them; and, when he looked for them again on a later date, was unable to see them at all. The story is that he gave them up as inexplicable.

Nearly half a century later, in 1656, Huygens published a book *De Saturni Luna Observatio Nova,* in which he announced the discovery of a satellite, and also gave a correct explanation of the mysterious *ansæ.* But Huygens was not quite certain that his explanation was right. He was most anxious to secure for himself the priority of discovery, and yet he was unwilling to make a premature and possibly incorrect announcement. So he resorted to the ingenious device of a "logogriph," or puzzle. It appears in the *De Saturni Luna* as follows:[1]

aaaaaaa ccccc d eeeee g h iiiiiii
llll mm nnnnnnnnn oooo pp q rr s
ttttt uuuuu

It was not until 1659, three years later, in a book entitled *Systema Saturnium,* that Huygens rearranged the above letters in their proper order, reading:[2]

[1] It may be found in 's Gravesande's edition of Huygens, *Lugduni Batavorum,* 1751, p. 526.

[2] Same edition, p. 566.

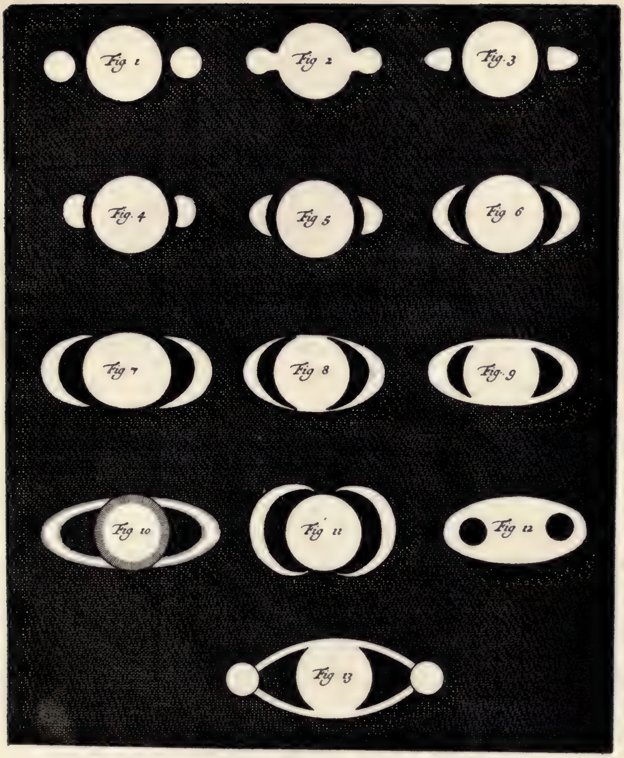
"*Annulo cingitur, tenui plano, nusquam cohaerente, ad eclipticam inclinato.*"

At the same time,[1] he re-published a series of drawings exhibiting several incorrect interpretations of the ring phenomena, as observed by various older astronomers. These are reproduced in Plate 10: Fig. 1 is by Galileo, observed 1610; Fig. 2, by Scheiner, 1614; Figs. 3, 8, 9, 13, by Ricciolus, 1640–1650; Figs. 4, 5, 6, 7 by Hevelius; Fig. 10, by "Eustachius de Divinis," 1646–1648; Fig. 11, by Fontana, 1646; Fig. 12, by Gassendi and Blancanus. Under these reproductions from Huygens we have placed a fine drawing made by Barnard with the Yerkes 40-inch telescope, Dec. 12, 1907.

It will be observed that by the publication of the logogriph of 1656, Huygens secured for himself the credit of what he had done. If any other astronomer had published the true explanation after 1656, Huygens could have proved his claim to priority by re-arranging the letters of his puzzle. On the other hand, if further researches showed that his explanation was wrong, he would never have made known the true meaning of his logogriph, and would thus have escaped the ignominy due to publishing an erroneous explanation. So the method of announcement was comparable in ingenuity with the Huygenian explanation itself.

The ring phases admit of easy explanation. The rotation axes of all revolving bodies maintain constant directions in space, unless disturbed by attractions such as cause the earth's axis to produce the precession of the equinoxes (p. 129). Therefore the plane of the rotating rings must likewise always maintain an unvarying direction in space. Now if this plane of the rings is imagined extended outward,

[1] Same edition, p. 634.



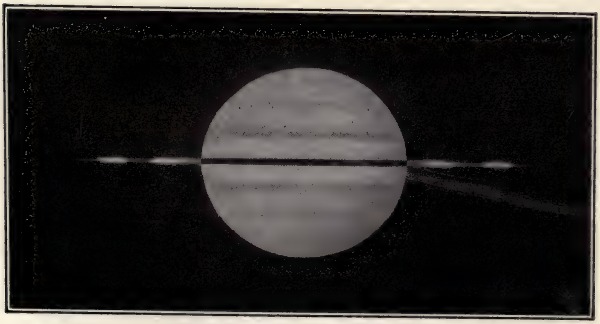


PLATE 10. Saturn.

until it cuts the celestial sphere, it will trace out a great circle there. This circle necessarily meets the ecliptic circle in two opposite points (cf. Fig. 6, p. 35), which are called nodes; and it so happens that the angle between the great ring circle and the ecliptic is 28° on the celestial sphere.

Saturn revolves in its orbit around the sun in a period of about 30 years. Therefore, it must pass one of the nodes every fifteen years, approximately. When Saturn is thus

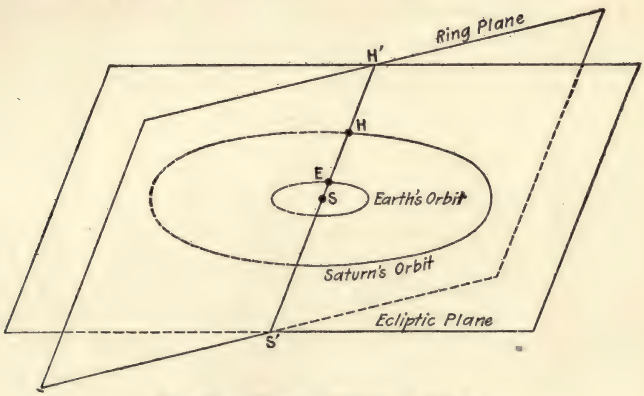


FIG. 63. Phases of Saturn's Ring.

projected at one of the nodes, the sun, in its apparent motion along the ecliptic, may happen to appear in the other node at the same time. These positions are illustrated in Fig. 63; but in using this figure, it must not be forgotten that Saturn's orbit around the sun is very nearly in the ecliptic plane, in which the earth's orbit is also located. Let the sun, then, be at S, and the earth at E. Thus the sun appears projected on the celestial sphere at the node S'. Saturn, located at H in its orbit, is projected at the same time in the other node at H'. It is evident that the earth must then lie on the line HS', the intersection of the two planes,

so that it is temporarily in the plane of the ring as well as in the ecliptic plane.

When the earth is thus in the ring plane, we must see the ring edgewise; and it is so thin that it then becomes quite invisible, except to the most powerful modern telescopes. It disappears, as Galileo found. Furthermore, near these times of disappearance, the earth may be for a short time on either side of the ring plane. And unless Saturn is quite accurately at the node, the sun will also be a little on one side of the ring plane or on the other. But the ring is illuminated on one side only,— the side toward the sun. Consequently, if the sun happens to be on one side of the ring plane while the earth is on the other, we observe the dark side of the ring-system. It should then also be invisible; but powerful telescopes will still show it, appearing like a fine line of light. This is well seen in Barnard's drawing (Plate 10, p. 242), together with certain condensations, or thick places in the ring. This drawing was made with the ring in the edgewise phase: in Plate 11, we have added a fine photograph, also by Barnard, showing the open phase. This negative was made Nov. 19, 1911, with the 60-inch reflecting telescope at the Mt. Wilson observatory in California.

As we have found, these times of ring-disappearance occur about once every fifteen years. In years near the periods of disappearance, the ring is seen nearly edgewise: it then looks like a very narrow oval or ellipse; and it opens out to the widest extent about seven or eight years on either side of the date of disappearance. But the ring can never open into a circle, for the earth can never be elevated more than 28° above the plane of the ring, since 28° is the angle between the ring plane and the ecliptic plane, in which the earth is always situated. And the earth would need to be

Photo by Barnard.

PLATE 11. Saturn.

elevated 90° above the ring plane to enable us to see the ring as a circle.

As to the constitution of the rings, we have very certain and most interesting knowledge. They are neither solid, liquid, nor gaseous, but consist of a dense swarm of tiny satellites, moving in orbits closely interwoven, and all lying in, or near, the plane of Saturn's equator. They are so numerous, and their orbits so closely packed and intertwined, that we cannot see between them, and so they look like a solid disk. They are not very unlike the group of planetoids, which are known to encircle the sun between the orbits of Mars and Jupiter (p. 231). In 1857, Clerk-Maxwell proved mathematically that it is impossible for a system of solid or liquid rings to exist permanently. They would be in unstable equilibrium, and must infallibly break into a series of satellites. And this mathematical demonstration was abundantly verified observationally in 1896, by Keeler. He observed the ring on both sides of the planet with the spectroscope. With this instrument, to be described later, it is possible to measure the linear velocity with which the edges of the ring approach the earth, or recede from it, as the ring performs its axial rotation around the polar axis of Saturn. Now it can be shown mathematically that if the ring is really a mass of satellites, its outside edge should rotate more slowly than its inside edge.[1] On the other hand, if the ring is solid, of course the outside must move faster than the inside. Keeler found by actual measurement that the outside of the ring was moving 10 miles per second; the inside, 12½ miles; and he thus verified observationally the correctness of Clerk-Maxwell's mathematical conclusion.

This observation of Keeler's is destined to rank as a

[1] Note 32, Appendix.

classic observation. We are given to regard astronomy as an ancient science, long since perfected, and incapable of further progress of importance. But this analysis of the ring constitution by methods purely mathematical; this theoretic prediction of invisible relative velocities of rotation; and, finally, the complete observational verification by a method essentially novel, — all this constitutes a chain of scientific research worthy of standing at the side of the master work of the seventeenth century.

Saturn has ten moons, in addition to the swarm composing the ring system. The largest (discovered by Huygens) is visible in small telescopes. Five were found before 1700; Herschel found two in 1789; Bond one in 1848; the other two were discovered photographically at Harvard College observatory within recent years.

The next planet, Uranus, was discovered by Sir William Herschel in 1781. The history of Herschel, and of this discovery, is not without interest. He was the son of a German musician, was born in 1738, and came to England in 1757 to seek his fortune. He settled at Bath, where he supported himself successfully as a music teacher. Although he worked very hard at his music, he found time to study also his favorite sciences of mathematics and astronomy. Having no instrument, he decided to make one; but it was not until 1774 that he succeeded in constructing a tolerable reflecting telescope. He wrote in 1783: "I determined to accept nothing on faith, but to see with my own eyes what others had seen before me." Four times he made a new telescope, each of greater size than the last; and with each he made a re-survey of the entire visible heavens. On his tomb is graven the epitaph:

"Cœlorum perrupit claustra."

It was in the second of his celestial reviews, made with an instrument only seven feet long, that, as he says, "in examining the small stars in Gemini, I perceived one that appeared visibly larger than the rest. I suspected it to be a comet." Within a short time, Lexell was able to show that the motions of the new object could not be explained by any cometary orbit, and that it must be a new planet.

It was perhaps the most startling discovery ever made in astronomy: Herschel named it the *Georgium Sidus*, in compliment to the English king, who promptly honored him with an appointment at court, and made him rich with a pension of £200. He removed to Slough, near Windsor, where he built "Observatory House," and made it memorable as the scene of endless important astronomical discoveries. Long afterwards, Arago characterized it as "*le lieu du monde ou il a été fait le plus de découvertes.*"

Uranus has four satellites, two discovered by the same Herschel in 1787, and two by Lassell in 1851. They have one important peculiarity: they revolve in their orbits around the planet from east to west instead of west to east, the usual direction of orbital motion in the solar system. They are thus an exceptional case, and constitute in a way an unexplained difficulty in the Laplacian nebular hypothesis (p. 235), which would seem to require all satellites to revolve in the same direction.

The outermost known planet is Neptune, remarkable principally on account of the interest attaching to its discovery. Shortly after Uranus had been found, astronomers searched their old records, and ascertained that good observations of it existed as early as 1690. But it had always passed for a star, its disk not being big enough to betray its

planetary character on sight, even in the telescope. But no orbit could be found which would bring these early observations into accord with the numerous ones which began to be accumulated immediately after discovery. And the planet soon refused to live up to its modern observations, also. More than one astronomer suggested that there must be an unknown planet exterior to Uranus, and perturbing its motion, so as to throw it alternately in advance of its proper orbital position and behind it.

In 1845, a young Englishman, Adams, who had graduated from Cambridge University only two years before, succeeded in constructing an orbit for the hypothetical exterior planet, basing his calculations simply on the observed discrepancies in the orbital motion of Uranus. He wrote to the astronomer royal at Greenwich, asking him to look for the new object with his big telescope in a certain definite position on the sky. We now know that this position given by Adams was correct within 2°, so that a little careful "sweeping" with the telescope would undoubtedly have revealed the planet. But the astronomer royal made an unfortunate mistake; the story is that he delayed attending to Adams' letter.

But another astronomer, Leverrier, was also working at the problem in France; by August, 1846, he, too, had worked out the new orbit. On the 23d of September a letter from him arrived in Berlin and was delivered to Galle at the observatory there. Galle had a new and very complete star-chart of the proper region of the sky; and it was for this reason that Leverrier had written to him, rather than to any other astronomer. As soon as it became dark, Galle went into the observatory dome, and began to compare his chart with the sky. He very soon found a strange body within less than 1° of the exact spot indicated in Leverrier's

letter. It was an exciting moment; the new planet had been seen at last.

One curious fact is that both Adams and Leverrier made use of Bode's law (p. 196) to obtain an approximate value for the supposed planet's distance from the sun. This law has no foundation in theory; but it had proved to be fairly exact for all planets then known, including Uranus. But it fails for Neptune; and accordingly both the computers were very largely in error as to this important element of their new planet's orbit. There is ground for supposing that their success was due in some degree to accidental favoring circumstances. But the result was unquestionably a great triumph for mathematical science and for Newton's law of gravitation: at this distance of time it is proper to divide the honor of the discovery equally between Adams and Leverrier. It is certainly great enough for two men, but in the middle of the last century, an ascerbitous controversy raged about the assignment of priority in this matter.

Neptune is so far away from the earth that but few details have been discovered concerning it. There is but one known satellite. And beyond Neptune, no further planets have been found, though the existence of such "ultra-Neptunian" bodies has often been suspected. But none has ever been revealed, even to the most careful photographic surveys so far made in the heavens.

But there is one other material substance in the solar system that requires mention here. The mysterious Zodiacal Light is observable as a faintly luminous band traceable along the ecliptic circle outward from the sun for a considerable distance both east and west. There is also at times a faint glow called by the German name "Gegenschein" discernible in the part of the ecliptic opposite the sun. The

whole thing may perhaps be best explained as a ring of excessively minute planets, revolving around the sun in an orbit larger than that of the earth. Those near the sun would, of course, be the brightest; and the Gegenschein would be the combined effect of an infinitude of these particles acting like tiny full-moons in their position of opposition (p. 163) to the sun.

CHAPTER XIII

THE TIDES

KEPLER was probably the first man to notice that the tides of ocean are due to some form of attraction exerted by the moon. He looked upon the moon as a personal ally of the earth, and in his quaint Latin remarks that "a mutual affection between allied bodies tends towards their union." It is possible that Kepler may have had some hazy idea of gravitation as a species of personal characteristic of celestial bodies.

Let us begin by summarizing the facts easily observable by any one who examines the behavior of the ocean along its shores. In the first place, it will be found that the water level changes considerably. During six hours, approximately, the waters rise; and again, for about six hours, they fall. In each day there are ordinarily two high tides and two low tides. Furthermore, in addition to merely rising and falling, the water also flows along the coast, in one direction during a period equal to the time of rising tide, more or less, and in the opposite direction during a period corresponding in duration to the falling tide. Thus strong tidal currents exist; and navigators frequently take advantage of them to increase the speed of ships, especially in the case of sailing vessels engaged in the difficult business of beating (as it is called) against an adverse wind.

We shall consider the earth for a moment as a globe uniformly covered with a shallow ocean. The most im-

portant cause of tides on such an imaginary globe would be the gravitational attraction of the moon. We know, from Newton's law, that such gravitational attraction diminishes rapidly with an increase in the distance separating the attracted particle from the moon. Consequently, the moon attracts the water on the earth's surface more strongly than it does the more distant solid earth beneath it. We should therefore expect the moon to heap up water at that point on the earth which is nearest to the moon. But there is also water on the earth on the side opposite the moon; and this water is attracted less than the solid earth; it is attracted least of any terrestrial material, because it is farthest of all from the moon. In other words (Fig. 64), the moon should pull the water away from the earth at M, tending to heap it up; and at O, it should pull the earth away from the water, again tending to heap it up.

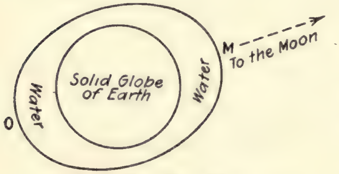


FIG. 64. The Tides.

But the tidal forces exerted by the moon do not act in the above very simple way; in fact, the actual heaping up of the water due to the above cause would be quite insignificant. A far greater effect is produced at points not directly under the moon. Here the tidal force is not vertical, because the moon is not directly overhead; it may be regarded as divided between the vertical and horizontal directions. And the horizontal fraction is then usually the important one. It tends to move particles of water horizontally along the earth's surface, and to move them toward the place where the moon is overhead. But owing to the earth's axial rotation, the moon rises in the east and sets in the west. While it is east of the meridian in any given place, it is pulling the

water particles eastward with the horizontal fraction of the force. After crossing the meridian, it of course pulls them westward; and therefore the result should be an oscillation of the water particles backward and forward, occupying approximately half a day to go and come. And of course this means half a lunar day (p. 176); not twelve ordinary solar hours.

Since the moon, at any given moment, is east of certain places on the earth, and west of other places, it follows that different parts of the ocean will be oscillating different ways at the same time. This must produce a tidal wave, with high tide at the place where the crest of the wave is situated. The crest would follow the moon around, as the earth rotates, but it would not be under the moon. It can be shown that it would, in fact, ordinarily be 90° distant from the point under the moon. And there would be a second crest opposite the first, according to reasoning similar to that of Fig. 64; and therefore two daily high tides, following the moon around.

Having thus outlined the explanation of the semi-diurnal tide, of which the period is half the lunar day, it is now possible to explain that the two tides each day are of unequal size. In general, one rises higher than the other. Although we have seen that the high tidal crest is not under the moon, we can still reason as if it were, in order to explain the above inequality. As the earth rotates on an axis perpendicular to the plane of the terrestrial equator, every point on the earth's surface must rotate in a circle parallel to the equator. Now, in Fig. 65, supposing

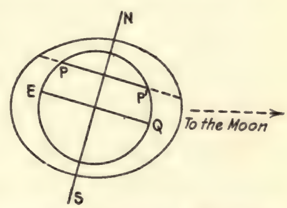


FIG. 65. Diurnal Inequality of Tides.

the tidal crests to be under the moon and opposite it, and the earth rotating around the axis NS, a person at P will not have as high a high tide as he will have twelve hours later, when the earth's rotation has carried him around to P'. The difference is shown by the dotted lines at P and P', and is called the diurnal inequality of the tides. It is due, of course, to the moon's not being situated in the plane of the equator. If the moon were in the equator plane, the tidal crests would be placed symmetrically with respect to the equator, and the two high tides would be practically equal. In fact, it must happen on two days each month that the moon really is in the equator plane. For the moon's apparent motion on the sky, due to its orbital motion around the earth, appears to take place in a great circle of the celestial sphere (p. 160), which must, of course, cut the celestial equator at two points. And it is a fact in accord with actual tidal observation that the diurnal inequality disappears twice each month. Then the two tides are equal.

To complete this part of the subject, two more details must be mentioned. First, we recall that the lunar orbit around our earth is an oval or ellipse, with the earth at one focus, at some distance from the center. The moon will therefore be especially near the earth at certain times. When it is at the nearest point of its orbit, the perigee (p. 169), its tide-raising force will be greater than at any other time. In fact, this force is about $\frac{1}{5}$ greater at perigee than at apogee. Perigee and apogee, of course, both occur each month; consequently, the high tides are by no means equally high during the entire month.

The other matter requiring mention is the effect of the sun on the tides. It operates in a manner precisely similar to the moon, but its greater distance diminishes the tidal

force, so that the solar tide is only about $\frac{5}{11}$ of the lunar tide. The sun is far larger than the moon, but its greater gravitational attraction due to mass or bulk is more than counterbalanced by its greater distance. But it is clear that when the sun and moon are so placed that they act together, we shall get especially high tides; and when they act against each other, the tides will be especially feeble.

Of course these two bodies pull together when sun, earth, and moon are situated more or less in a single straight line; and this occurs at the dates of full and new moon (p. 163). We then have the great tides called Spring Tides; and when the moon is in the first and third quarters we have the little tides called Neap Tides. Spring and neap tides have relative heights in the ratio of $11 + 5$ to $11 - 5$, or 8 to 3, because the solar pull is $\frac{5}{11}$ of the lunar.

The above brief outline of tidal theory is greatly modified when applied to the actual earth, upon which the oceans are deep bowls, large, but still limited in size; and the gulfs, sounds, etc., small, shallow limited cups. The laws of wave motion in areas limited as to size and depth come into play: according to these laws, the rate of progress of a wave, or its time of oscillation, depends on the depth of water. For instance, in a basin like the north Atlantic Ocean, a wave would move 500 miles per hour if it were set in motion, and then left to itself. It would pass from Europe to America in about six hours. Thus its period for going and returning would be nearly equal to the tidal period of half a lunar day. It can be demonstrated that when these two periods are thus about equal, the tides will be large. The water will practically oscillate about a neutral line in the middle of the ocean, giving high tide at the European coast when it is low tide at the American.

But this explanation is complicated still further by the configuration of the coasts, whereby the Atlantic does not act as a single basin with a single neutral line, but as several basins overlapping, more or less. But the general result is nearly as stated.

When we come to the peculiar tides belonging to limited areas of the coast, — such a basin as Long Island Sound, for instance, — still a different explanation is required. Here the tidal wave is not really due directly to the moon; it is a special local oscillation, set up by contact or impact from the lunar tide in the ocean outside the sound. In such cases, conditions become quite complicated, and often lead to tides much higher than the ocean tides. For instance, in Long Island Sound the rise and fall is about seven feet; high tide occurs at nearly the same time throughout the sound; and the wave motion produces a rapid current, or motion of the water particles along the sound. Another interesting tidal modification is found in the funnel-shaped Bay of Fundy, where the tides rise and fall as much as forty feet, or even more.

Tidal phenomena produce results of importance other than recurrent changes in the oceans. Tidal evolution is a term used to describe effects produced on the earth as a whole by tidal action continued throughout vast ages of time. It is clear that the tidal motions of great masses of water must consume a vast quantity of mechanical energy, especially where the great tidal currents occur along the ocean coasts. In such cases there must also be much friction between the land and water. Friction will generate heat, and consume more energy.

Now all this energy must be derived from some source: the law of the conservation of energy (p. 2) tells us that there

can be no manifestation of new energy, such as we have just mentioned, without an equal and corresponding diminution of the manifestation of energy somewhere else. The place where we should expect to find this diminution is in the earth's rotation. In other words, we should expect tidal friction, etc., to act as a sort of brake on the earth's axial rotation, and to bring about a consequent minute lengthening of the terrestrial day, after the lapse of sufficient ages of time. But the most delicate astronomic observations have failed to detect any such lengthening of the day. It must therefore be extremely small, certainly not more than $\frac{1}{10000}$ of a second in a century.

But there is another interesting consequence of these considerations: how does terrestrial tidal friction affect the moon's motions? In Fig. 66, the moon is shown in the celestial equator, and the two great tidal protuberances at *H* and *H′*. According to theory, as we have seen (p. 253), these protuberances or tidal crests should be at *A* and *B*, 90° from the point under the moon. But so much of the tidal effect as acts like friction, to retard the terrestrial rotation, must also make the two protuberances lag behind their proper positions at *A* and *B*. This brings *H* nearer the moon than *H′*; increases the lunar attraction at *H*, as compared with *H′*; and therefore accelerates the moon's motion in its orbit, as shown by the arrow.

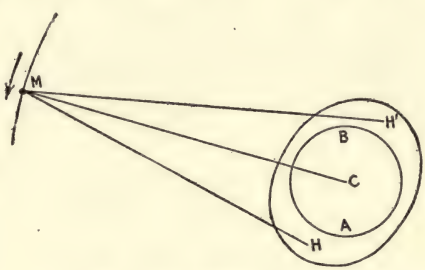


FIG. 66. Tidal Effect on the Moon.

Now it can be demonstrated from the principles of

mechanics, that increasing the velocity of a body moving in an orbit will increase also the size of the orbit and the period of revolution of the body in the orbit. Therefore, tidal friction must make the moon recede from the earth, and must also lengthen the lunar sidereal period. In the hands of G. H. Darwin, these simple principles have led to an extremely plausible theory as to the formation of our moon. According to Darwin, the moon once formed part of the earth; the entire mass was in a semi-liquid or plastic condition; and was in quite rapid rotation about an axis. There was a tremendous flattening of the earth at the poles, due to plasticity. It can be shown mathematically that such a rotatory flattened plastic body may assume any one of several shapes. One of these possible figures is pear-shaped. The fact that we have a moon is thought by Darwin to prove that the pear-shaped figure actually was the one that happened to prevail. The rotating pear-shaped figure should then pass over into an hour-glass; from that to a dumb-bell, with unequal weights at the ends. Finally comes a separation; a true planet with a moon, both revolving rapidly about their common center of gravity, and very near each other.

Now come gigantic tides; tides compared with which our present ocean tides are absolutely insignificant. For the plastic earth was subject to great bodily tides, not merely little oscillations of a thin shell of ocean. Frictional forces then produced no mere slight perturbative action; they were dominating forces. The moon was driven farther and farther from the earth; and the lunar sidereal period was lengthened, until both bodies reached the condition now existing.

If this theory is correct, it enables us to predict for future

ages the final condition of our moon, when the last stage of equilibrium shall finally prevail. When that occurs, the lunar sidereal period and the terrestrial day will be equal, the earth rotating on its axis in 55 of our present days, and the moon making an orbital revolution around the earth in precisely the same period. The moon should then be always opposite the same point of the earth's surface; and both bodies should revolve as though both were attached rigidly to the ends of an unbending bar.

CHAPTER XIV

THE SOLAR PARALLAX

We have had occasion to mention several times the importance of a correct knowledge of the distance separating the earth from the sun. In our discussions of planetary motions we have considered this distance to be known; in fact, we have assumed all the elements (p. 200) of the earth's orbit to be within our knowledge.

Until the latter part of the eighteenth century, astronomers had only a very rough knowledge of the sun's distance, or of its Parallax. This last term may be defined easily; it is exactly analogous to the corresponding term already defined (p. 169) in the case of the moon. By the solar parallax we simply mean half the angular diameter (p. 118) of the earth, supposed to be seen from the sun.

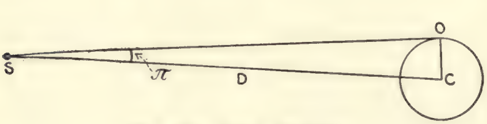


Fig. 67. Solar Parallax.

Thus, in Fig. 67, imagine an observer situated on the sun at S. Draw a straight line from S to the center of the earth at C and another to the surface of the earth at O. Then the angle at S between these two lines is half the angular diameter of the earth as seen from the sun, and is therefore the solar parallax. A simple equation exists,[1] by means

[1] If, in Fig. 67, we let D represent the sun's distance; π, the parallax angle; and r the earth's radius; we have at once, from the triangle SCO: $\tan \pi = \frac{r}{D}$, or $D = \frac{r}{\tan \pi}$. (Cf. Note 20, Appendix.)

of which we can calculate the solar parallax from the solar distance, or the distance from the parallax. If either be known, we can at once find the other. So the term "solar parallax" is really a substitute for "solar distance." The two terms are interchangeable, in a way; but they are not synonymous. One is an angle, the other a linear distance. The present accepted value of the solar parallax is 8.″80.

We shall now consider various ways of measuring it. The reader will remember the method already mentioned (p. 168) for ascertaining the moon's distance by simultaneous observations of that body from two observatories, one in a high northern latitude on the earth, and the other in a high southern latitude. Of course this same method might be applied to the sun, but there is an objection that renders it almost useless. This objection arises from the small size of the parallax angle, which is really the quantity to be measured. The lunar parallax is about 1°, the solar only 8.″8; consequently, any small error of observation, such as one-tenth of a second of arc, will have a considerable effect in the case of the solar parallax, while it would be quite inappreciable in the case of the moon.

This difficulty can be obviated in some degree by measuring the solar distance in an indirect manner. The distance is really only used to ascertain the scale, or size, of the planetary orbits. For with the aid of Kepler's harmonic law (p. 188), we can find the relative distances of the various planets from the sun, after we have observed their periods. Then, knowing the distances, we can make a map of all the orbits, here once more supposed to be concentric circles. And, again with our knowledge of the periods, we can locate the planets themselves in their orbits on the map, for any

date, if we have also observed the dates of conjunctions, etc. (p. 209). Such a map will be correct in every respect, except that the scale remains unknown. The heavenly bodies will be shown in their proper relative places on the date in question, but we do not know the number of miles corresponding to an inch on the map; in other words, the scale of the map. To ascertain this, it will be sufficient to measure observationally the distance from the earth to any other planet on the date for which the map was drawn. This distance once known from the observation in miles, every other distance on our map of the solar system also becomes known in miles.

This work will be most accurate, if we select for measurement a planet which comes comparatively near the earth, and make our observations and our map at the time of closest approach. For, after all, distance from the earth can be measured only by using the earth's own diameter in the way surveyors use what they call a "base-line." Thus, in Fig. 68, for a planet at P, we can at best only measure the angles POM and PMO, so as to ascertain the planet's distance by constructing the triangle POM from the known base OM (the earth's diameter) and the two measured angles.[1] But the base OM is always necessarily wofully short, compared with the planet's distance; the slightest errors in the measured angles produce very large errors in the distance. Therefore

FIG. 68. Base-line for Computing Planet's Distance.

[1] We can, of course, substitute a solution of the triangle by trigonometry for the geometrical construction.

we must do this work when we can observe a planet that is as near to us as planets are ever found.

The planet Mars has been used with advantage. A time is selected when Mars is in opposition (p. 212) so that it comes to the meridian at midnight, and can therefore be observed almost all night. And an opposition is chosen, too, when the earth has one of its closest approaches to Mars. This combination of conditions gives the most favorable state of affairs for the desired measurements. Figure 69 shows the positions of the sun, earth, and Mars at the time of opposition. The orbits are not circles, and therefore the distances of the two planets from the sun are variable. If the opposition is one at which the earth happens to be at its greatest distance from the sun (aphelion), and Mars at its closest possible approach to the sun (perihelion, p. 120), the distance between the earth and Mars is as small as it can ever be; and the conditions are especially favorable for its precise measurement.

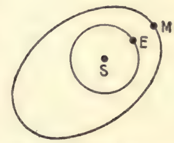


FIG. 69. Favorable Opposition of Mars.

Having thus secured a favorable opposition, there are two different ways in which the Martian distance can be observed. We may employ a modification of the method already described for the moon (p. 168), and observe Mars from two terrestrial observatories situated as far apart as possible. In that case our base-line is the line joining the two observatories. Or we can use the "diurnal" method. In this method, the planet is observed from the same place on the earth at two different times on the same night: first, shortly after sunset; and second, shortly before sunrise. In the interval, the rotation of the earth on its axis will carry the observer to a different position in space; and the line joining his two positions becomes the base-line.

Thus, in Fig. 70, let us disregard the slow orbital motions of Mars and the earth, since these will amount to but little in the few hours elapsing between the two observations; and consider the earth's diurnal rotation only. Let the earth's center be at C, with the rotation axis passing through

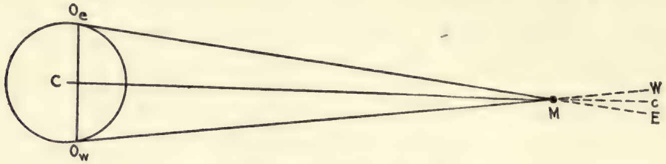


FIG. 70. Diurnal Method.

C perpendicular to the printed page; and the observer at O_w. Then the observer will be carried in about ten hours from O_w to O_e by the diurnal rotation, and the length of the line O_wO_e can be calculated easily from the known dimensions of the earth, and the time elapsing between the two observations. This line O_wO_e becomes the base-line. From the point O_w we see the planet projected on the celestial sphere in the direction MW; from O_e we see it in the direction ME. The difference of the two measured directions is the angle O_wMO_e; from this, together with the known base-line O_wO_e, we can calculate the distance MC from Mars to the earth.

In making the observations, both for the diurnal method and for the method with two observatories, the most accurate way to observe is to use as an auxiliary some small star appearing near Mars on the sky.

FIG. 71. Observation of Mars.

In Fig. 71, which represents a part of the sky, such a star is shown. In either method of observation, owing to the observer's change of position from one end of the base-line to the other, the observations show the planet

projected at two slightly different positions on the sky, as M_1 and M_2. The star itself is always seen in the same position, because the stars are all practically infinitely distant in comparison with any base-line available on the earth.

The angular distance on the sky between the star and Mars (or its equivalent, the difference in direction of the star and Mars as seen from the earth) can be measured in the same way that the angular diameter of a planet is measured (p. 203), with an instrument called a Micrometer, to be described later.

Thus we obtain from the two observations the angular distances of M_1 and M_2 from the star. Their difference is the arc M_1M_2 on the sky; and this is the angle O_eMO_w of Fig. 70, or the angle subtended by the base-line O_eO_w at the distance of Mars from the earth. Comparatively simple calculations will then transform this angle into a knowledge of the Martian distance, the Martian parallax, and thence the solar parallax.

The diurnal and the two-observatory methods each have advantages and disadvantages. In the diurnal method, the observations can all be made by one man in one place with one instrument. This eliminates those errors that arise from personality of the observer, and differences between different instruments, etc. On the other hand, the two observations are necessarily separated by several hours in time, while they can be made quite simultaneous in the two-observatory method. In our present discussion, we have neglected totally the slight orbital motions of Mars and the earth. This is without effect if the two observations are simultaneous; but they never are so in the diurnal method. The two planetary motions must be taken into account by calculation; and thus any slight existing errors in our supposed knowledge of

the two planetary orbits produce slight indirect inaccuracies in the resulting parallax determination.

But in the two-observatory method it is by no means easy to secure perfectly simultaneous observations, either. The two stations are very far apart on the earth. The weather is quite likely to be cloudy at one station when it is clear at the other, thus preventing simultaneous results. Indeed, vagaries of weather sometimes seem especially designed to hinder astronomers in their work, particularly when simultaneous observations are required. But in the diurnal method, the astronomer carries his weather with him, as it were, while he is rotated by the earth from one end of his base-line to the other. If he begins with a good clear night, he is quite likely to secure the necessary corresponding second observation.

The best measurements of Mars by the diurnal method were made by Gill in 1877. He organized a special astronomical expedition to the island of Ascension in the south Atlantic; and this was an especially favorable spot for his purpose. It was desirable to be near the equator, so that the diurnal base-line might be a long one. For at the pole, of course, the diurnal circle shrinks into a mere point.

Gill obtained the value 8.″78 for the sun's parallax from the Ascension expedition, but it appeared that various causes interfered to render the result less exact than was desired. Chief among these causes was the difficulty of measuring the angular distance between the planet and a neighboring star in the manner we have described. This difficulty arises from the fact that Mars appears in the telescope as a disk, while the stars, of course, show only tiny points of light: and there seems to be some kind of personal error introduced by the effort to measure from a

disk to a point. For this reason, Gill decided to repeat the work, using certain of the planetoids (p. 231) whose orbits are located between Mars and Jupiter. These little planetoids, of course, appear in the telescope like star points, and the above cause of personal error does not arise.

Having been appointed director of the great observatory maintained by the British admiralty at the Cape of Good Hope, Gill attacked the problem on a large scale, using three different planetoids, Iris, Sappho, and Victoria, all of which have orbits suitably situated for the purpose. He caused to be constructed a special instrument for measuring the angular distances between the planetoids and neighboring small stars. This instrument is called a Heliometer. Four were made, and mounted respectively at the Cape of Good Hope, New Haven, Leipzig, and Göttingen. With all these special instruments simultaneous observations were made in such a way that both the diurnal and the two-observatory methods could be used in the subsequent calculations. The final result of the whole campaign was to fix the solar parallax at 8.″80: this value is now regarded as the best, and has been adopted by all authorities to determine the scale of the solar system, and to perform calculations of every kind relating to planetary motions.

Since this work of Gill's was completed, a certain newly discovered planetoid, Eros (p. 236), was found to have an orbit so placed that it can at times approach the earth nearer than any other object in the heavens except our own moon. Consequently, its distance from the earth must admit of very accurate determination. One of the close approaches of Eros, or favorable oppositions (cf. p. 263), occurred in 1900; and extensive observations were then made,

this time by the newly perfected method of photographic observation. Results have been published only very recently, and they confirm Gill's value, 8.″80.

This method of minor planet observation is so superior that all other methods are of historic interest only; but, historically, the famous determinations from transit of Venus observations (p. 221) are well worth a careful study. If we consider the motion of an inferior planet like Venus, and assume the orbits of both Venus and the earth to lie in a single plane, then, at each inferior conjunction, Venus will pass between the earth and the sun. This is evident from Fig. 72, which once more shows Venus at inferior conjunction. In point of fact, the orbits do not lie exactly in a plane, but, nevertheless, a passage between us and the sun does sometimes occur. It will happen whenever the inferior conjunction takes place at about the time when Venus is at one of the nodes (p. 200); or, in other words, when the conjunction happens while Venus is on the line of intersection of the two orbit planes of Venus and the earth. When on this line of intersection, Venus is for the moment in both planes; and if there is also an inferior conjunction at the same time, there must be a transit.

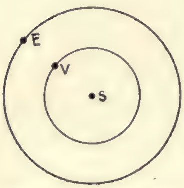


FIG. 72. Venus at Inferior Conjunction.

Venus, during transit, is seen as a small round black dot projected on the bright disk of the sun. This dot appears to enter the solar disk on the western edge, transits the sun in a line approximately straight, and finally passes away from the sun again at the eastern edge. It then disappears; for, of course, we cannot see Venus against the sky background, when near the sun, since the illuminated side of the planet is then turned toward the sun. We see the dark side;

it is "new Venus," if we may borrow a term from the analogy of the moon.

Halley, a famous astronomer royal of England, showed how to determine the solar parallax from transit of Venus observations.[1] His method is shown in Fig. 73.

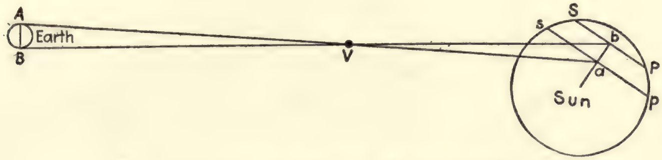


FIG. 73. Transit of Venus, Halley's Method.
(After Herschel's *Outlines of Astronomy*, London, 1851, p. 289.)

Two observers on the earth, located at the points A and B, widely separated in latitude, are provided with good clocks, and observe the exact quantity of time required by Venus to complete a transit across the sun's disk. But these two observers will see Venus crossing the sun along two different lines or "chords" SP and sp. The lengths of these two chords can be calculated in seconds of arc; and thence the solar parallax can be determined.[2]

Halley thought it would be possible to observe the durations belonging to the two chords within two seconds of time. In actual observable transits of Venus this would give the solar parallax correct within one-hundredth of a second of arc. But no such precision of observation has ever been possible, principally because Venus has an atmosphere (p. 220) which introduces errors in the observed durations of the chords. Even in the modern transits of 1874 and 1882, these errors were as great as 10 seconds of time.

[1] See Phil. Trans. Roy. Soc. Lond., Vol. XXIX, p. 1716, or Hutton's Abridgment, Vol. VI, p. 243.

[2] Note 33, Appendix.

There is an interesting astronomical story connected with the first observed transit of Venus. It seems that in the year 1639 there was a young curate in England; a man living in miserable circumstances, but nevertheless inspired with an extraordinary zeal in the study of astronomy. This man, Horrocks by name, and at the time but 22 years of age, united in his own person two of the most poorly paid professions, that of an unbeneficent clergyman of the established church, and an astronomer. A diligent student of Kepler's writings, he had been able to correct an error of the latter, and to predict that a transit of Venus would occur on Nov. 4, 1639. He was unable to fix the exact hour, but he had a little telescope, and prepared himself to watch the sun through the entire day.

Now comes the peculiarly human part of the story. He found that the eventful date would occur on a Sunday. It seemed of the last importance to secure the observation, which was at that time an unprecedented one; the circumstances therefore found him undecided between his duty at church and his keen desire to secure fame as an astronomer. His sense of duty prevailed: he decided to give to the telescope only the intervals between services. And he had his reward, after afternoon service. Hurrying to his poor home, he was in time to see the black round planetary dot on the sun just before sunset, which happens at a very early hour in the northern latitude of England, and in November. To-day, a tablet may be seen in Westminster Abbey, bearing a Latin inscription commemorating this famous observation.[1]

[1] A very good account of it is to be found in Cassini's *Éléments d'Astronomie*, published in 1740. At that time the Horrocks observation was still unique, and Cassini founds many calculations upon it. The tablet in Westminster bears a quotation from Horrocks' own work, *Venus*

Having thus outlined several methods of determining the sun's parallax by observations of planets, it remains to mention certain indirect methods of arriving at its value. For instance, the sun's distance can be computed from the theory of the aberration of light (p. 136).[1] Another quite independent way is called the perturbation method. Briefly stated, it consists in measuring the slight perturbations (p. 403) produced in the regular elliptic motions of the planets. These perturbations are caused by gravitational attractions between the bodies concerned, and the mathematical equations expressing them involve the distance from earth to sun as a factor. If this distance is known, it is possible to compute the perturbations; or, the perturbations being measured observationally, the solar distance may be computed.

in Sole Visa, published by Hevelius in 1662 at Dantzig. The quotation reads, "Ad majora avocatus quae ob haec parerga negligi non decuit."

[1] Note 34, Appendix.

CHAPTER XV

ASTRONOMIC INSTRUMENTS

It is not easy to understand the details of instruments from printed descriptions and illustrations. A short verbal explanation, by an astronomer in an observatory, with the instrument under discussion before him, is the very best way to gain an insight into the methods and machinery of observation. For those who have no opportunity to visit an observatory, we give here a brief account of the most important kinds of astronomic apparatus, prefacing it with Plate 12, a photograph of the famous Lick observatory buildings on the summit of Mt. Hamilton, in California.

To begin with the telescope itself. In the popular imagination, it is a big tube more or less filled with lenses from end to end. But this notion is quite wrong. Theoretically, the telescope has two lenses only, one at each end of the tube. The large lens, which is turned toward the sky, is called the Object Glass: upon it falls the light coming from the celestial object under observation. This light is concentrated or "focused" by the object glass, and forms an image of the celestial body near the small end of the tube where the observer places his eye. Between this focal image and the object glass, the tube is empty.

The other telescope lens is placed at the small end of the tube, between the observer's eye and the focal image, but very near the latter. This lens is simply a magnifying glass, or microscope, and is intended to enlarge the focal image, so

PLATE 12. The Lick Observatory, Mt. Hamilton, Cal.

that the observer will see more detail than would be possible with the eye alone. This eye-end lens of the telescope is called the Eye-piece. In modern instruments, both telescope lenses are of the kind called "Compound" lenses. Each is made up of two or three separate lenses, placed close together, or even in actual contact. By this compounding of the lenses it has been found possible to eliminate partially certain optical imperfections from which all lenses suffer. But each compound lens really acts like a simple lens, except that it does its work better. Galileo's telescope of 1610, which found the moons of Jupiter and the spots on the sun, had single lenses only.

Telescopes intended for terrestrial use have an extra lens in the eye-piece, called an "erecting" lens. For the simple astronomic telescope reverses the image of the object we look at; we see it with the top and bottom interchanged, and the right and left sides likewise inverted. This is of no consequence in astronomy, since there is no up or down in space, and a round planet may be observed just as well one way as the other. But for terrestrial purposes, we must have objects represented to the eye in their true positions. This extra erecting lens diminishes slightly the efficiency of the telescope, because it introduces two additional glass surfaces, the two sides of the erecting lens itself. And as human hands cannot grind lenses with absolute accuracy to their correct theoretic shape, it follows that the erecting lens causes slight errors that do not exist in the astronomical eye-piece. Plate 3, p. 17, shows the moon as seen in an astronomical or inverting telescope (cf. p. 166).

The question is often asked: "What is the magnifying power of a given telescope?" Or the same question occurs in another form: "How near does this telescope bring the

moon?" These two questions are really one. The moon's distance is 240,000 miles (p. 169); a telescope magnifying 1000 times would therefore bring it within a distance of 240 miles; or give us as good a view, approximately, as we would get with the unaided eye if the moon were only 240 miles away.

From what has been said before, it is perfectly clear that, within certain limits, the magnifying power of a telescope is just as great as we care to make it. The magnifying power comes from the power of the eye-piece regarded as a microscope: it is evident that we can use a microscope of high power, or one of low power, on the same telescope, at different times; and thus we can vary the magnification afforded by the instrument as a whole. But there is a definite practical difficulty that limits the available power of the eye-piece.

Suppose we are observing a planet, such as Mars, for instance. The quantity of light received from Mars may be regarded as constant, and therefore a constant quantity of light from Mars reaches the focal image. This light is there spread uniformly over the surface of that image. Now if we double the magnifying power of the eye-piece, we shall see twice as large an image. The same quantity of light from Mars is therefore spread over a larger surface, and so the image is dimmer than before. Increase of magnifying power in the eye-piece enlarges the image and brings out more detail; but it makes that detail fainter (cf. p. 230).

If we continue to increase the magnification, there must come a time when we shall increase the detail, but will be unable to see it on account of faintness. For these reasons, astronomic telescopes are provided with a "battery" of eye-pieces of different powers. It is customary for the

astronomer to try gradually increasing powers until he finds, by experiment, the one that gives the best result. It will not be the same one each night, because a little higher power than usual may be employed when the terrestrial atmosphere is especially clear.

As soon as the above limit of power is reached, no further increase is possible, unless we enlarge the object glass; or, in other words, make the whole telescope bigger. With a larger object glass we can gather more light from Mars, because the "light-gathering" power of an object glass must increase with an increased area of the glass itself. And if we gather more light, we can have a larger focal image without making it too dim for practical use. Experience has shown that under the most extremely favorable terrestrial atmospheric conditions, it is possible to use a magnification of about 100 for each inch in the diameter of the object glass. In the case of the great Yerkes telescope, the diameter of that glass is 40 inches; a power of 4000 should therefore be conceivably possible; and the moon should be brought within the equivalent of $\frac{240000}{4000}$, or 60 miles. But this theoretic result is never quite attained in practice, because all imperfections of the atmosphere are magnified by increased optical power. We see the moon as we would see it with the unaided eye at a distance of 60 miles; but through an atmosphere more like water than air.

Among the most important accessories of a telescope, when it is to be used for accurate measurement, is a pair of "cross-threads" at the focus. These threads are usually made of spider web; for they must be extremely delicate, so that magnification by the eye-piece will not prevent accuracy of observation. The field of view of a telescope provided with cross-threads would look like Fig. 74. When the focal image

of a star is brought to the exact intersection of the cross-threads, by moving the telescope, the instrument is aimed or "pointed" accurately at the star; and if the telescope is attached to brass circles divided into degrees and minutes, it is possible to measure the exact direction in which we see the star projected on the celestial sphere. This kind of measurement is fundamental in the astronomy of precision (cf. p. 197).

FIG. 74. Cross-threads in a Telescope.

Sometimes the single pair of fixed cross-threads is replaced by a pair of parallel threads *aa′* and *bb′* shown in Fig. 75, together with a cross-thread *cc′*. In such an arrangement the two parallel threads *aa′* and *bb′* are made movable, while *cc′* is fixed. The two parallel threads can be moved nearer together or farther apart by means of suitable screws outside the tube of the telescope; and a method is also provided for measuring accurately their distance asunder. This arrangement is called a micrometer (p. 265), and with it short angular distances on the sky can be measured with very high precision. Examples of such measurements are the observation of Mars to ascertain the solar parallax (p. 265), the angular diameter of the sun (p. 118) or of the planets (p. 203), etc.

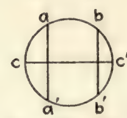


FIG. 75. The Micrometer.

The methods of mounting telescopes for astronomical purposes are most interesting. It is of course essential that the tube be movable: it must be possible to turn it about pivots or "axes," in order that it may be pointed toward different parts of the sky. The most simple form of mounting is indicated in Fig. 76, which shows an instrument called a Meridian Circle. *OE* is the telescope, *O* being the object-glass and *E* the eye-piece. At *f* is the focus, containing the cross-threads. *AX* is a

rotation axis, firmly attached to the telescope. There is no motion of the instrument, except rotation around this one axis; but a complete rotation about that axis is possible. The line AX is made to point due east and west when in proper adjustment; and it is made perfectly level. So the telescope must point north or south accurately, since it is placed at right angles to the axis AX. It follows that if the "sight-line" EfO be continued outward indefinitely beyond O, until it reaches the celestial sphere, it will meet that sphere at some point of the celestial meridian (p. 36). And if the telescope is turned through a complete rotation around the axis AX, the sight-line EfO may be imagined to trace out the celestial meridian on the sky.

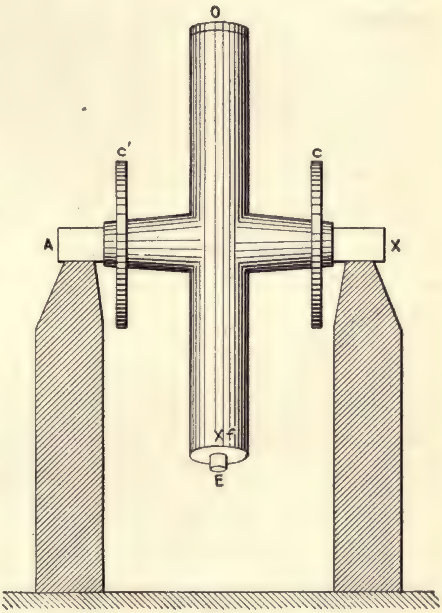


FIG. 76. Meridian Circle.

It results from these considerations that the meridian circle can observe stars on the celestial meridian only; and, conversely, if a star be observed on the cross-threads of the telescope, the observer knows that it is at that moment projected on the celestial meridian. If the exact time of the observation be noted, too, it is possible to calculate the right-ascension (p. 34) of the star; and thus are the right-ascen-

sions of stars and planets determined observationally by astronomers.

This meridian instrument is provided also with two brass circles c and c', divided into degrees and minutes of arc. By the aid of these circles it is possible to measure the altitude (p. 36) of the star, or its angular elevation above the horizon at the moment when it is observed to be on the meridian. From this measurement of altitude it is possible to calculate the star's declination (p. 34). Thus the meridian circle makes known both the right-ascension and declination of the star or planet, and these give us its exact location on the sky at the moment of observation.

It will be noticed that a precise record of the time of such observations is most essential. For this purpose astronomers employ "standard" pendulum clocks of the most extreme accuracy, usually kept in vaults and air-tight cases where the temperature and barometric pressure are not allowed to vary, so as to produce inaccuracy in the running of the clocks. For the actual record of the time of observation, the clock is connected with an electric "chronograph." This instrument maintains an automatic record of the running of the clock upon a sheet of paper attached to a revolving brass drum; and upon this same sheet the observer can record electrically the instant of time when he makes his observation; and he can make this record without his error ever exceeding one-fifth of a second of time. By taking the mean of several observations, the average error can even be reduced below this small amount.

The above process, as outlined, indicates the method of observing stars of unknown location on the sky in order to make known their right-ascensions and declinations. But the same meridian telescope, clock, and chronograph can

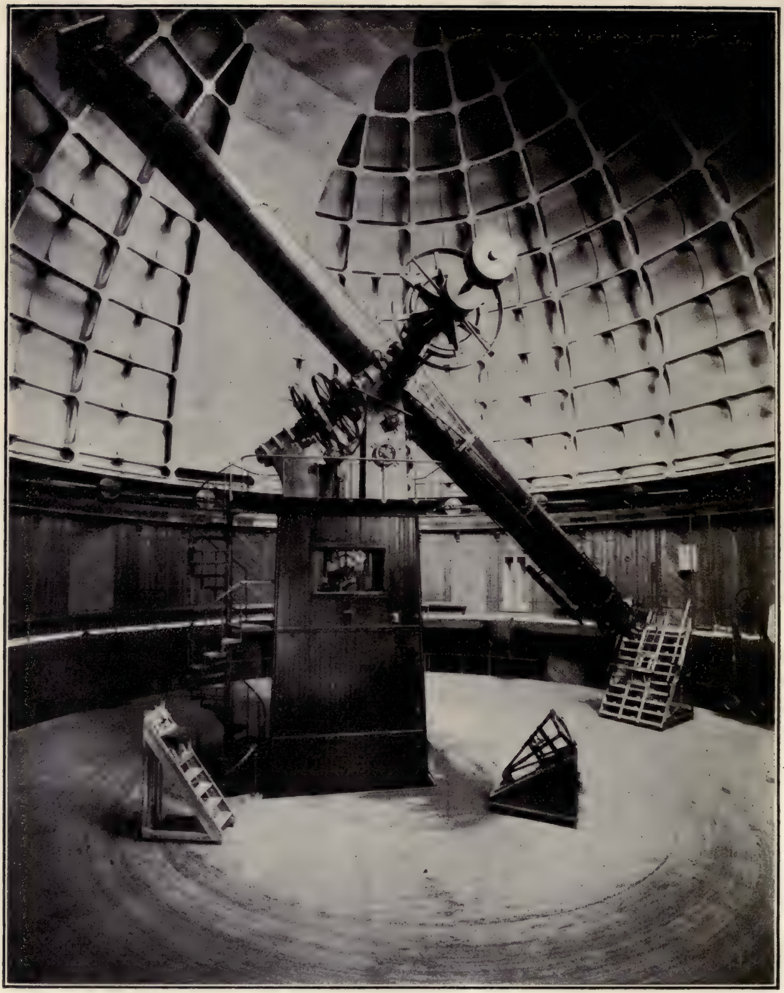


PLATE 13. The Lick Telescope.

be used for the observation of known stars; and will then furnish a check upon the time indicated by the standard clock. It is by this latter process that the astronomic standard clocks are kept correct in order that the observatories may be able to distribute correct time telegraphically for the control of the "regulator" clocks that are to be found in most jewelers' shops, where people "set" their watches (cf. p. 18).

Having thus described briefly the astronomer's instrument of precision, the meridian circle, we must next consider the "equatorial" mounting, the arrangement with which almost all ordinary telescopic observations are made. This is the form of mounting usually fitted with a micrometer for the measurement of small angular diameters, distances, etc. A photograph of a large telescope, mounted equatorially, is reproduced in Plate 13. This instrument is set up at the Lick observatory; the diameter of the object-glass is 36 inches; and the whole observatory floor is built like an elevator, so that it can be moved up and down, to accommodate the observer when the tube is directed to various parts of the sky. The supporting pillar of the telescope mounting passes down through a hole in the floor, so that the instrument itself is not disturbed, when the floor is raised or lowered. James Lick, donor of the telescope, is buried under it.

The first essential of such a mounting is some form of "universal joint," so that the tube may be aimed at any part of the sky. A single axis, such as that of the meridian circle, is not sufficient. Accordingly, in the equatorial, shown again in Fig. 77, we find two axes, A and A', perpendicular to each other. The telescope can be rotated around the axis A'; and this axis itself, with the telescope attached, can be rotated around the axis A. A combination of the two rota-

tions furnishes a universal motion, giving access to any part of the sky. By means of these rotations, the tube can be moved from the position of Fig. 77 to that of Plate 13.

The axis A is called the polar axis; and the instrument is so constructed and adjusted that this axis points directly toward the celestial pole (p. 32). Since the stars perform their apparent diurnal rotations (p. 33) around that pole, it follows that they will *seem* to perform them around the polar axis of the equatorial. This simplifies the use of the instrument; for a star once brought into the field of view of the telescope, we can keep it there by moving the instrument around the polar axis only. For the telescope must be kept moving to prevent the diurnal rotation of the stars from carrying the object under observation out of the field of view. The necessary rotation around a single axis can be accomplished automatically by means of clock-work, shown inside the vertical supporting pillar in Plate 13. The astronomer is thus left free to pursue his observations without any further attention to the telescope. If there were no inclined polar axis, but, in its place, a pair of axes, one verti-

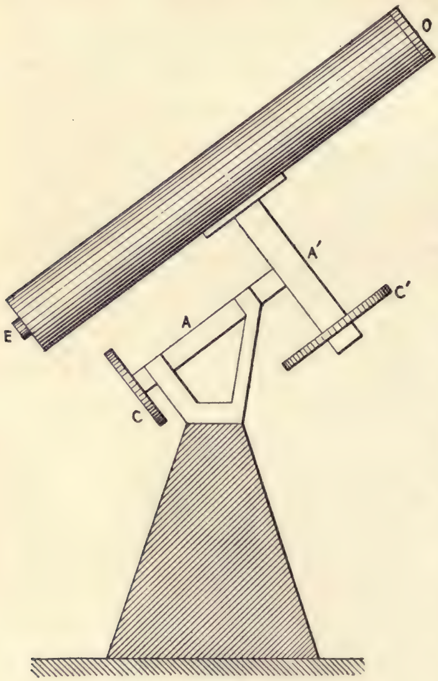


Fig. 77. The Equatorial.

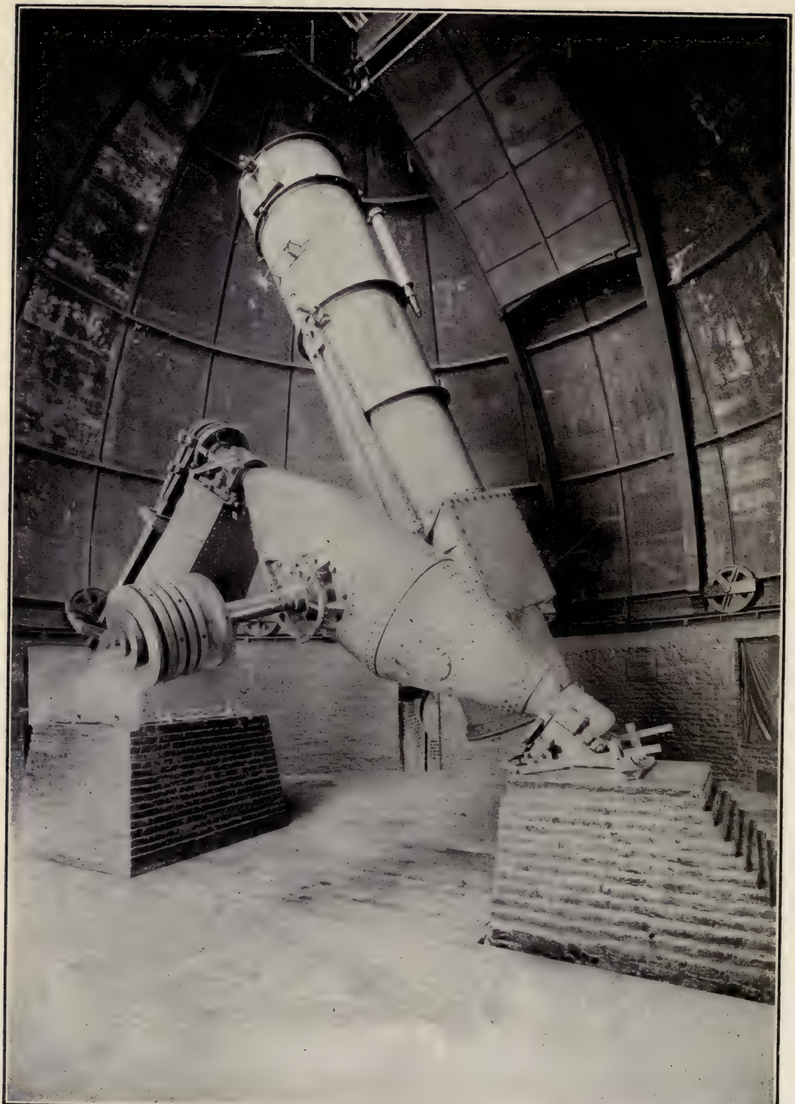


PLATE 14. The Crossley Reflector.

cal and the other horizontal, this simple clock-work plan would be impossible.

The equatorial carries two circles, c and c', divided into degrees, and attached to the axes A and A'. With the circle c' it is possible to measure the declination (p. 34) of an object in the field of view; and with the circle c its hour-angle (cf. p. 68) can be measured. And if we wish to find a known object which is invisible to the unaided eye, we have merely to turn the telescope around the two axes, until the two circles indicate the object's known declination and hour-angle, when it will at once appear in the field of view.

The foregoing description of the telescope applies to the "refractor" with its object-glass and eye-piece. But there is another form of instrument, the "reflector," in which the object-glass lens is replaced by a curved mirror. This forms a focal image similar to that given by a lens; and the image is again examined with an eye-piece lens as before. Plate 14 shows the equatorially mounted Crossley reflector of the Lick observatory. The polar axis is shaped like a double cone; and the reflector is at the bottom of the big tube. The focal image is formed near the top of the tube, and is there examined with an eye-piece shown in the plate. It was with this instrument that Keeler made his famous photographs of spiral nebulæ, one of which is shown in Plate 2 (p. 4). Others will be described in a later chapter.

Telescopes intended for astronomic photography are always mounted equatorially, and their clock-work mechanism is made especially precise. For when it is desired to make long photographic exposures, the telescope, which takes the place of a camera, must of course be kept in motion, so as to neutralize the diurnal rotation of the stars. If this is not done quite exactly, we obtain only a worthless "moved

negative." To be certain of this essential element in astro-photography, such telescopes are usually made with two tubes, like a pair of opera-glasses. The one tube is a photographic telescope, the other a visual one, provided with cross-threads. With this arrangement, the astronomer can watch an object with the visual telescope, while it is being photographed in the other.

If the clock does not move quite perfectly, the error will at once show itself; for the object will move away from the cross-threads in the visual telescope. The slightest tendency to such motion must be prevented; and for this purpose the clocks of such instruments are provided with certain adjusting screws, or other devices, with which the astronomer can correct any possible errors of the clock while it is actually running, and while the photograph is being exposed.

The last important astronomic instrument to be described here is called the Spectroscope, of which Fig. 78 shows the essential parts. The reader will recall that if we look at a source of light through a glass prism, it will be spread out into a band of colors, violet, indigo, blue, green, yellow, orange, red. The actual spectroscope consists of a prism P (or a succession of several prisms) and two brass tubes. One, the "collimator" C, admits light to the prism. It has a narrow slit s, at one end, so that the light may enter as a thin line, parallel to the edge of the prism; at the other end there is a lens, O, to render the rays of light parallel. The

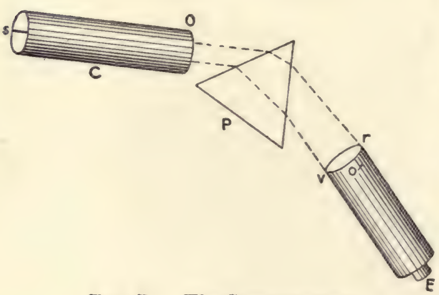


FIG. 78. The Spectroscope.

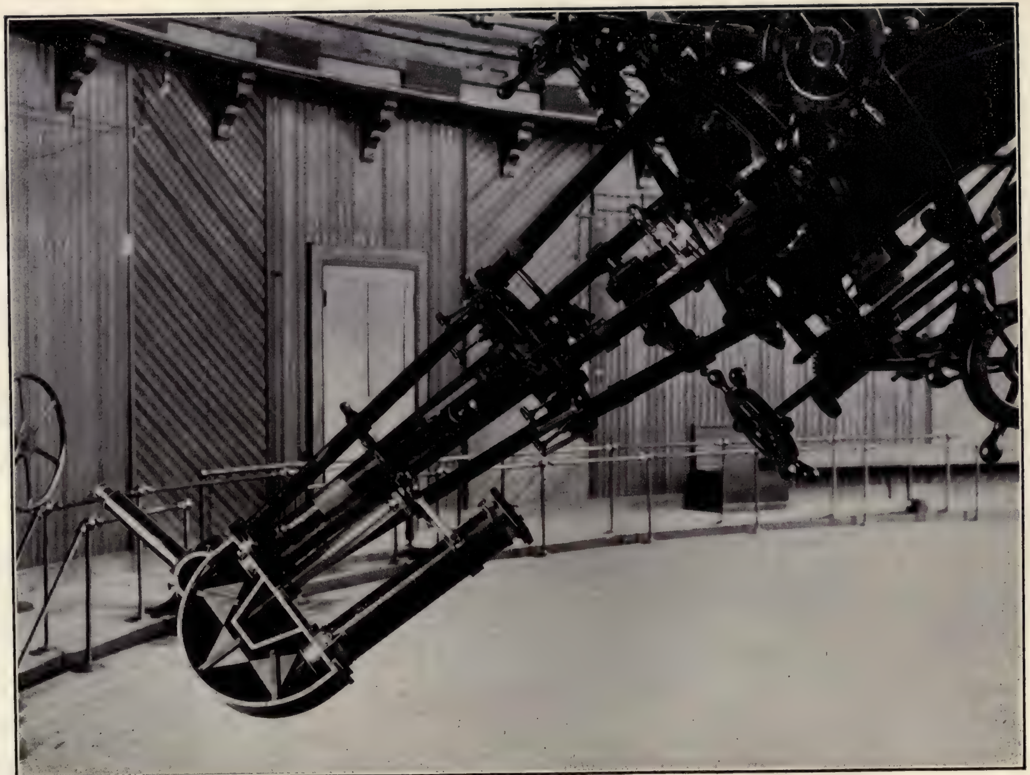


PLATE 15. Lick Spectroscope.

other brass tube is merely a little view telescope *O′E*, with which to examine the "spectrum" and to magnify it. Frequently it is better to substitute for the prism a glass plate on which a great number of very close parallel lines have been ruled with a dividing engine. Such a glass spreads light into a spectrum similar to the one obtained with a prism. Plate 15 shows a spectroscope attached to the eye-end of the big Lick telescope. Three prisms are used; and the eye-piece of the view telescope is replaced by a plate-holder, so that stellar spectra may be photographed.

If we send colored light into the spectroscope, the result is as follows: If the light is red, it goes to the red part of the spectrum, where it belongs, and we there see a bright red line, which is merely an image of the spectroscope slit. But if we send in white light, which is really a mixture of all kinds of colored light, the prism analyzes it: the yellow part goes to the yellow part of the spectrum, etc. We then see the colored "continuous spectrum," made up of an enormous number of slit-images side by side (see Plate 16, 1). But if the light comes from an incandescent gas, instead of a solid or liquid, we see no continuous spectrum, but a series of bright-colored lines, or images of the slit, variously located throughout the spectrum; and the combination of lines is different for every different gas. We can actually determine the name of the incandescent gas from the positions of the lines. This is well shown in Plate 16, 3, 4, and 5, for the incandescent vapors of sodium, hydrogen, and potassium.

But the most singular thing of all relates to the "absorption" of light by gases; and this is one essential thing in astronomic spectroscopy. If a beam of white light is passed through a layer of gas or vapor before entering the spectroscope, this vapor will sift out, and absorb, precisely those

light-rays or colors which the gas or vapor would itself emit if it were incandescent, and which would then appear as a bright-line spectrum in a spectroscope.

From the foregoing principles we shall find it possible to study the chemical constitution, or the nature of the vapor existing in the sun and stars. And there is also another principle, due to Doppler, by means of which we can obtain information of still another kind. It is a fact that the spectral lines may at times be shifted out of their proper positions as ordinarily seen in the spectrum. We are taught in the science of physics that red light has comparatively long light-waves; violet light, the shortest waves. Now if a source of light, such as a star, happens to be increasing its distance from us at the time of observation, we shall receive fewer light-waves per second than would be the case if the distance were stationary. But if we receive fewer light-waves per second, they will seem to be longer waves, and therefore more like the waves from red light. The effect is to shift the observable lines toward the red end of the spectrum. This shift can be measured with a micrometer, and from such measurements it is actually possible to determine the velocity with which a star is approaching us or receding from us in space (cf. p. 245).

The planet Venus has often been observed to test this principle; and the photographed spectrum of Venus in Plate 16, 6, shows plainly the shift of the spectral lines. The middle spectrum belongs to Venus; the two outer ones are artificial spectra produced in the observatory for comparison. Microscopic measurement of this "spectrogram," as it is called, enables us to compute that Venus was increasing its distance from the earth at the rate of eight miles per second when the spectrogram was made. From our knowl-

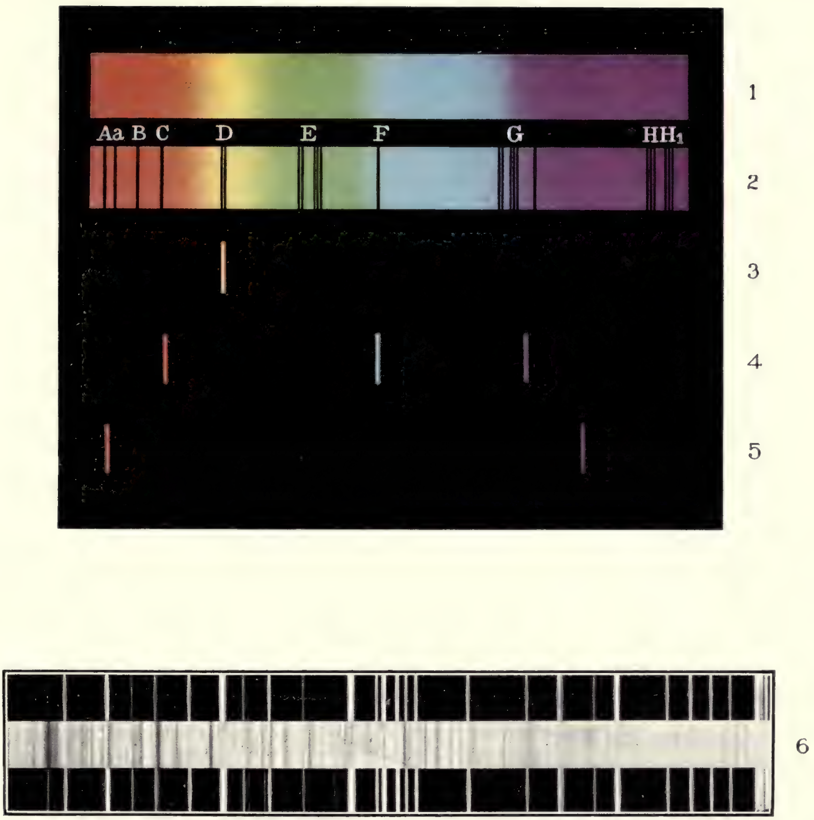


PLATE 16. Various Spectra.

edge of the orbital motions of Venus and the earth, it is possible to check this result by an entirely independent method of calculation; and thus no doubt remains as to the correctness of the Doppler principle.

In addition to the spectroscope just described, astronomers also employ a "slitless" instrument. This is made by mounting one or more prisms outside the object-glass of a telescope, prisms large enough to cover the entire object-glass. With such an instrument it is possible to photograph on a single plate the spectra of all stars in the telescopic field of view, while the slit spectroscope will give only one star-spectrum at a time. It is therefore clear that the slitless instrument is best for statistical researches intended, for instance, to classify all spectra; but the slit instrument is more accurate, and is also the only form permitting the measurement of line shift in accordance with Doppler's principle.

CHAPTER XVI

SUNSHINE

In Chapter XIV we have considered at length the ancient problem of determining the distance of the sun from our earth: let us next attempt to describe the sun itself. Here we meet something distinctly modern in the venerable science; for almost all knowledge we have of the sun is knowledge obtained during the last hundred years. The ancients knew little or nothing about it.

Our subject falls readily into two parts: first, information obtained by the use of the spectroscope (p. 282); and secondly, investigations other than spectroscopic. Let us begin with the chemistry of the sun. We have already had an explanation (p. 283) of the manner in which gases absorb certain light-rays while passing through them, each gas absorbing its own particular combination of such rays. This principle, applied to the sun, gives the following result. The body of the sun sends out white light which the spectroscope would naturally split up into a continuous spectrum (p. 283). But before this white light can pass through the outer gaseous layers of the sun, the absorption phenomena take place; and so the regular solar spectrum appears in the spectroscope as continuous, but crossed by a vast number of black lines. These lines are simply the dark places where absorption has occurred. Thus, for instance, if there is iron vapor in the outer solar atmosphere, we shall see dark lines in the solar spectrum at exactly the points

where we should see bright lines (p. 283) if we vaporized some iron in the laboratory, and examined the spectrum of its light. All these dark lines are merely bits of nothingness occupying the places where there should be images of the spectroscope slit, if absorption were absent. Such are the famous Fraunhofer dark lines, named from their discoverer.

Plate 16, 2, shows a number of the principal ones, together with the letters by which spectroscopists designate them. The double *D*-line, for instance, arises from absorption due to sodium vapor, as may be seen from a comparison with the sodium vapor spectrum, 3.

So much being premised, we can now explain the method of using the spectroscope to ascertain the sun's chemical composition. It is merely necessary to attach a spectroscope to an ordinary telescope in such a way that the slit will be in the telescopic focal plane (p. 272). Arrangements must then be made, by means of a tiny reflecting prism attached to the slit, to throw into the view telescope the spectrum of any desired substance vaporized and heated to incandescence in the observatory, near the telescope. We then see the solar spectrum and the artificial spectrum side by side.

Now this artificial spectrum is a bright-line spectrum (p. 283): and if opposite each of its bright lines we find a dark Fraunhofer line in the solar spectrum, we have conclusive proof that the substance vaporized in the observatory is actually present as a gas in the outer atmosphere of the sun. Many terrestrial chemical elements have been thus found in the sun: the general conclusion is that earth and sun have a similar constitution, which was to be expected, if we accept any hypothesis postulating a common origin for the sun and earth. (Cf. nebular hypothesis, p. 235.)

Next we must consider a very interesting phenomenon

called the Reversing Layer. We have seen that the Fraunhofer dark lines are due to absorption in the outer gases of the sun. But these gases are themselves so hot as to be incandescent. It is only because the inner sun is so much hotter and so much more luminous that we ordinarily see only the light from the inner sun and not from the outer gases. The latter are dark by comparison only.

There is just one occasion when it is possible to observe the light from the outer gases separately and directly. This occurs during a total solar eclipse, when the moon happens to pass accurately in line between the earth and the sun. On such an occasion, when the lunar globe, advancing in its orbit around the earth, has almost covered the sun, just before it is covered absolutely, there must be a moment when a tiny sickle of the outermost layer of the sun is alone visible. At that exciting moment, and at that moment only, can we look upon the outermost incandescent gases.

But their light suffers no further absorption; and so, like all incandescent gases, should give a spectrum consisting of bright lines only. And this is precisely what occurs. If we observe the advancing eclipse, just for an instant before totality, the continuous solar spectrum with its myriad black Fraunhofer lines is suddenly replaced by a bright-line spectrum. Each bright line corresponds accurately to one of the vanished dark lines, since the dark lines were caused by absorption due to the very gases that are now furnishing the bright-line spectrum. The critical instant over, the sun is covered totally, and the bright lines in turn disappear, too. This phenomenon is appropriately termed the Flash Spectrum.

The surface we see when we turn a telescope upon the sun is called the Photosphere. It is not uniformly brilliant,

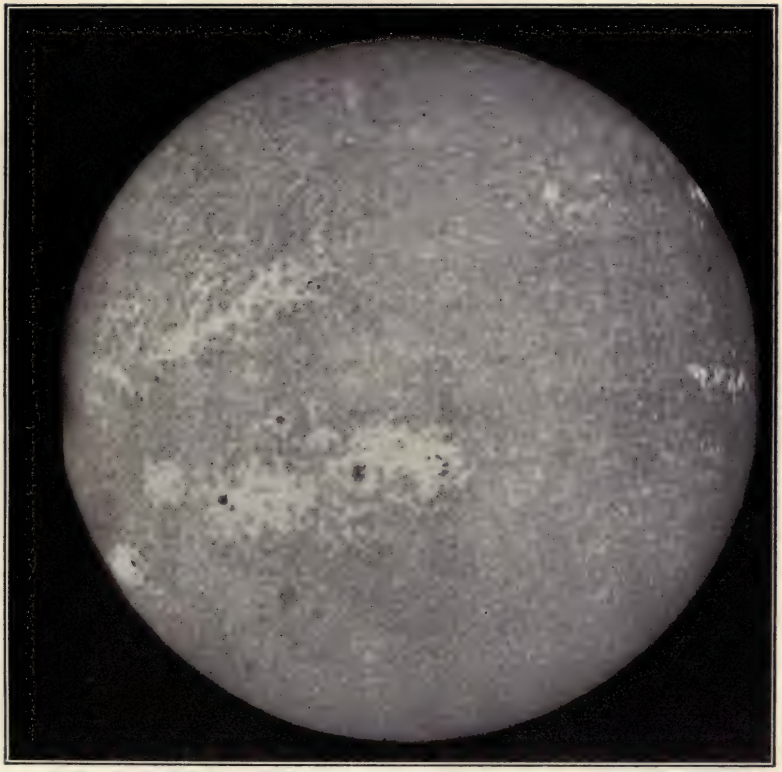


Photo by Fox.

PLATE 17. The Sun.

but shows certain brighter "nodules," and also especially bright points called "faculæ,"[1] which appear mostly at the edges of the sun and near the sunspots. They are perhaps suspended in the solar atmosphere at a high level, and owe their extra brightness to our observing them through a somewhat thinner layer of solar atmosphere. Probably everything we see when we examine the sun is "atmosphere." Young aptly compares the state of affairs to a gas-burner, in which the heated particles of the mantle are far more luminous than the flame of gas which heats them.

Let us next enumerate some of the principal facts known about the sunspots (p. 17). We are on sure ground when we speak of their size; for we can, as usual, measure their angular diameters, and then compute their linear dimensions from our knowledge of the sun's distance. At times they are 50,000 miles in diameter; and exceptionally large ones can even be seen without a telescope. But our knowledge is less certain when we attempt to explain their cause. They are to be regarded probably as solar atmospheric disturbances or storms. In that case we should expect them to shift their positions on the sun's surface, much as storm-centers move across our earth. And we find by observation that all spots have a common drift; and those near the solar equator also drift toward it, while those far from the equator drift toward the solar poles. This might be analogous to our phenomena of the trade-winds, especially as spots never occur near the solar poles, or exactly at the equator.

So the real cause of the spots must be regarded as unknown. They may be eruptions from the interior; they may be gases rushing downward into hollows. But we cannot help thinking they are vast storms of some kind;

[1] Plate 17.

storms of which the materials are incandescent gases, moving with great velocities, and at enormously high temperatures.

The duration of individual spots is not great, never more than 18 months; and the central, apparently blackest part of the spots, called the "umbra," is not really dark, but appears so only though contrast with the much more luminous surrounding solar material. They have also a periodicity, discovered in 1843 by Schwabe; and this is perhaps the most interesting of the many unexplained observations of the sun. Schwabe found, by constant watching of the solar surface, that every eleven years there is a period of extra great spot frequency. This discovery owes its importance to the known fact that there exists also an eleven-year period in the frequency of terrestrial magnetic storms: and especially great sunspots are always accompanied with very strong magnetic disturbances and auroral displays on earth. This establishes the existence of some intimate magnetic relation between earth and sun; but it has not yet been possible to reach a satisfactory explanation of it. Nor have astronomers been able to make certain that any other terrestrial meteorological phenomena exhibit a real connection with the spots, though many efforts have been made to do so on account of the assistance such investigations might give in the matter of weather prediction.

The accompanying Plate 18 is a photograph of a particularly large sunspot which occurred July 17, 1905. It had an unusual and very brilliant "bridge" across the umbra.

The next important question requiring consideration relates to the size of the sun. We have already determined its distance to be about 93 million miles. Observations very similar in principle to methods already explained for the moon and planets enable us to measure the sun's apparent

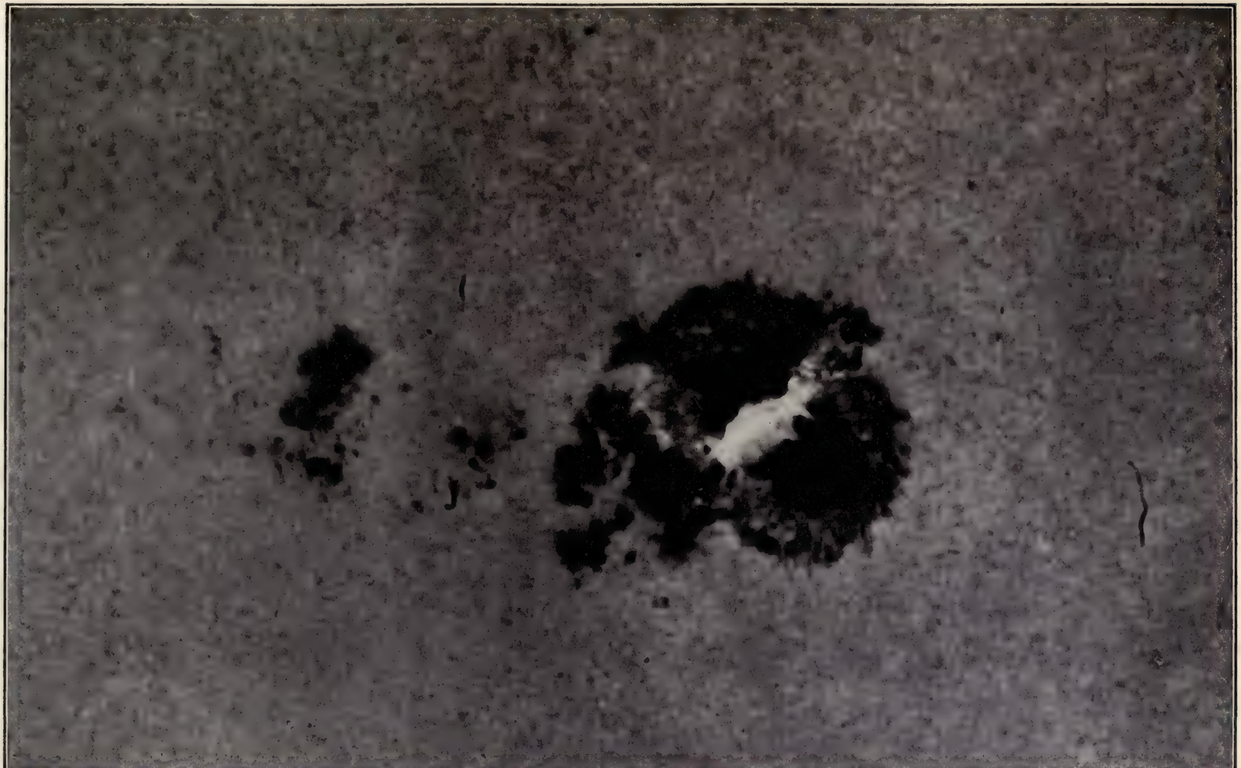


Photo by Fox.

PLATE 18. Great Sunspot.

angular diameter (cf. p. 203); and this we find to be 32′ 4″, on the average. In Fig. 79 we then have, as usual, a long, narrow triangle, of which the base is the sun's linear diameter *AB*. This we can calculate readily, because we know the sun's angular diameter, or the angle 32′ 4″ at the vertex

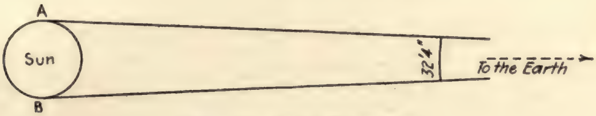


FIG. 79. Sun's Diameter.

of the triangle, situated on the earth. The result comes out nearly nine hundred thousand miles for the sun's linear diameter.

Comparing this with the known terrestrial diameter (p. 97), we find that the sun's diameter is approximately 110 times that of the earth.

To ascertain the sun's mass is a little more difficult than to find its diameter; but it can be estimated by simple mathematical methods,[1] which show that it is about 330,000 times the earth's mass.

We can also calculate the force of gravity that must exist on the solar surface as compared with the gravitational attraction existing on our earth. For the gravity force on the surface of a sphere is, by Newton's law, proportional to the mass of the sphere, divided by the square of its radius. If we then consider all solar quantities expressed in terms of the corresponding terrestrial quantities as units, we have:

$$\text{Solar force of gravity} = \frac{\text{solar mass}}{(\text{solar radius})^2} = \frac{330000}{(110)^2} = 28, \text{ approximately.}$$

This means that an ordinary one-pound weight would

[1] Note 35, Appendix.

weigh 28 pounds, if transported to the sun's surface, and there weighed with an ordinary terrestrial spring-balance.

The solar volume or bulk compares with that of the earth in the proportion of the cubes of their radii; that is, as 1 to $(110)^3$. This makes the solar volume 1,300,000 times the earth's. And since density or specific gravity is proportional to mass divided by volume, it follows that the solar density, as compared with the earth's, is:

$$\frac{\text{solar mass}}{\text{solar volume}} = \frac{330000}{1300000} = 0.25,$$

or only about $\frac{1}{4}$ the earth's density. The latter, compared with water, is about 5.5; so the solar density is only about $1\frac{1}{2}$ times that of water. This means that a cubic foot of average solar material, transported to the earth's surface, would there weigh only about $1\frac{1}{2}$ times as much as a cubic foot of water.

The quantity of light and heat received by us from the sun is certainly enormous; and yet it cannot be more than a small fraction of the total quantity actually radiated into space. A most interesting question arises in connection with this matter: How does the sun maintain through the ages so gigantic an output of heat energy? What is the source of the sun's heat? Helmholtz has proposed a plausible possible cause,—the shrinkage of the sun's vast bulk under the influence of its own gravitational attraction. He computed that an annual shrinkage of only 300 feet in the solar diameter would be transformable into enough heat energy to keep radiation active as it now is. And it would require 8000 years for this diminution of size to reduce the sun's observable angular diameter by a single second of arc; nor could any smaller diminution be discovered by observation with actual astronomic instruments.

Photo by Slocum.

PLATE 19. The Prominences.

The fact that we have not observed a reduction of the sun's size is therefore no argument against the Helmholtz theory.

Up to this point we have supposed the sun to consist of a highly heated interior of more or less unknown constitution, surrounded by an atmosphere of incandescent gases which produce the Fraunhofer lines by absorption, and the bright-line flash spectrum during an eclipse. But we know much more than this. Beyond the photosphere and the reversing layer of gases is the Chromosphere, or color sphere, composed principally of great flaming masses of red hydrogen vapor. Sometimes great red jets burst outward to immense elevations from the solar surface. These are the Prominences (Plate 19); and while these various solar layers have different names, it must not be supposed that they are distinct. They intermingle, doubtless, at their boundaries, and melt into each other without sudden interruptions.

The hydrogen prominences were first seen during a total solar eclipse, when the photosphere was completely covered by the moon. But just after the eclipse of 1868, Janssen and Lockyer for the first time succeeded in observing them without an eclipse. We cannot see them by merely looking at the sun with a telescope, and covering the central part of the solar disk at the telescopic focus, because the terrestrial atmosphere is strongly illuminated by the sun itself, and so the prominences become invisible by contrast. But if we bring the slit of a spectroscope tangent to the sun's disk at the focus of a telescope, and open the slit wide, the prominences become visible.

For we then see two spectra superposed, one upon the other. The first is an ordinary continuous solar spectrum derived from the diffused sunlight in the terrestrial atmos-

phere, the second a bright-line spectrum from the incandescent hydrogen of the prominences. Now, if we employ in the spectroscope a number of prisms, instead of a single one, both these spectra will be spread out to a great length. The continuous spectrum will be thereby rendered dimmer, but the bright-line spectrum will have its lines separated further, without rendering them less brilliant. If we continue thus increasing the "dispersion" of the spectroscopic prisms, we shall finally diminish the luminosity of the atmospheric continuous spectrum until it disappears practically, and we see only the bright-line spectrum of the prominence.

Now, as we know, these bright lines are ordinarily merely images of the slit. But if the slit has been opened wide enough to be wider than the angular diameter of the prominences, the bright lines become images of the prominences, instead of images of the slit. We have therefore merely to point the view telescope at a place in the spectrum where there is ordinarily a bright hydrogen line, and we shall see the prominence, if there happens to be one on the sun's edge at the point where we have placed the widely open slit tangent to the sun's image at the telescopic focus.

In 1891 Hale invented an instrument called a spectroheliograph, with which the prominences may be photographed without an eclipse. Plate 19 was made with such an instrument. It utilizes the light of calcium gas, which, like hydrogen, is plentiful in the prominences; and can be made to give a fine bright line in the middle of the usual dark Fraunhofer calcium line due to the photosphere and reversing layers. The spectrum is allowed to fall on a screen having a second narrow slit corresponding accurately to the bright calcium line from the prominence. Through this slit

Photo by Barnard and Ritchey.

PLATE 20. Total Solar Eclipse, with Corona.

the calcium light which originated in the prominence passes to a photographic plate, so that the plate receives prominence light only. Now, by mechanical means, the original slit of the spectroscope is moved across the solar image at the telescopic focus; and the second slit in the screen is moved in unison. The result is to build up a picture of the sun on the photographic plate with light from the outer solar layer only, and thus to secure a photograph of the prominences.

Still another extraordinary solar phenomenon has been discovered during total eclipses. This is the Corona, which bursts into view when the sun is completely concealed by the moon, and appears as a faint luminous ring, of more or less irregular shape, around the sun. We know that it belongs to the sun, because its spectrum is that of an incandescent gas, not a continuous solar spectrum, such as it would be if we had here to do merely with solar light reflected in some way from the moon; and also because the coronal form always appears the same at the same moment, even when seen from observatories widely separated on the earth. Beyond this little is really known for certain as to the corona.

Plate 20 is a photograph of the corona during a total eclipse, the sun's disk being entirely covered by the interposed moon. The form of the streamers indicates that the phenomenon may be electric or magnetic. A large prominence appears near the lower part of the plate, jutting out from the obscured solar disk. The height of this prominence is estimated by comparing it with the photographed diameter of the sun: this would make the prominence about 40,000 miles high.

The question of solar axial rotation can be examined best by studying the apparent motions of the spots. (Cf. the

case of the planets, p. 202.) It is found that they always seem to cross the solar disk from east to west; and when one of them makes a complete rotation, disappearing at the western, and later re-appearing at the eastern edge of the sun, the whole revolution takes about $27\frac{1}{4}$ days. But this is not the true period of solar axial rotation: the above observed period, and the true one, are related by an equation analogous to the equation connecting the synodic and sidereal periods of the planets (p. 209), since we must correct the observed period for the effect of the earth's orbital motion around the sun during the time occupied by the latter in turning on its axis.[1] This correction makes the true axial rotation period of the sun about $25\frac{1}{3}$ days.

These sunspot motions not only tell us the period of the sun's axial rotation; they also enable us to ascertain the direction in space of the rotation axis (cf. p. 203). The apparent paths of the spots are generally curved; but on June 3 and December 5 they appear quite straight. Referring to Fig. 80, we see that this straightness determines on these dates the axial or polar points A and B on the sun's edge, and also the angle by which the solar rotation axis is inclined to the plane of the ecliptic. The angle is about 83°.

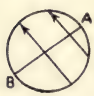


FIG. 80. Sun's Rotation.

[1] The planetary equation here takes the form:

$$\frac{1}{\text{true period of rotation}} - \frac{1}{\text{length of year}} = \frac{1}{\text{observed period of rotation}}.$$

CHAPTER XVII

ECLIPSES

ECLIPSES are occasional phenomena; they are usually defined as temporary obscurations of the sun or moon, either wholly or in part. We have seen that these two bodies are visible from two very different causes: the sun is self-luminous, — it is visible because it sends us its own light; the moon is merely rendered visible when illuminated by the sun. Therefore a solar eclipse can occur only if the moon interposes between us and the sun, thereby preventing our seeing it; but a lunar eclipse happens when the earth passes between the moon and sun, so that solar light cannot reach the moon, and render it visible. Thus there is not necessarily any actual obstruction in the way of our seeing the eclipsed moon; it is invisible merely because it is dark for the time being. Figure 81 makes all this plain.

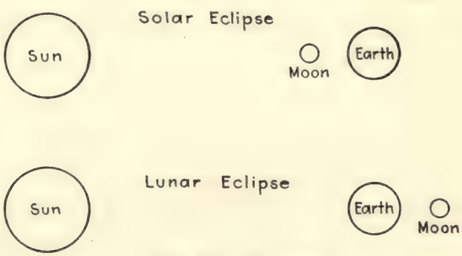


FIG. 81. Eclipses.

Hipparchus was the first to explain eclipses, and the method of making a fairly good approximate prediction of their occurrence. Figure 81 shows that if the orbits of the earth and moon were both situated in a single plane surface (here represented by the plane of the paper) there must result one eclipse of the sun and one of the moon during

each revolution of the moon in its orbit around the earth. This simple state of affairs is modified and complicated by the fact that the lunar orbital plane is actually inclined about 5° to the ecliptic plane, in which the earth's orbit is situated (p. 160). Figure 82 is supposed to represent a portion of the surface of the celestial sphere. *NS* is part of the ecliptic circle, in which the sun is always seen; and *NM* is part of the great circle cut out on the sphere by the plane of the moon's orbit, in which great circle the moon is seen, for the same reason that the sun is always seen in the ecliptic (p. 160). *N* is the point of intersection of these two great circles on the celestial sphere, the angle between them being 5° only. The point *N* is called the "node" of the lunar orbit; and there is another similar node on the opposite side of the sky, because any pair of great circles must necessarily intersect at two opposite points on the sphere.

FIG. 82. Moon's Circle.

We have already seen that the moon moves around the earth, and therefore appears to travel around the sky among the stars, at the rate of about 13° per day, so that it overtakes and passes the sun once in each synodic period or lunar "month" (p. 161). When it thus passes the sun, the two bodies are said to be in conjunction (cf. p. 209). If this conjunction happens to occur exactly at the nodal point *N*, then sun, moon, and earth will lie in a single straight line, and a "central" eclipse will occur. Furthermore, it will be an eclipse of the sun; for if the sun and moon appear projected at a single point of the sky, they must both lie on the same side of the earth (Fig. 81, solar eclipse).

But an eclipse can also happen when the sun and moon are in opposition, or 180° apart, as seen projected on the sky. If such an opposition takes place when the moon is exactly in the node N, and the sun in the other, or opposite, node, we once more have sun, earth, and moon in a single straight line, and a central eclipse takes place. Only, in this case, the earth is between the sun and moon (Fig. 81, lunar eclipse), and the central eclipse is a lunar eclipse.

The problems connected with eclipse prediction would present but little of interest beyond the above, if the node N always remained at the same point of the ecliptic. But this node is constantly moving along the ecliptic, — a phenomenon somewhat analogous to precession of the equinoxes (p. 126) in the case of the earth. Only, the lunar node, unlike the equinoxes of the terrestrial orbit, moves quite rapidly, making a circuit of the entire ecliptic once in about 19 years. It is this phenomenon that complicates the eclipse problem and makes it interesting.

Up to this point we have supposed eclipses to occur at the exact nodes only; and this would be the case if the sun and moon appeared to us on the sky as mere mathematical points, like the fixed stars. But both sun and moon have quite large disks, as we see them projected on the sky. Each disk has a diameter of about half a degree of arc. Consequently (Fig. 83), when a conjunction occurs, these two disks may just touch if their centers are half a degree apart, provided we suppose the terrestrial observer located at the earth's center.

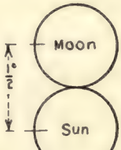


FIG. 83. Contact Eclipse.

But an observer on the surface of the earth, as at the point O in Fig. 84, will see the lunar and solar disks in contact when

the moon is at M_1; while to an observer at the earth's center c, there would be no contact until the moon had advanced to M_2. It is clear from Fig. 83 that the angle ScM_2 is $\frac{1}{2}°$. The angle ScM_1 (from the center of the sun to the center of the moon, as seen from the center of the earth) is $1\frac{1}{2}°$, approximately,[1] thus enlarging greatly the possibility of an eclipse being actually visible from some point or other on the earth's surface.

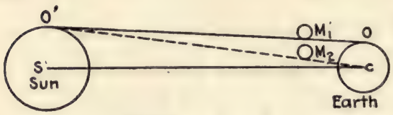


FIG. 84. Observer on the Surface of the Earth.

Now referring again to Fig. 82, we see that if the conjunction occurs when the centers of the sun and moon are at the points S and M, just far enough from the node N to make these two points $1\frac{1}{2}°$ apart, the two disks will just touch, and we shall barely escape the occurrence of an eclipse, visible from some point of the earth's surface. If M and S happen to be a little nearer the node, the two disks will overlap, and there must be at least a partial solar eclipse.

Knowing the angle at N to be 5°, there is no difficulty in calculating how great must be the angular distances NM and NS, to make the distance SM just $1\frac{1}{2}°$. This distance NM is thus found to be about 17°; so that when conjunction occurs within about 17° of the node, there is an eclipse. But this number 17° may vary all the way from 15° to 19° in different years, largely on account of small periodic changes of the angle N, between the two orbital planes. These changes are of course due to orbital perturbations (p. 206). The number 17° is called the "solar eclipse limit."

As we have seen, these eclipses at conjunction are solar eclipses; but corresponding eclipse limits exist also in the

[1] Note 36, Appendix.

case of oppositions of the sun and moon, when lunar eclipses occur. Referring again to Fig. 81, we see that a solar eclipse is possible, only if the disks of the sun and moon, as projected on the sky, actually overlap. But Fig. 85 makes plain that a lunar eclipse will happen if the moon enters or touches the shadow cast into space by the earth. But the apparent angular diameter of this shadow, as seen from the earth,

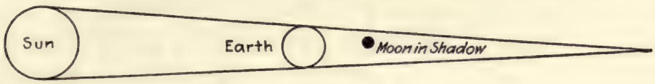


Fig. 85. Lunar Eclipse.

and at the distance of the moon from the earth, is larger than the moon's angular diameter. The result is that a lunar eclipse takes place if the center of the moon and the center of the shadow are separated by an angular distance of less than about 1°, as seen from the earth. We can, of course, again calculate how far the sun must be from the node, at the time of opposition, to make this angular distance less than 1°. We thus find the lunar eclipse limit of about 11°, with a variation between 10° and 12°, in round numbers.

Since conjunction always happens at new-moon, and opposition at full-moon, it follows from the foregoing simple considerations that there will be a solar eclipse at the time of new-moon, if the sun is within about 17° of the node; and there will be a lunar eclipse at the time of full-moon if it is within about 11° of the node. Two other simple conclusions follow at once: (1) Since the sun appears to move about 1° daily in the ecliptic, there will be a solar eclipse if the date of new-moon falls within 17 days of the date when the sun appears in the node. And there will be a lunar eclipse if the date of full-moon falls within 11 days of the same date, all in round numbers. (2) The solar eclipse limit being

the larger, solar eclipses must be more frequent, on the whole, than lunar eclipses.

Hipparchus was the first to explain correctly the seeming paradox that lunar eclipses are seen much more often than solar eclipses, although the latter occur more frequently. The reason is perfectly simple. When the earth interposes between the sun and moon, and the moon thus enters the earth's shadow, it becomes dark at once, because it gives no light of its own. Consequently, any one on the earth who should be able to see the moon will fail to see it on account of the eclipse. But at any given instant, the moon should be visible from half the earth's surface; therefore, if there is a lunar eclipse, at least half the earth's inhabitants will see it.

But in the case of a solar eclipse, the sun is not made dark. The sun's light is actually cut off by the interposed moon; and it is never at any one time thus cut off from observers living in more than a small part of the earth's surface. In Fig. 86, an observer on the earth at A will see the moon projected in his zenith, just as it would be seen by an observer at the earth's center C. But an observer at B will see the moon in the direction BM, instead of AM. This will project it in the sky for the two observers at points whose angular distance apart is equal to the angle BMC. Now this angle may be as great as 1°, approximately; so that the disks of the moon and sun might easily overlap for an observer at A, but not for an observer at B (cf. Fig. 84, p. 300). In other words, solar eclipses are by no means

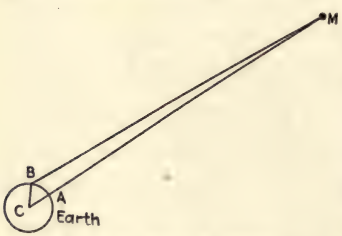


FIG. 86. Solar Eclipse.

visible throughout an entire hemisphere of the earth, like lunar eclipses. In fact, the distances of the sun and moon from the earth are such that any total solar eclipse can be seen from a very narrow strip of the earth's surface only: not more than 70 miles wide at the most, and extending through a distance of much less than a hemisphere. Plate 20 (p. 295) is a photograph of a total solar eclipse.

Having thus outlined the general nature of eclipses and their causes, we shall next describe certain special phenomena which are of sufficient interest to be mentioned. The earth's shadow, into which the moon enters when eclipsed, is not uniformly dark throughout. It is, in fact, made up of two parts, — the "umbra," or shadow proper, and the "penumbra," or partial shadow. Figure 87 shows how the penumbra is formed. There are two penumbral regions, as it were, and one umbral region. The latter is a central cone which re-

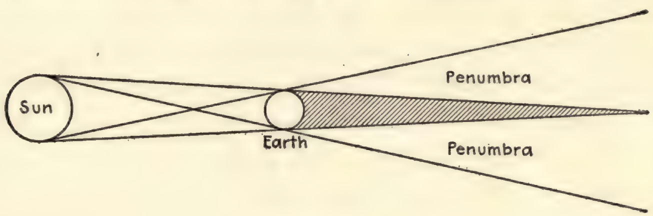


FIG. 87. The Penumbra.

ceives light from no part of the sun. The two penumbral regions receive light from part of the sun, while the rest of space behind the earth receives light from the entire solar surface. It is evident that the darkness of the penumbra will increase gradually from its outer edges to the boundary lines, where it gives the black umbra. The moon, when about to be eclipsed, will therefore enter the penumbra

first, and be partially darkened; and the darkening will increase gradually until it becomes practically complete, as the moon enters the umbra. The same gradual phenomena will be repeated in the inverse order towards the end of the eclipse.

In the case of solar eclipses (Fig. 88), if we consider the interposed moon as casting a shadow on the earth, the eclipse will be total where the true shadow cone cuts the earth, and partial where the penumbral regions meet the terrestrial surface.

Owing to the ellipticity of the moon's orbit, and consequent variation in the distance between the earth and moon, it sometimes happens that the true shadow cone of Fig. 88 does not quite reach the earth. In such a case, in that part of the terrestrial surface for which the eclipse is central, the sun will appear as a luminous "annulus," or ring, with the central part dark. Such eclipses are called Annular eclipses, and occur, of course, for the sun only. A total solar eclipse can never last longer than eight minutes at any one place on the earth, but totality in the case of the moon may last a couple of hours.

FIG. 88. Solar Eclipse.

There exists a peculiar periodicity in the recurrence of eclipses called the Saros. It was discovered by the Chaldæans, who found, by actual observation, and comparison with ancient records in their possession, that after the lapse of a period of 6585 days after an eclipse, the phenomenon will be repeated; and eclipse occurrences can thus be predicted easily. The explanation is as follows:

The reader will remember that in the case of the earth's

motion around the sun we found the tropical year to be shorter than the sidereal year by about twenty minutes, on account of precession of the equinoxes (p. 126).

In a similar way, on account of the motion of the moon's nodes, the time required by the moon to travel in its orbit from one node back to the same node again is shorter than the lunar sidereal period (p. 161). This nodal period is called the Draconitic period; it is three hours shorter than the sidereal period, and two days seven hours shorter than the synodic period. So we have:[1]

$$\left.\begin{array}{lrr} \text{Sidereal period} & = 27^{d} & 8^{h} \\ \text{Draconitic period} & = 27 & 5 \\ \text{Synodic period} & = 29 & 12 \end{array}\right\} \text{approximately}$$

An inspection of these figures shows that 223 synodic periods equal 242 draconitic periods, very nearly; and either includes 6585 days. But successive full-moons, or successive new-moons, follow each other at intervals of one synodic month, because the synodic month is the interval between two successive overtakings of the sun by the moon, in their respective apparent motions around the celestial sphere. And successive passages through either node succeed each other at intervals of one draconitic period. Therefore, any period of days like the Saros, containing exactly a definite number of synodic periods and also a definite number of draconitic periods, — after the lapse of such a period of days, both the lunar phase and the node passage must both repeat. Therefore, if there was an eclipse at the first new or full moon of the Saros period, there must also be an eclipse at the first new or full moon of the succeeding Saros period.

[1] Note 37, Appendix.

In the light of the above explanation of eclipses, it may be possible to make somewhat clearer the allied phenomenon called a transit of Venus (p. 268). Such transits also occur in the case of Mercury, but they are then of lesser interest. The planetary nodes do not move around the ecliptic rapidly, like the lunar nodes; they remain almost stationary at a definite point. The sun, in its apparent motion around the ecliptic, reaches the nodal points of Venus on June 5 and December 7; so that transits of that planet happen (if at all) within a day or two of these dates. To ascertain the interval between successive transits, we note that:

5 synodic periods of Venus = 8 years, nearly;
152 synodic periods of Venus = 243 years, very nearly.

Since conjunctions of Venus occur at intervals of one synodic period, any given transit may be followed by another at the same node eight years later. But there could not be a third transit sixteen years later; the eight-year period is not exact enough for that. We should then have to wait for the 243-year period to become effective. But at the other node, a transit, or an eight-year pair of transits, may happen after half the 243-year cycle has elapsed.

Photo by Barnard.

PLATE 21. The Morehouse Comet, Nov. 18, 1908.

CHAPTER XVIII

COMETS

The comets, *stellæ cometæ*, or stars with hair, must next receive our attention. These bodies usually move in very elongated orbits, with the sun at one focus. They often come as mere occasional visitors to the solar system, are seen during a short period only, while they are traversing that part of their orbit which is near the sun and therefore also near the terrestrial orbit. Occasionally comets have been as luminous as the brightest planet (Venus); have sometimes been seen in daylight; and very often have a long appendage streaming out from the head, — the comet's "tail."

It is easy to enumerate the chief known facts concerning the comet's physical appearance. The head usually consists of a "coma," or hazy nebula, containing a "nucleus," or central condensation. Attached to it is the tail, or, as it was sometimes formerly called, the "beard." The coma is the part that generally becomes visible first, as the body begins to approach the solar system. The nucleus forms later, or at least becomes visible later. The tail, strange to say, is always directed away from the sun; so that when the comet is receding from the sun, after passing the perihelion point (p. 120) of its orbit, it pushes its tail out ahead of it. But many comets were not discovered until after perihelion. It was then that the astronomers of old used the name "beard" for the tail.

Comets are big; their volume is sometimes incredibly

large. The heads run up to a million miles in diameter, and the tails may be ten million miles long, or even much longer. But they have little mass, as is evidenced by the total absence of gravitational perturbations (p. 206) in the motions of the earth and Venus, even when a big comet passes very near these planets. Owing to this vastness of bulk and extremely small mass, comets have, of course, a very low density, especially in their tails. Moreover, stars have at times been seen through the comets, even through their heads.

In view of this extreme lack of mass, it may seem strange that modern science is compelled to admit the possibility, at least, of danger resulting from collision between the earth and a comet. If the cometary particles are infinitesimally small, no injury would follow; but if the particles are rocks weighing tons, they might cause considerable local damage at the point of collision on the earth.

But, on the whole, the most plausible theory is to suppose these bodies to be composed of tiny particles traveling together in swarms, and separated by distances many times greater than the diameter of the particles. And the particles may be surrounded by atmospheres of incandescent gases; for we know that comets are partly self-luminous, although they send us also a certain amount of reflected sunlight, like the planets. This is made certain by observations of their spectra, which generally show the existence of hydrocarbon gas in a luminous state, as well as a dim continuous spectrum containing Fraunhofer (p. 287) lines, — the sure indication of solar light.

We have a good theory to account for the repulsive forces that must come from the sun, so as to make the cometary tail always point away from that body. The researches of

Clerk-Maxwell (the same who proved mathematically the satellite construction of Saturn's ring, p. 245) have brought out the fact that light-rays exert a slight physical pressure upon any object they reach. This pressure is theoretically proportional to the area illuminated.

Now if we imagine a small spherical cometary particle, and suppose its radius to diminish, the light pressure will diminish in proportion to the square of the radius, because the area of the circular cross-section illuminated diminishes in that proportion. But the volume and mass of the particle will diminish as the cube of the radius. Therefore the mass diminishes more rapidly than the area. But the solar gravitational attraction is proportional to the mass; consequently, the solar attraction diminishes more rapidly than the light pressure, as the particle grows smaller. We have therefore merely to imagine the particle small enough, and the light pressure will balance the attraction. Still smaller particles will actually be repelled.

If we now suppose the tail composed of particles smaller than those in the head, everything is explained: the head attracted, the tail repelled, by the sun. Probably the comet has no tail until it approaches the sun, when the light pressure sifts out the small particles; repells them in an increasing degree as the comet comes near perihelion; and thus makes the tail "grow." Perhaps the tail particles never rejoin the head, but are left scattered throughout the length of the cometary orbit. If we finally suppose both gravitational attraction and light repulsion to be exerted on the several particles, both by the sun and the comet's head, we have a combination of forces sufficiently flexible to account for the most complicated observed forms of cometary tails. (See Plate 21, p. 307.)

The number of comets is very great; while only some four hundred were observed before 1610, when Galileo first used the telescope, at least four hundred more have been found in the succeeding three centuries. Of course this greater abundance of discovery in modern times has been brought about by the possibility of recording comets too faint to be seen by the unaided eye. Only thirteen naked-eye comets belong to the nineteenth century.

The method used in discovering comets is interesting. This work is carried on by specialists; except as a result of unusual chance, one can expect to find new comets only after a number of years' severe study of the heavens. The usual process is to "sweep" the sky with a telescope of moderate size and low magnifying power. Any hazy object may be a comet; for when they are distant and dim, comets always look more or less like small nebulæ. The only sure test to distinguish them is to watch for an hour or two, and ascertain whether there is motion relative to the fixed stars. If there is motion, a comet has been found. But this motion test occupies much time; and at this point the experience of years is of value. For the comet hunter learns at last to know all the tiny configurations of stars seen in the telescope; he knows the telescopic constellations at sight, as well as most astronomers know Orion and the Great Bear. It is said that Olbers, for instance, could tell the approximate right-ascension and declination of the point on the sky toward which his telescope was directed, by simply looking through the eye-piece, and noting the diagram of stars appearing in the field of view.

And the comet hunter must know all the little nebulæ too, as well as their positions on the sky relative to the surrounding small stars. When he sweeps a faint nebulous ob-

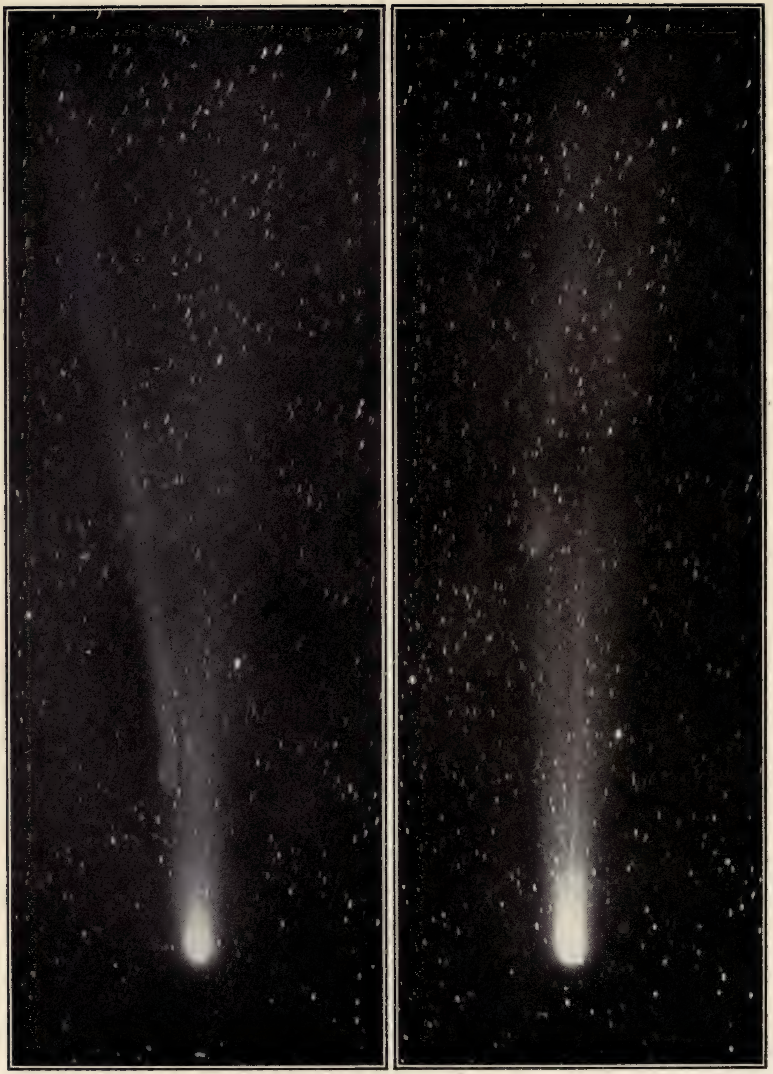


Photo by Curtis.

PLATE 22. Halley's Comet.

ject into view, he does not ordinarily need to delay his work by testing for motion; he recognizes the object at once, if it is one of the known small nebulæ.

Comets are usually named after their discoverer, though a few bear the name of some person who has explained their motions or peculiarities by a special investigation. Thus the famous comet of Halley, a photograph of which is shown in the accompanying Plate 22, was the first comet for which future returns to the solar system were predicted by means of orbital calculations. These were made by Halley (cf. p. 269); and his name was accordingly assigned to this comet.

Small telescopic comets are at first designated by the year of discovery and a letter; as, 1899 *a*, etc. Later, when orbits have been computed, they take the number of the year in which their closest approach to the sun, or perihelion, occurs; and a number in addition to show the order of cometary perihelion passages during that year. Thus Donati's great comet of 1858 was 1858 *f*, or the sixth comet discovered in 1858; and later it became 1858 VI, or the one whose perihelion passage was the sixth perihelion passage in 1858.

The period of visibility does not usually last more than a few months, although its average duration has been lengthened considerably in recent years, because modern giant telescopes can observe the comets long after their orbital motion has carried them quite beyond the range of ordinary glasses.

Concerning these orbital motions, the mists of antiquity certainly enshroud some very singular notions. There was a time when comets were regarded as material thrown out from the earth, possibly through volcanoes. It was not until 1577 that Tycho Brahé for the first time proved from actual measurements that the great comet of that year was

surely farther from the earth than is the moon. Kepler thought comets are alive. Hooke, in 1675, a century after Tycho Brahé, suggested that comet orbits might be parabolic: a very few years later, Newton showed that they are "conic sections," and Halley calculated actual orbits for all the comets observed up to that time.

There are three kinds of conic sections,—the ellipse, parabola, and hyperbola; and it is easy to draw a figure illustrating these three curves as comet orbits. Parabolic orbits are four times as frequent as elliptic orbits: hyperbolas are

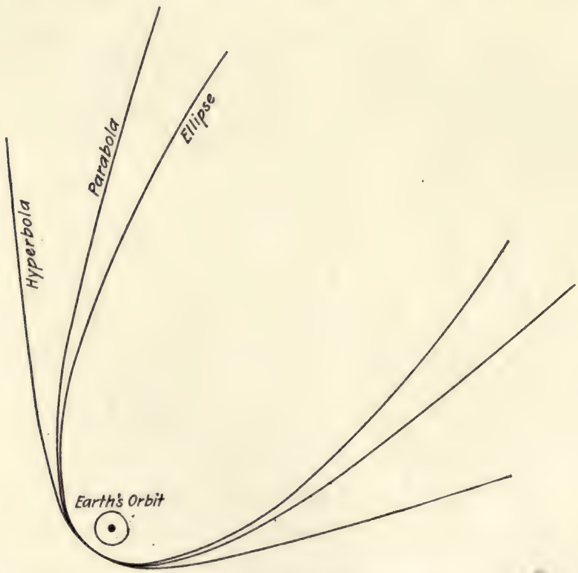


Fig. 89. Forms of Comet Orbits.

very rare, and it is not absolutely certain that any such orbits really exist. Figure 89 exhibits the three orbital forms, together with the comparatively tiny circular terrestrial orbit, and the sun-dot at its center. We see especially how nearly alike all three kinds of comet orbits are, while

the comet is near the earth's path; and, after all, it is only then that we can see a comet, and observe its position, as projected on the sky. To construct a comet orbit from observations is often as difficult as trying to draw a circle of large radius through three points very close together. Thus, in Fig. 90, it is easy to draw the circle A accurately, through the three points P_1, P_2, P_3. But if we were asked to draw a circle through P_1', P_2', P_3', we might not be able to decide between the circle B and the circle C.

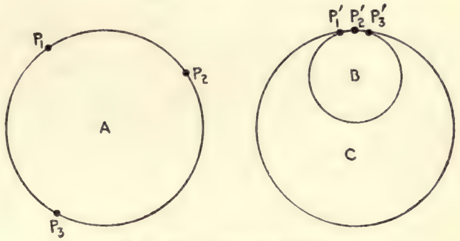


FIG. 90. Comet Orbits.

Now the ellipse is a closed curve; the others are open curves: therefore only elliptic orbits will produce so-called "periodic" comets, with future returns to the solar system. The parabolic comets visit us once, and never return. But since parabolic orbits closely resemble extremely elongated ellipses, it is not always possible to make certain whether any given object is periodic or not. But it is, after all, really immaterial whether a given comet orbit is truly parabolic, or elliptic, with a period of several hundred thousand years.

The exact details of any comet orbit are defined by means of elements exactly analogous to the elements of a planet's orbit (p. 200).

So much being premised about these orbits, we can now consider one of the most interesting things known about comets,—their "families." For there are in existence most curious kinships between various groups of comets. Coming back to Kepler's amusing notion that they are alive, we must expect to find close relationships among them, and

also some that are merely distant cousins, as it were. The most remarkable "blood-relations" are the great comets of 1668, 1843, 1882, and 1887. They must be brother-comets, for they all pursue practically the same orbit, though traveling in different parts of it. They approached the solar system from the direction of the bright star Sirius, and left again in nearly the same direction, in a parabolic orbit.

On the other hand, there are the comet-families belonging to the great planets, especially Jupiter. Here all the comets of a family have the peculiarity that the points of their orbits farthest from the sun, the aphelion points, all lie near the orbit of Jupiter. In other words, they recede from the sun just far enough to reach Jupiter's orbit. If Jupiter happens to be in the neighborhood when they get out there, he must exert a powerful gravitational attraction upon them. It is supposed that on their first arrival, perhaps in parabolic orbits, this attraction pulled them around into ellipses, having their node and aphelion point near the place where this disturbing pull took place. This is the well-known capture theory of comets, due to Laplace.

So we see that the comets did not originally belong to our solar system; they come to us from outer space, possibly from among the fixed stars, possibly from some nearer region. If they come from interstellar spaces, we should, on the whole, expect to find a preponderance of orbits having their aphelion points lying in the direction of that point on the sky toward which the solar system's own motion in space is tending. For the solar system, as a whole, is drifting through cosmic space, as will be explained in a later chapter. But we have only slight indications of such a clustering of aphelion points: our whole theory as to comet origins is till hazy, very hazy.

CHAPTER XIX

METEORS AND AËROLITES

THE consideration of comets leads us directly to the closely related subject of meteors or "shooting stars." These look like stars falling from the sky; actually, they are small particles of matter traveling in space, and passing through the earth's atmosphere. They give a bright light, and usually leave a long visible trail behind them. Sometimes they do not appear merely as isolated bodies; but regular showers occur, with the bright intermittent trails almost covering the sky, or a portion of it, for a considerable time. When this happens, it has been found that all the meteor trails are directed away from some single point on the sky. This point is called the Radiant; and it has the peculiarity that the trails are always short in its vicinity. Figure 91 exhibits this state of affairs. The point R on the celestial sphere is the radiant. All the trails are directed away from it, as shown by the arrows; and the longer trails originate at points farther from the radiant than do the short trails.

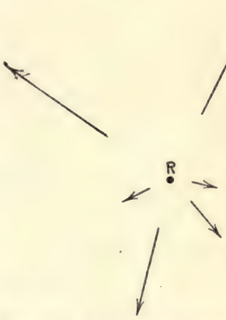


FIG. 91. Radiant of Meteor Shower.

Figure 92 shows that the whole appearance is due simply to perspective. The meteors move in parallel lines to meet the earth. Suppose the observer to be on the surface of the earth, at O, and two meteors moving along parallel lines,

such as A_1B_1 and A_2B_2. To the observer at O the meteors will seem to move along the lines A_1C_1 and A_2C_2. And all the meteors will seem to move along lines that will appear to radiate out from a single point R, where A_1C_1 and A_2C_2 intersect, if produced backwards from A_1 and A_2. And this point will appear in the sky near the meteors that seem to have short trails.

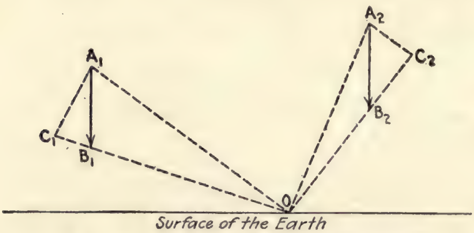


FIG. 92. Radiant of Meteor Shower.

Each meteor shower can be distinguished from all others; not by a difference in the appearance of its constituent meteors, but by the position on the sky of its radiant. Thus the shower called the Leonids, the greatest of all the showers, has its radiant in the constellation Leo (Fig. 20, p. 62). These meteors occur always about November 12, and have been found especially numerous at intervals of 33 years.

The cause of this fixity in the dates and recurrences of individual showers is quite simple. Each shower travels in a definite orbit around the sun, just like a periodic comet (p. 313). This orbit somewhere intersects the orbit of the earth; or, at least, passes very near it. But the earth must reach that point of intersection on the same date each year. Therefore the shower must occur on that particular date, if it occurs at all. And it will occur if the meteors happen to be at the intersection point of the two orbits on the date when the earth also reaches that point. In the case of the November Leonids this happens only once in 33 years; but the Perseids, or August meteors, are ready for us every year. We conclude, of course, that the Perseids are spread

out all along their orbit, so that we meet some of them whenever we strike the orbit. But the Leonids must be concentrated in a certain region in their orbit: this region comes around to the point of intersection in the proper way only once in 33 years.

A very interesting fact about the meteors is that we ordinarily observe more of them per hour just before sunrise than we do just after sunset. The reason is shown in Fig. 93. E_1 is the position of the earth in its orbit at about six in the morning, just before sunrise. The sun is seen projected on the ecliptic at S_1, which is therefore near the east point of the horizon. The earth's orbital motion is for the moment directed towards the point P, 90° west of S, where the sun appears on the ecliptic. The earth's orbital motion takes place in the direction of the curved arrow, so that at six in the morning we are on the front of the earth in respect to its orbital motion; we advance to meet the meteors. But on the opposite side of the earth it is at the same instant 6 o'clock in the evening. There they see only such meteors as overtake the earth, while on the front side we see them all. After the lapse of twelve hours, the earth has made a half-turn on its rotation axis; conditions are reversed; and we then have our clocks at six in the evening.

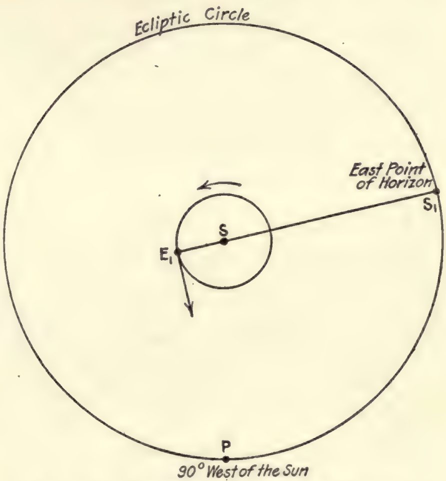


FIG. 93. Meteors at Sunrise and Sunset.

We are then, in our turn, on the back of the earth with respect to the direction of its orbital motion.

Meteors are never seen until they enter the atmosphere of our earth, but their heat and light are not due to atmospheric friction in the ordinary sense. Sometimes it is said, erroneously, that they are "set on fire" in the same way as a friction match is lighted by being rubbed on a rough surface. Their light is really caused by the compression of air in advance of the moving meteor. It is harder for the meteor to move against the compressed air; this retards its motion, and the motion energy is transformed into heat energy (p. 2). Doubtless both the meteor and the air are heated.

When a moving body is retarded by the resistance of an atmosphere, the heat engendered is proportional to the square of the velocity of motion. At the usual meteoric velocity, the temperature produced is probably equivalent to several thousand degrees Fahrenheit; and this, of course, will melt almost any substance. The rapid motion through the air then tears off particles of heated incandescent matter from the melted meteoric surface; and these particles are left behind to form the tail or trail of the meteor. It is not known just why it sometimes remains visible for many minutes. Plate 23 is a photographic reproduction of a meteor trail, showing two remarkable variations of brilliancy. The negative also contains a couple of interesting nebulæ of irregular form.

It is altogether probable that the meteors, and especially the meteoric showers, are nothing else but fragments of disintegrated comets. As soon as periodicity in the recurrence of showers was recognized, and it thus became plain that the meteors travel in definite orbits, it was but a short step to compare those orbits with known cometary paths. And

Photo by Barnard.

PLATE 23. Meteor Trail.

soon after the great Leonid shower of 1866, Schiaparelli showed that the Perseids, or August meteors, are in the same orbit as a comet discovered by Tuttle in 1862. And it was not long before the Leonids were similarly identified with the comet of 1866, discovered by Temple.

At least eight different meteor showers are now known to be connected with comets. The conclusion is possible that the comet is itself but a condensed place in the meteoric procession; the meteors themselves the disintegrated part of the material involved in the whole transaction. Certain it is that at least one comet (Biela's) has actually been seen to break up. It was discovered in 1826, re-appeared in 1846 according to prediction, and was seen to break in two during this period of visibility. In 1852 it was seen again, the two parts being now widely separated; and it has never been visible since. In its orbit, however, moves one of the big meteor swarms.

It is very important to determine as accurately as possible the height of meteors above the earth's surface. For this is about the only direct method we have to ascertain observationally the extent of the terrestrial atmosphere. Since the meteor becomes visible only when it penetrates the earth's envelope of air, we shall know something about the height of the air if we can measure the height of the meteor. The only way to do this is to select a couple of stations on the earth and make simultaneous observations of the same meteor. Figure 94 shows how this is done.

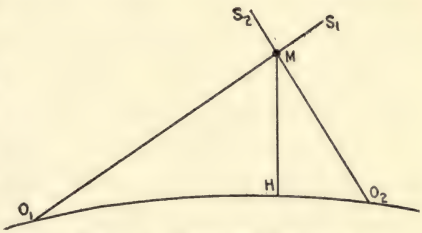


Fig. 94. Height of Meteors.

If observers on the earth's surface, at O_1 and O_2, see the meteor M projected on the celestial sphere near the stars S_1 and S_2, it is clear that we can calculate the height MH of the meteor from the known distance O_1O_2 between the observers, and the angles $S_1O_1O_2$ and $S_2O_2O_1$, which are, of course, known, if we know the positions of the stars S_1 and S_2 on the sky.

But there is great difficulty in securing these observations in such a manner as to be certain that they apply to the same meteor. It is necessary to make the attempt on some night when numerous meteors are to be expected; and it is then by no means easy to be certain that the observations of M from the two stations are really simultaneous, and apply to but a single meteoric object. Such as they are, observations of this kind indicate an extreme height of 75 miles for our atmosphere.

Having thus explained the principal facts about the meteors, we come next to the Aërolites. These are stones, or pieces of iron mixed with other materials, which fall upon the surface of the earth from time to time. There is little doubt of their being meteors that actually strike the earth, probably on account of unusually large size. For a very big meteor would not be entirely consumed, as it were, in its passage through the air, and might be attracted down to the surface of the earth by the gravitational pull of the planet. Those that are completely consumed perhaps fall on the earth finally as dust: certain it is that dust, probably meteoric, has been found on the surface of ancient Arctic ice. This is the so-called "cosmic dust." Many specimens of aërolites are preserved in museums. A number have actually been seen to fall, so there is no doubt whatever as to their origin being outside the earth itself. When seen

at night they exhibit a bright round head, with a luminous trail. Occasionally there is an audible explosion.

The outer surfaces of the aërolitic specimens in our museums seem to have been melted, showing the effect of the high temperatures produced through their impeded motion in the air. Chemically, they contain only elements or substances known on the earth.

CHAPTER XX

STARSHINE

In the preceding chapters we have completed a somewhat detailed description of the solar system, and are now ready to proceed outward into space, to study the distant universe of stars. Astronomers believe that an analogy exists, more or less close, between our sun and the stars (p. 6). Together with its system of planets, the sun may be regarded as a small isolated group suspended in space, and separated from other similar groups by distances almost incomparably greater than any existing within the solar system itself. We may be quite sure that the stars are all excessively distant: we are troubled by no doubts in this respect, as were the ancients. For we now have the law of gravitation; from it we know that if there were a celestial body of any kind in space, as massive as the sun, and not more than ten thousand times as far away as the distance separating the earth from the sun, — that body would surely reveal its existence through observable perturbations (p. 206) produced in the motions within our solar system. And this it would do, even if it were a dead sun, no longer luminous, and quite invisible. To produce gravitational attraction, and consequent perturbative effects, merely the presence of matter would be necessary, not visible matter.

Furthermore, actual measurements, to be described later, have shown that the nearest fixed star so far observed is more than 200,000 times as far from the sun as is the earth.

PLATE 24. The Constellation Serpentarius.

(From Hevelius' *Prodromus Astronomiae*, Gedani, 1690.)

But this last argument is not conclusive, because we can measure only visible stars, and the nearest one might conceivably be non-luminous. This objection, of course, does not apply to the gravitational argument.

We have seen (p. 6) that the stars are classified according to their magnitudes, and that this term "magnitude" does not here have its usual meaning. It has no relation to size or bigness, but simply indicates the degree of luminosity or brightness of a star. We shall now consider this matter somewhat more in detail. Old Hipparchus (pp. 127, 189, 297) was the first to divide the stars into magnitude classes; he simply selected arbitrarily the twenty brightest stars he could see, and called them first-magnitude stars. He then designated as sixth-magnitude all objects that belonged at his absolute lower limit of vision, — that he could just see, though with difficulty. Stars of intermediate luminosity he placed in intermediate classes, also somewhat arbitrarily. This gave a rather rough classification; but it is still in use (with some improvements) down to the present day.

Adopting Hipparchus' magnitude scale, we find the number of stars of the various magnitudes situated between the north pole of the heavens and the circle of declination 35° south of the celestial equator to be as follows:

1st mag. 14,	3d mag. 152,	5th mag. 854,
2d mag. 48,	4th mag. 313,	6th mag. 2010,
	Total, 3391.	

The above rough system of classification was replaced by a more exact one about 1850. Sir John Herschel had remarked from his photometric observations that first-magnitude stars average just about 100 times the luminosity of sixth-magnitude stars. So it was suggested that we take the exact "fifth root" of 100 as the ratio between the lumi-

nosities of any two successive star-magnitudes. And this ratio is called the "light-ratio."

In this way, we make the fifth-magnitude stars $\sqrt[5]{100}$ times as bright as the sixth-magnitude stars; the fourth $\sqrt[5]{100}$ times as bright as the fifth; etc. Consequently, the first-magnitudes would be $\sqrt[5]{100} \times \sqrt[5]{100} \times \sqrt[5]{100} \times \sqrt[5]{100} \times \sqrt[5]{100}$, or 100, times as bright as the sixth-magnitudes, in exact accord with Herschel's observation.

Now the fifth root of 100 is about $2\frac{1}{2}$, so that stars of any magnitude are approximately two-and-one-half times as bright as those of the next fainter magnitude. To fix a definite zero for this scale, it has been decided to select certain stars as standards. Thus, Aldebaran is a standard first-magnitude: other stars can be compared with it; their light-ratio measured by observation; and the magnitude difference then ascertained.[1] This process, of course, sometimes assigns zero-magnitude, or even a negative magnitude, to an exceptionally bright star, like Sirius. On the above scale, Sirius actually comes out from photometric observations as *minus* 1.4. The sun's stellar magnitude is about -26, the enormous luminosity being in this case due to proximity, not to intrinsic light-giving power.

Observations of the relative luminosities of stars are made with an instrument called an astronomic photometer. The ordinary telescope may be so used in a very simple way. To estimate a star's brightness, we have only to place diaphragms pierced with holes of various sizes outside the object-glass (p. 272) until we find one that will just allow us to glimpse the star. It is obvious that this method is possible, for if we use successively a series of diaphragms with apertures diminishing gradually, we shall in effect be

[1] Note 38, Appendix.

making the telescope smaller and smaller, and there must come a time when it has been made so small that it will just fail to show the star under observation. From the size of the aperture in this last diaphragm, it is possible to calculate the luminosity of the star.[1]

There are also other, and perhaps better, forms of photometers, in which the star under examination is compared with an artificial star produced by a light in the observatory placed behind a screen having a very small hole. Varying the artificial star until it appears of the same luminosity as the real one enables the observer to measure accurately the brightness of the latter. Magnitudes may also be measured photographically. The little dots produced on a sensitive plate by prolonged exposure in the telescope vary in a sort of proportion to star-magnitudes: the bright stars produce larger dots. Therefore a microscopic measurement of the dot diameters on an astronomic negative enables us to estimate star-magnitudes.

But all astronomic photometric measures are subject to considerable error on account of "light-absorption" in the terrestrial atmosphere. Some of the stellar light is lost in passing through our air. This effect is, of course, smallest for stars near the zenith; for there light passes straight through the atmospheric layer, and at right angles to it. The path through the air is thus the shortest possible. But for stars near the horizon the light enters the atmosphere at a rather small angle, and its path is much longer, before it reaches the observer's eye. Consequently, stars are brightest when they are near the zenith.

How much is the total light received from the stars? This question has been widely studied, but only the very

[1] Note 39, Appendix.

roughest results have been obtained. Possibly the whole sidereal heavens give about as much light as 2000 stars like Vega. This is approximately $\frac{1}{30}$ of full moonlight; and it includes the considerable quantity of starlight coming from objects below the sixth magnitude, and therefore invisible to the unaided eye.

The heat received from the stars is almost evanescent; only the very slightest indications of it have been rendered perceptible by the most delicate thermometric apparatus so far invented.

It is also possible to obtain a very rough comparison between the total light actually emitted by the stars and by the sun. It is found, for instance, that Vega emits about 49 times the light sent out by the sun.[1] Other stars give similar results: we see, therefore, that our sun is really a rather faint star, but not an infinitesimally small one, in comparison with Vega.

There exists still another very remarkable phenomenon in connection with stellar luminosity, — its variability. For it must not be supposed that the brightness of all stars is strictly constant: many increase or diminish their light from time to time; and these are called "variable stars." There are several distinct kinds. Some vary their light steadily, ever increasing or diminishing it; others show no regularity whatever in their rise and fall; and a few, the "temporary stars," or *novæ,* appear suddenly in the heavens, and last but a short time. Finally, there are stars that wax and wane with more or less accurate periodic regularity; and wonderful variables, in which changes are caused by some form of eclipse phenomenon. Of these last the best known is the star called Algol (the Demon).

[1] Note 40, Appendix.

The number of stars whose light alters steadily in one direction is very small: demonstrated permanent variations of brilliancy since the time of Hipparchus are extremely infrequent. But we have records that Eratosthenes saw β Librae brighter than Antares, though the contrary is surely the fact at present. Similarly, in 1603, Bayer recorded Castor as brighter than Pollux, in the constellation Gemini; but now Pollux gives us more light than Castor.

The most prominent irregular variable is η, in the constellation Argo Navis, in the southern celestial hemisphere. Sir John Herschel saw it as bright as Sirius in 1843, on the occasion of a visit to the Cape of Good Hope. It has been of the seventh magnitude since 1865, and is still in process of change.

Temporary stars have blazed up about eighteen times since men began to write their records of the skies. The most famous is Tycho Brahé's star of 1572, which was brighter than any other star, and lasted only sixteen months all together. This is the star that first interested Tycho in astronomy: it is reported that he refused for a long time to describe his observations, because he thought it beneath the dignity of a Danish nobleman to write a book.

A peculiarly interesting recent object was *nova Aurigæ* (the new star in the constellation Auriga), which appeared in 1891. It has been variously explained as the result of heat engendered by a collision between two dark stars, or as an explosion occurring in a single dark body.

Another important new star was found in the constellation Perseus in 1901. Within a few days after discovery its brightness grew to the first magnitude, but it faded again with almost equal celerity. It developed a surrounding nebulosity after a time; and is all together one of the most

wonderful astronomic objects ever observed. The accompanying Plate 25 shows the manner in which this nebulosity grew in size. If this growth is the result of motion outward from the central star, the velocity must have been incredibly rapid. What cosmic process or catastrophe there occurred before our eyes, we can neither describe fully nor attempt to explain.

The periodic variable stars are of three kinds. First, those like the star Mira (the Wonder) in the constellation Cetus, which is ordinarily invisible without a telescope, but increases rather suddenly every eleven months to the third magnitude, when it is of course a naked-eye star. The second variety of these variables includes stars like β, in the constellation Lyra, with changes completed in a very short period; and with the alteration of light apparently compounded of two different variations superposed. And the third class of these stars are like Algol, in the constellation Perseus, which, at regular intervals, undergoes a partial eclipse.

A possible explanation of regular variations might be offered from the analogy of sunspots. Certain stars may have very large permanent spot zones that are carried around by axial rotation; and when these are turned toward the earth we may properly expect for a time a considerable diminution of light.

The eclipse theory for the Algol stars is quite old; but it was not proved correct in a convincing way until 1889, when Vogel successfully observed the spectrum according to the Doppler principle (p. 284). He showed that the visible star Algol has a velocity of recession from the earth before its period of minimum luminosity, and an equal velocity of approach after such minimum. The explanation now pos-

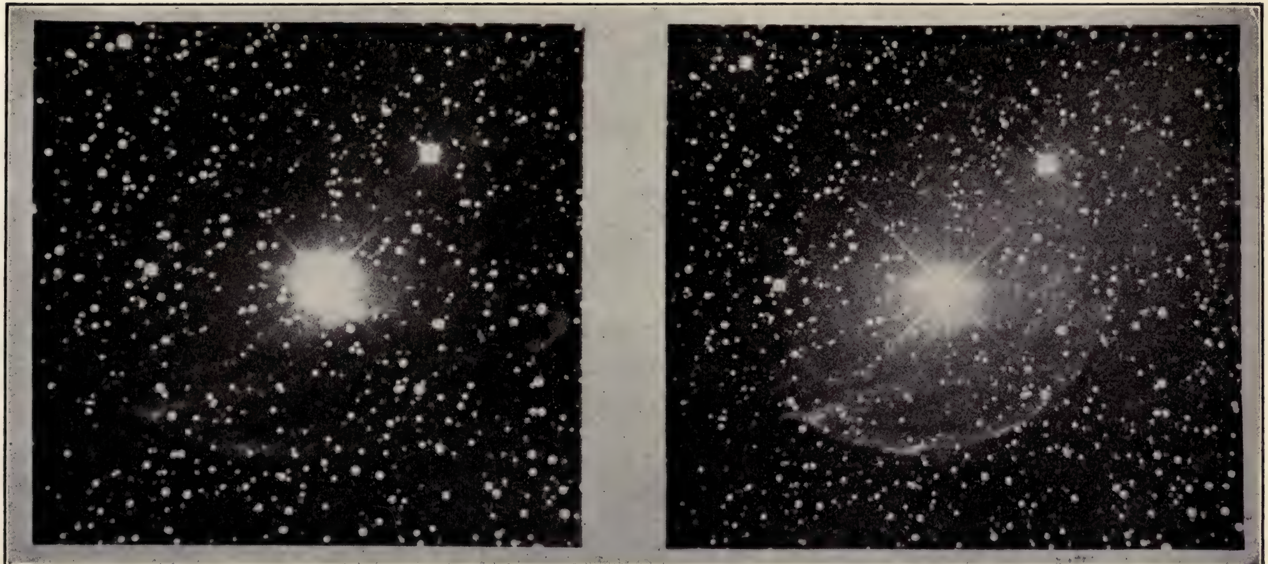


Photo by Ritchey.

Plate 25. Nova Persei.

tulates (Fig. 95) a dark but massive companion to the visible star, and both revolving in nearly circular orbits about their common center of gravity. The orbit plane is supposed to be directed towards the solar system, so that we see the orbits edgewise. The dark body is supposed to be smaller in size than the luminous or visible one.

In the course of their orbital revolutions, there will come a time when the smaller dark body will be at D, and the luminous body at L. An observer on the earth, in the direction shown by the straight arrow, will see the luminous body partly covered or eclipsed by the smaller dark body. As soon as the eclipse is over, the luminous body will regain its full brilliancy, as in the positions D' and L'.

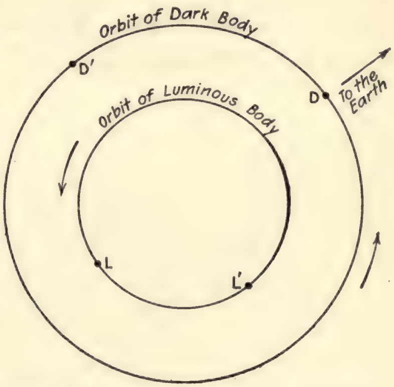


Fig. 95. Explanation of Algol Stars.

From a combination of the velocities of approach and recession, observed spectroscopically, with the known light-variations of Algol, Vogel was able to prove that the distance between the two bodies is $3\frac{1}{4}$ million miles, and their diameters respectively 0.8 and 1.1 million miles. It will thus be seen that they are so near each other as to be almost in rolling contact; but there exists no cosmic law preventing the occurrence of orbital motion of this kind.

The combined mass of the two bodies Vogel found to be about $\frac{2}{3}$ that of the sun; his calculations were made by a process analogous to the method of determining a planet's mass from the observed orbital period of its satellite (p. 204).

The more complicated light-variations of certain other stars may also be explained on the eclipse theory, the orbital planes being supposed inclined to the direction of the earth, and neither body dark. If both are luminous, there will still be a diminution of light during the eclipses. For one luminous body will then appear superposed on the other, and the total light will be the same as the larger body would emit alone.

The next thing we have to consider is the important question of stellar distances from the solar system. After all, this is in a way the most interesting matter we have to discuss in connection with the stars, for the question at issue really relates to the actual dimensions of the magnificent sidereal universe.

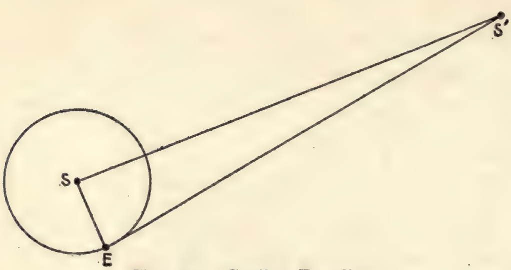


FIG. 96. Stellar Parallax.

In the first place, let us once more define "stellar parallax" (p. 192). Solar parallax (p. 260) has been explained to be half the earth's angular diameter, as seen from the sun. In a similar way, stellar parallax is the angle $SS'E$, in Fig. 96, between two lines, one drawn from the star S' to the sun S, and the other from the star S', tangent to the earth's annual orbit around the sun at E. In other words, when the earth reaches such a position in its orbit around the sun that there is a right angle at the earth E, between the sun S and the star S', then the angle $SS'E$ is the star's parallax.

We are compelled to use the radius of the earth's orbit in defining stellar parallax, whereas we used the radius of the

earth itself in the case of solar parallax. For the earth's own radius is far too small to subtend an angle at all appreciable, if the vertex is situated at the vast distance of the stars. Even when we use the orbital radius, the very nearest star, so far as we now know, has a parallax angle of only three-quarters of a second of arc.

It is clear, from Fig. 96, that the star's parallax angle really equals the difference in direction of the star as seen from the earth, and as it would be seen by a supposed observer on the sun. As the earth goes around its orbit, this parallactic displacement, or change of the star's direction due to the observer's being on the earth instead of the sun — this parallactic displacement must change its direction. At intervals of six months, during which the earth traverses one-half of its orbit, the displacement reaches equal amounts, but opposite directions. In the interval, there is a constant change in the direction of the displacement; so that, if a star is projected on the sky at a point perpendicular to the plane of the earth's orbit, it will appear to describe in a year a little circle on the sky, which is a miniature replica of the earth's own orbit around the sun.[1] A star in the plane of the earth's orbit will simply appear to swing back and forth through the year in a short, straight line, instead of describing a circle. Stars in intermediate positions will have apparent "parallactic orbits," which will be small ovals, intermediate in form between the circle and the straight line.

The measurement of a star's parallax is therefore nothing more than a measurement of the form and angular diameter of its little apparent parallactic orbit on the sky. This may be measured by an "absolute" method, or a "differential"

[1] A star so situated is, of course, at the "pole" of the ecliptic circle on the sky.

one. The absolute method requires a determination of the star's right-ascension and declination at various dates during a year. These being located on a celestial chart give the parallactic orbit at once. But the method is practically of little value, because we possess no instruments capable of measuring these declinations, etc., within the small fraction of a second of arc which is here necessary.

The differential method is better, since it enables us to use a micrometer (p. 276) as illustrated in Fig. 97. The parallactic orbit of a star is here shown as an ellipse or oval (p. 331). S_1 and S_2 are apparent positions of the star in its parallactic orbit on two different dates. S_1' and S_2' are two small stars in the vicinity. The observer measures, on various dates through the year, the small angular distances, from the parallax star to the small stars S_1' and S_2'. These latter, on account of their minuteness, may reasonably be supposed situated at practically an infinite distance from us, and therefore to have no appreciable parallaxes of their own, and no parallactic orbits. So the measures determine a series of points on the parallactic orbit; and, from the size of the orbit, the parallax of the star S.

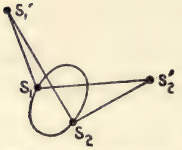


FIG. 97. Differential Parallax.

It is clear that this method should bring out a value of the parallax possessing a high degree of precision: and microscopic measures of an astronomic photograph may here replace actual visual micrometric work at the telescope with advantage. If the small stars are not really at an infinite distance, the differential method furnishes what may be called "relative parallaxes"; or, in a sense, the parallax excess of the star under examination over the small stars supposed infinitely distant. The first

successful stellar parallax measurement was made by Bessel in 1838. He used the differential method, and applied it to a star of moderate magnitude in the constellation Cygnus (cf. p. 192).

When we come to translate stellar parallax measures into terms of linear distances, we arrive at numbers so large as to be unmanageable. For this reason astronomers have invented a new and large linear unit to be used in sidereal measurements. It is called the "light-year," and is defined as the linear distance light would travel in a year. As the velocity of light is about 183,000 miles per second, the light-year, in miles, amounts to 183,000 multiplied by the number of seconds in a year. It is therefore an enormous unit of distance; but it is none too large for use in describing stellar distances in space. Its length is about 60,000 times the distance from the earth to the sun, and corresponds to a parallax of $3\frac{1}{4}$ seconds of arc.

Closely connected with the above subject of stellar distances is the question of the stars' motions. We have already seen (p. 7) that the objects called fixed stars are not really fixed in space. They are all actually drifting across the sky; it is only because of their vast distance that their motions seem to us small and slow. In reality, their velocities are of the same order of magnitude as those existing among the planets of our solar system: but to the rough instruments of the ancients these motions remained unrevealed; and so the stars received their designation of "fixed," to distinguish them from the wandering planets. And within the last half-century it has become possible to do even more than merely measure this stellar drift across the face of the sky; the drift which shows itself as a change of the star's right-ascensions and declinations. As we

have already seen (p. 284), we can now also evaluate, with the spectroscope, stellar velocities of motion in the "line of sight"; or, in other words, the star's linear velocities in a direction perpendicular to the celestial sphere.

Now changes in a star's right-ascension and declination do not necessarily prove the existence of motion. For the precession of the equinoxes (p. 126) moves the point from which we count right-ascensions, and it also shifts the celestial equator from which we count declinations. Since right-ascension is angular distance from the vernal equinox, measured on the equator, and declination angular distance from the equator, it is clear that precessional changes of equinox and equator will change both these quantities.

But all precessional effects can be calculated easily: and even after these effects have been eliminated from our observations by a suitable process of calculation, we still find small "residual" changes of right-ascension and declination. These may be ascribed wholly or in part to real motions of the stars. It is this residual motion that is called a star's "proper motion." This term is now a century old in astronomy: it is applied only to motion across the celestial sphere; not to motion in the line of sight, revealed spectroscopically. The latter has been separately denominated "radial velocity." Proper motion is measured in seconds of arc per annum; radial velocity in linear miles per second.

Less than two hundred stars have proper motions as large as one second of arc per annum. The largest known motion of the kind belongs to a little star numbered V 243 in a great catalogue of stars made at Cordova in South America. It drifts nearly nine seconds annually. The next largest and, until 1898, the largest known belongs to a star numbered 1830 in a catalogue observed by Groombridge in England

about the year 1790. This star is often called the "runaway"; it travels seven seconds in a year.

The relation of proper motion and radial velocity is indicated in Fig. 98. If a star at S_1 moves to S_2 in a unit of time (a year, let us say), and if the solar system is at E, we can draw S_1S_1' perpendicular to S_1E, to indicate the star's apparent motion across the celestial sphere as seen from E. In the same unit of time the star will have receded from the earth by a distance S_1S_2'; and this will represent its annual radial velocity. The true motion of the star, S_1S_2, may be regarded as really made up of two parts, — the radial motion and the proper motion. Now that we are able to measure these radial velocities, it is at last possible to ascertain from observation both parts of the distance S_1S_2, which is the star's true motion in a unit of time. Before we had the spectroscope and Doppler's principle, we never knew, or could know more than the one part S_1S_1'. These considerations emphasize the importance of the spectroscope in sidereal research: it has not only created a celestial chemistry; it has also given us new and essential knowledge in the oldest department of astronomic science, — the theory of sidereal motions.

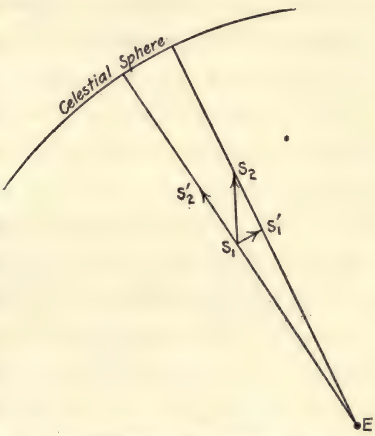


FIG. 98. Proper Motion and Radial Velocity.

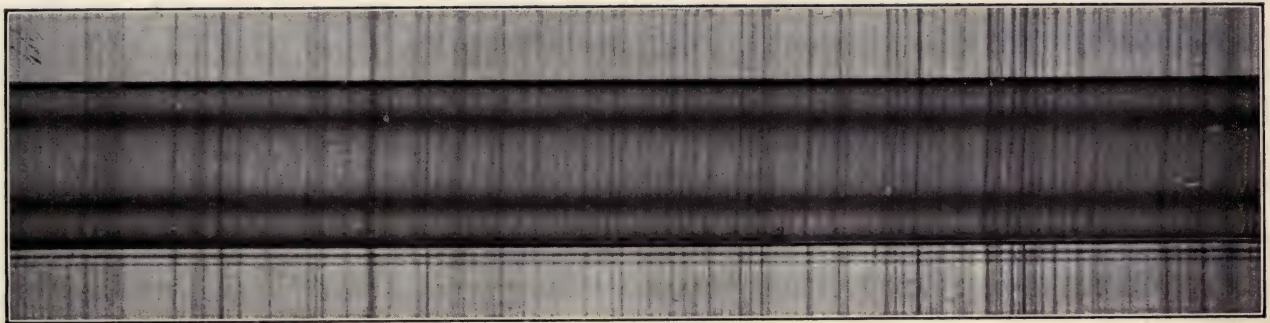
Not infrequently it happens that a certain number of stars are found to have proper motions, and probably real motions in space, nearly identical, both in amount and direction. For instance, Proctor pointed out some years ago

that five of the bright stars in the Dipper (p. 52) probably possess such a community of motion. There can be little doubt that they are proceeding through space in parallel lines, and that they belong together. The same is true of a number of the brighter stars in the Pleiades group. They are not merely an apparent group, but a real cluster.

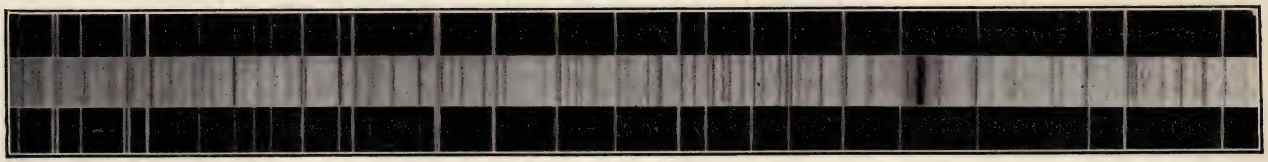
A very surprising thing, discovered in 1909, is that Sirius is also probably a member of the Dipper group of stars. If this be so, the group in question is not a distant congeries passing us at a distance, perhaps, of billions of miles; but it is passing so close that we are actually now within the drifting group of stars itself. This follows from the fact that Sirius is in quite a different part of the sky from the Dipper region.

We have seen quite enough to make clear the high value to science of these very modern measures of radial velocity: unfortunately, it has not been found possible as yet to apply the method to faint stars, because their light is not sufficient to give a measurable spectral image. This class of work was first attempted by Huggins in 1867; and he began by measuring the radial velocity of Sirius, the brightest of the stars, with a visual telescope and spectroscope. It was not until Vogel began to use photography in 1888 that any considerable extension of the process became possible. Fainter stars then first became observable, for the exposure of a photographic plate can be lengthened within reasonable limits, so as to give even a small quantity of light time to impress a spectral image on the plate.

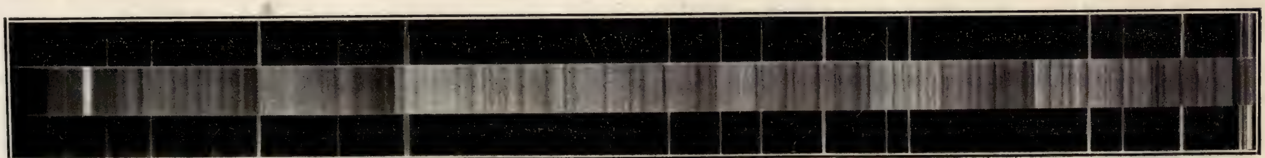
Good examples of radial velocities are found in the star α Carinæ, receding from the solar system 17 miles per second; and o Ceti, receding 54 miles per second. As the earth's own orbital velocity around the sun is about $18\frac{1}{2}$



Spectrum of Saturn.



Spectrum of α Carinæ.



Spectrum of o Ceti.

Photo Lick Observatory.

PLATE 26. Spectra.

miles per second, we see that the stellar motions are only of the same order of velocity as those existing within the confines of the solar system.

Spectrum photographs of the above stars are shown in the lower part of Plate 26. In each case the stellar spectrum is placed between two artificial spectra, produced in the observatory for comparison. It will be seen that the stellar spectral lines of both stars are displaced in the same direction with respect to the artificial spectra, because both stars are receding; but the displacement is much greater in the case of *o* Ceti, on account of its greater velocity of recession. The upper photograph of Plate 26 is a comparison of the lunar spectrum with that of Saturn and its ring (cf. p. 245). It shows, as it should, that the outer part of the ring is moving slower than the inner part.

This matter of stellar spectroscopy was first taken up by Huggins to study the chemical composition of the stars, though it led him also to the measurement of radial velocities, as we have seen. At about the same time, Secchi undertook similar investigations, and to him we owe a sort of classification of stellar spectra, as follows:

1. White and blue stars, with strong evidence of hydrogen. Examples are Sirius and Vega. These stars are believed to be in an early stage of cosmic development.

2. Solar stars, showing in their spectra many dark lines due to absorption, as in the solar spectrum. Capella is one of those stars; they are supposed to be somewhat older than those showing the hydrogen lines.

3. Red stars, with spectra showing dark, broadened lines.

4. Faint red stars, probably very old; the spectra having a few bright lines.

All the spectroscopic observations indicate a stellar

chemistry similar to that of the solar system. The entire sidereal universe seems to contain but one set of chemical elements; and these are very widely distributed. So we see once more that our sun is a star; and if the other stars are traveling through space, our sun should also be in motion, carrying its planets with it, much as the earth moves in its orbit around the sun and carries the moon with it. Analogy would lead us to expect such a solar motion.

Of course, the simplest way to study this question is to examine the radial velocities of a large number of stars. Suppose we find that those near a certain point on the sky are approaching us; those near the opposite point receding; and those halfway between, neither approaching nor receding. Then we may conclude that this is merely a result of the solar system's own motion; and that we are approaching the stars projected near the point of the sky where they seem to be approaching us. Towards this point on the sky, then, the solar motion is for the moment directed.[1]

Campbell made such a research a few years ago, using spectroscopic results derived from 280 stars. The point on the sky indicated by his work is not very far from the first-magnitude star Vega. It is called the "apex of the sun's way." Naturally, Campbell used only bright stars whose spectra could be observed; and of course brightness indicates nearness, other things being equal. It may therefore be a fact that stars having a common drift with the sun predominate in Campbell's series of observations; and if so, this might partly invalidate his result. But however this may be, he finds the region near Vega to contain the apex, and 13 miles per second as the "cosmic linear velocity" of the solar system.

[1] Note 41, Appendix.

Another fact that may cause a slight error in such an investigation as the foregoing is the necessity of making some assumption as to the average real motions of the stars whose radial velocities are observed. For we do not find all stars near the apex approaching us; only a preponderance of motions of approach. So astronomers assume that in the average of so large a number of stars (in this case, 280) there will be as many motions in any one direction as in any other. Therefore, a preponderance of motions of approach near the apex must be due to solar motion, not to motion of the stars themselves.

It is singular that as far back as 1783 Sir William Herschel obtained almost the same result from a discussion of the proper motions (p. 334) of various stars across the sky. His method is well illustrated in Fig. 99. If there are two lamp-posts, L_1 and L_2, on opposite sides of a street, the angular distance between them will seem much larger when they are viewed from H_2 than from H_1. A person walking from H_1 to H_2 will see this increase of the angular distance. Applying this principle to the sky, Herschel concluded that near the apex the constellations must be opening out, as we approach, and at the opposite point of the sky they must be closing in.

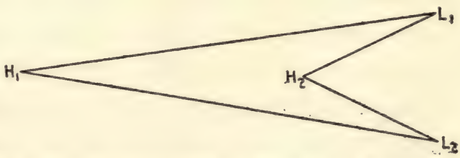


FIG. 99. Determination of the Apex (Herschel).

In other words, near the apex stellar proper motions directed away from that point must predominate; and near the opposite point, called the "anti-apex," proper motions directed towards the critical spot must be most in evidence. Herschel had at his disposal the measured proper motions of only

thirteen stars all together. Yet with the insight of rare genius, he so sifted this meager evidence that he was able to find the right-ascension and declination of the apex with some approximation to correctness.

Later investigators have of course repeated this work with much more elaborate modern material at command. They find a result in very fair accord with the spectroscopic one. But they also find this important peculiarity. If the proper motions are divided into groups, and the calculations made separately with stars of large and small proper motions, the apex comes out farther south on the sky for the stars of large proper motions.

Now it is evident that any investigation of this kind must assume that if there were no solar motion, the stellar proper motions would be quite casual, and free from any tendency to congregate in direction towards or from any apical point. But such a tendency in the stars themselves is indicated by the peculiar result just mentioned. It is clear that the large proper motion stars must have a common path of their own. But largeness of proper motion should indicate nearness to us, other things being equal; for at a sufficient distance, even large proper motions would shrink into apparent nothingness. Therefore it is within the bounds of possibility that our sun is a member of a drifting stream of stars, to which, in general, the large proper motion stars belong also.

In the light of the above discussion of cosmic motions of the sun and stars, as well as stellar distances, it is possible to consider an interesting special problem which may be solved approximately with modern observational data. We have seen that the solar system is moving toward Vega at the rate of 13 miles per second (p. 338). Observations of Vega's

own radial velocity indicate that it is itself receding from us at the rate of 3 miles per second; so we are overtaking it at the rate of 10 miles per second. But the number of seconds in a year is, approximately, $365 \times 24 \times 60 \times 60$; and the actual annual approach of the two stars, therefore, $365 \times 24 \times 60 \times 60 \times 10$ miles. The parallax of Vega has been measured; it is $0.''11$. From the parallax, the distance between the solar system and Vega may be computed,[1] and it comes out, approximately:

$$\frac{93000000 \times 200000}{0.11} \text{ miles.}$$

To ascertain the time in years required by the solar system to reach the position occupied by Vega, we must divide the distance of Vega by the rate of approach per year. We thus obtain, for the number of years required to overtake Vega in space:

$$\frac{93000000 \times 200000}{0.11} \times \frac{1}{365 \times 24 \times 60 \times 60 \times 10},$$

or, approximately, 560,000 years; and after the lapse of that period of time, the solar system should reach Vega.

But in that interval Vega will have moved, too, for it has a proper motion across the sky of $0.''5$ per year, which is about four times its parallax angle. Figure 100 will explain this state of affairs. At a certain moment, the sun and earth are at S and E, with Vega at V_1. Then, by definition, the angle EV_1S is Vega's parallax. At the end of a year, the earth will be back at E, very nearly, but Vega will be seen at V_2, because its proper motion will have carried it across the sky, as seen from the solar system, through the angle V_1SV_2. Since this angle is four times the parallax

[1] Note 42, Appendix.

angle EV_1S, it must follow, approximately, that V_1V_2 is four times ES. Therefore, V_1V_2 is $4 \times 93,000,000$ miles, or 372,000,000 miles.

So we see that when the sun reaches the point where Vega should be in 560,000 years, Vega will have moved at

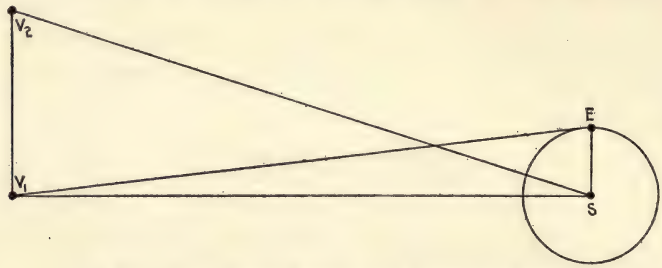


FIG. 100. Vega's Parallax and Proper Motion.

least $372,000,000 \times 560,000$ miles, and this is about as near as we shall ever approach Vega. What will be Vega's parallax at that time? We can answer this question by comparing the present distance of Vega, which we have found to be:

$$\frac{93000000 \times 200000}{0.11} \text{ miles,}$$

with its distance in 560,000 years as just obtained. The two numbers are not far from equal, and therefore Vega's future parallax will not be far from its present parallax. So there is no danger of a cosmic collision with Vega, so far as we may judge from the above rough calculation.

Our present discussion would not be complete without a brief account of certain quite recent researches, made principally by Kapteyn. His idea is that we need extensive statistical knowledge of stellar distribution, more than direct measures of a few parallaxes. He therefore undertakes to compute the average parallax of the stars of any given magni-

tude, separating the stars thus by magnitudes for the obvious reason that the fainter ones may be expected to average greater distances than the brighter ones, and therefore smaller parallaxes. Figure 101 shows his method of attack upon this problem. The arrow SS' is intended to represent the annual proper motion of a star S across the face of the sky, the arrow, of course, indicating both the direction and quantity of such motion. The position of the apex on the sky is shown at A. The line $S'S_1'$ is drawn perpendicular to SA, and the smaller arrow SS_1' then shows how much the proper motion SS' carried the star away from the apex. In fact, the proper motion SS' may be regarded as compounded of two motions: SS_1', which affects the angular distance from the star to the apex; and $S_1'S'$, which does not affect that distance, being at right angles to SA.

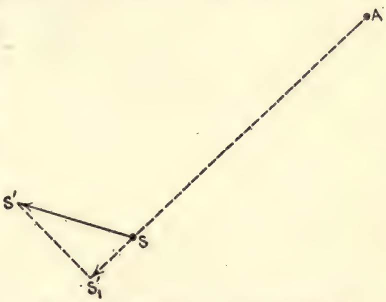


FIG. 101. Kapteyn's Researches.

Now Kapteyn "resolves" (as it is called) all known proper motions into two such "components," one directed away from the apex, the other at right angles to the first. But we have already seen (p. 339) that the effect of the solar system's own motion in space is to open out the constellations near the apex; therefore SS_1', the star's proper motion component away from the apex, must include the effect of the solar system's motion; but $S_1'S'$, the other component, is free from such effect. In the general average of a large number of stars of any given magnitude, the two components should be equal, if the sun were at rest. For the effect of the solar motion would then disappear; and we may assume

that the star's own motions are as likely to be in one direction as another, so that the average components would balance, as it were. It follows that any difference of the two components, derived from observed averages, must be an effect of the sun's motion alone.

Now this average difference is expressed in seconds of arc, being an observed angular proper motion across the sky. And we know the linear velocity of the solar system toward the apex to be 13 miles per second (p. 338), or $13 \times 365 \times 24 \times 60 \times 60$ miles annually. Figure 102 shows the further

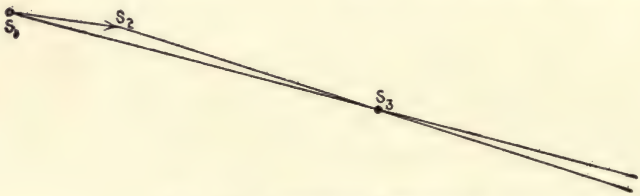


FIG. 102. Kapteyn's Researches.

procedure. The arrow S_1S_2 represents the solar system's motion toward the apex, in a year. Kapteyn's "average star" is shown at S_3. The little angle $S_1S_3S_2$ is the average star's proper motion component away from the apex, or the above-mentioned observed difference of the two components. Knowing, now, this little angle, and the linear velocity S_1S_2, we can calculate S_1S_3, or the average star's distance from the solar system S_1.

This beautiful method of investigation has enabled Kapteyn to obtain a table of approximate stellar distances. He gives the following results, for two types of stars separately (cf. p. 337): I, bluish white stars, like Sirius; II, solar stars, like Capella. The distances are expressed in light-years (p. 333), three of which correspond approximately to a parallax of one second of arc. The table shows that the

solar stars are decidedly nearer to us than are the Sirian stars.

TABLE OF KAPTEYN'S DISTANCES

(In light-years)

STAR'S MAGNITUDE	TYPE	
	I	II
1	101	43
2	130	56
3	166	71
4	213	91
5	273	117
10	948	405
15	3270	1404

Kapteyn has also obtained some further interesting results as to stellar distribution. He uses a "unit of sidereal space," and for this space unit he imagines a cosmic sphere of "unit radius," which he defines as a sphere such that a star on its surface would have a parallax of one second of arc to an observer at its center. The length of this radius would be about three light-years. Then, since parallax angles are inversely proportional to distances, it follows that stars whose parallaxes are greater than 0.″20 are all within a sphere whose radius is 5 units. About 20 stars with such parallaxes are known; and we may assume that these are probably all that exist.

Now the volumes of spheres are proportional to the cubes of their radii; so that the above-mentioned sphere with radius 5 must have a volume of 125. And as there are 20 stars in it, there must exist about one star to each six units of space; and this is the approximate "star-density" in the cosmic vicinity of the solar system. This conclusion may

not be very accurate; but it constitutes a most important addition to our knowledge of sidereal astronomy, since even a rough approximation is better than a total absence of information.

And Kapteyn has been able to proceed a step farther. Having found the average distance of stars of any given magnitude, and knowing the average proper motions and radial velocities as well, he has been able to compute the actual average linear velocities of the stars in space; and he finds them to be somewhat less than twice the cosmic velocity of the solar system.[1] Therefore the stars' average speed is about 26 miles per second; or, in a year, about seven times the distance between the earth and sun. But the sidereal unit, or radius of the unit sphere, is about 200,000 times the distance from earth to sun; so the stars move a sidereal unit, on the average, in 27,000 years.

Now we have found that, on the average, about one star exists in each six units of space. From this it may be computed, according to the theory of probabilities, that the average distance between the stars is about 3.5 linear sidereal units. Therefore the stars move through a distance equal to the 3.5 units that separate them in about $3.5 \times 27{,}000$, or 100,000 years, on the average. It follows from all these considerations that stars will approach each other infrequently, even within astronomical proximities, enormous as these are.

The above conclusions all relate to averages; and we know

[1] Knowing the star's parallax, or distance, and the angular annual proper motion across the sky, we compute the linear velocity component parallel to the celestial sphere in the same way that we obtain a planet's linear diameter from its measured angular diameter. Then, knowing the radial velocity, and the linear velocity at right angles to it, as computed from the proper motion, we can finally calculate the actual velocity.

one or two stars that probably have far greater velocities than the average. For instance, the star 1830 Groombridge (p. 334) has a velocity of perhaps 140 miles per second. We must conclude that this particular star is passing through our sidereal universe, and will leave it altogether in a few million years. But in general, from what has been said, it would appear that the stars in our universe are much like the molecules of a gas, as indicated in the kinetic theory of gases. The difference between a glass vessel full of gas and a universe full of stars is merely one of scale. In either case, each star or molecule moves in a more or less straight line of random direction, until or unless a couple of them happen to collide. In both cases, such collisions are extremely frequent: only, in the gas, the word "frequent" signifies a very minute fraction of a second; in stellar space, the same word may mean centuries.

Perhaps this kinetic theory of stars would undergo some material modification if we admit that all observed stellar motions are not necessarily random ones, and that there may be star-groups of common motion, and star-streams of vast extent. But however this may be, these magnificent researches are all inspiring in a high degree: it is extraordinary that such can still be made in our own day in the oldest and most completely perfected domain of human knowledge, the science of astronomy.

Not infrequent in the sky are the "binary" stars;[1] twin suns, they have been called (cf. p. 9). Each component star of such a pair moves around the center of gravity of both in an oval orbit, just as the earth and moon (p. 174) move around their center of gravity. The orbits are studied with a special micrometer (p. 276), with which astronomers can measure

[1] The lower part of Plate 27 is a photograph of a binary.

the angular distance between the two components in seconds of arc, and also the angle between the line joining them on the sky and a line drawn from the principal one to the celestial pole. Thus, in Fig. 103, S_1P is an arc of a great circle imagined on the sky, joining the principal component star S_1 and the pole. We then measure the small angular distance S_1S_2, between the two components of the double star, and also the angle PS_1S_2. The latter is called the "position angle." If we continue these measures, at intervals, for a number of years, and then draw the resulting orbital curve, we find it to be an oval or ellipse, often very much flattened.

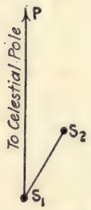


FIG. 103. Binary Star.

But this is only an apparent orbit; for what we see is the real orbit, projected on the celestial sphere. It is only when the plane of the binary star's orbit happens to be perpendicular to our sight-line that the apparent orbit coincides with the real one. But it is always possible to calculate the location of the true orbital plane; and, in fact, the elements of the real orbit, by applying Kepler's laws of motion (p. 187) to the apparent orbit. Yet, even after this has been done, we have only a "relative" orbit, representing the motion of one component star of the binary pair with respect to the other.

For a very few binaries, the actual orbit of both components has been separately determined, by means of micrometric comparisons with neighboring small stars. And when we are so fortunate as to know also the parallax of the binary, we can calculate the linear dimensions of the orbits in miles. Without the parallax, we can know only the angular dimensions of these orbits in seconds of arc. The relation of the two dimensions is like the relation of the

Photo by Barnard. *Photo Mt. Wilson Observatory.*

PLATE 27. A Star Cluster in Hercules and the Double Star Krueger 60.

angular diameter of the sun to its real linear diameter (p. 118). In the few cases for which such researches have been carried out with success, the linear size of the orbits appears to be comparable with the orbital radii we find in the solar system. So we conclude that the binary systems are not extremely large, speaking cosmically.

Certain binary stars have been recognized as such by spectroscopic instead of micrometric observations. We have already described Vogel's discoveries with regard to Algol (p. 328), where the binary character of the star was betrayed by an observed periodic change in the direction of its radial velocity. But Pickering also found that certain spectra photographed with a slitless spectroscope (p. 285) showed a periodical doubling of the lines. Ordinarily single, they became double at uniform intervals of time. Pickering explained this correctly as indicating a binary system, in which, unlike Algol, the components are both luminous stars. When one component is approaching us, and the other receding, in consequence of their orbital motions, the spectral lines are displaced in opposite directions, according to the Doppler principle, and we get two separate spectra and two sets of lines. When both components are moving at right angles to the sight-line, as they will do in another part of their orbit, the two spectra are superposed, and we get one set of lines only. It is remarkable that we can thus separate a pair of stars with certainty, although they appear so near each other on the sky that the most powerful telescope shows but a single object at all times. This was a great triumph for the spectroscope; the observation, together with the correct explanation of it, will surely have a place in the classic annals of astronomy.

It is interesting to note that we can calculate also the

masses of these binary stars for which both orbits and parallaxes have been observed. As we have just seen, we then know the linear dimensions of the orbit in miles, and so we can apply the method used (p. 204) for obtaining the mass of a planet having an observable satellite.[1] The masses of these binaries, in the few cases where they have become known, are found to be of the same order of magnitude as the sun's mass.

Before leaving the subject of binary stars, it may be of interest to touch on one possible theory as to their origin. It is not now believed by astronomers that the Laplacian theory of celestial development (p. 235) is the only possible or even probable one. For the Laplacian idea leads to a single central sun, with many planets of far smaller size than the sun. But it is possible that an original whirling nebula may have undergone changes more or less approximating the formation of two nuclei. These, revolving, gave rise, first, to an egg-shaped, — later, a dumb-bell shaped, — revolving body. The latter, finally separating, should produce twin suns, at first revolving with their surfaces almost in contact. Such a condition might even explain some of the peculiar light-changes of certain variable stars. Later, there might arise perturbative action, similar to the tidal effects produced between the moon and terrestrial oceans. These would drive the two bodies farther apart (cf. p. 258), and possibly lead to a visible binary star. Nor is there any objection to our imagining some of these distant suns to be attended by planets. Only, if such planets are no larger than Jupiter, we could not possibly hope to see them with the most powerful of our telescopes.

It is an easy transition from the binary systems to those

[1] Note 43, Appendix.

Photo by Barnard.

PLATE 28. The Pleiades.

still more wonderful, in which three or more incandescent suns revolve about their common center of gravity in plain view of the telescope. For instance, the constellation Lyra contains an important double star, the "double double," in which each component is itself a binary, forming all together a quadruple star.

And in addition to these "multiple stars," we find also various "clusters." Some contain comparatively few stars, spread over quite a considerable bit of the sky. The Pleiades group is a famous cluster of this kind. Two views of it are shown in the accompanying Plate 28: one contains the stars only; the other, made with a large telescope, indicates that most of these stars are still surrounded with nebulosities. Here and there we can see a nebulous lane running from one star to the next; nor have these peculiar formations ever been thoroughly explained. There are also other clusters, like the close-packed globular one in the constellation Hercules (Plate 27, p. 349), consisting of many thousand stars separated from each other on the sky by very small angular distances only.

We have two sure facts to indicate that the clusters are single objects, and not mere fortuitous groupings of stars, unconnected with each other, situated at all sorts of distances from the solar system, and appearing close together because they happen to be projected near a single point of the sky. First, in the Pleiades group, it is known that over forty stars have practically identical proper motions in the same direction on the sky, pointing to a community of real motion in space. And secondly, many close clusters have been found to contain a most unusually large percentage of variable stars, again indicating a community of origin for the whole cluster.

Unfortunately, it has not yet been possible to measure the parallax of any cluster. We can but make guesses at their distance from the solar system, rough estimates based on the size of their proper motions. These are so small that we must assume the clusters to be very distant,—probably not less than 400 light-years. If removed to that distance, our own sun would give us no more light than a star of the eleventh magnitude. It follows that the clustered stars are perhaps comparable in size with the sun, for they, too, average the eleventh magnitude, more or less.

With the above estimate of distance, we can also estimate the linear size of the clusters from their angular diameter, in the usual way; and we find them to be about two light-years in diameter. If such a cluster contained 10,000 stars, the average distance from one to another would be about 25,000 times the distance from the earth to the sun. At such distances gravitation would not be strong enough to bring all the constituent stars under the influence of a central force: it would not even produce velocities of interstellar orbital motion such as could become perceptible to our micrometric instruments during the comparatively short period since precise observations were commenced by terrestrial man (cf. p. 322).

Closely related to the clusters are the nebulæ (p. 3). Indeed, certain clusters, like the Pleiades, are so completely interwoven with clouds of nebulous matter, and with nebulous lines connecting the several stars, that one is almost inclined to regard them as nebulæ partly condensed into stellar nuclei. But the true nebulæ are undoubtedly gaseous: spectroscopic evidence on this point is conclusive (pp. 4, 283).

"Planetary nebulæ" are a class of nearly circular light-clouds possessing almost planetary central disks. Their

spectra contain certain lines belonging to nebulæ only, and ascribed to incandescence of a hypothetical substance "nebulium." At their centers, nebulæ of this class have at times nuclei that look like stars. One is tempted to imagine them in the last stage of nebular development, and on the verge of becoming starry. There exist also a few ring-formed nebulæ.

But the type-form of nebula is the spiral nebula (cf. Plate 2, p. 4). Keeler thought there are 120,000 objects of this form within the photographic range of his big reflecting telescope at the Lick observatory. His observations opened the eyes of astronomers to the probability that the Laplacian plan of cosmic evolution may not be the one generally active; that a single sun like ours is less likely to occur in space than some more complicated system of suns, resulting perhaps from a great apparently whirling complex by us seen as a spiral nebula.

The Andromeda nebula is by far the largest of the spirals: for modern long-exposure photographs have proved its spiral character, though it was always supposed to be elliptical or ring-formed until celestial photography came into general use. We cannot measure its distance from us by any of our parallax methods: but it is possible to fix for its parallax a limit of 0.″01. The parallax cannot be much larger than this, or traces of it would reveal themselves to our instruments. But assuming this parallax, and the known angular diameter of the nebula (one and a half degrees of arc), it must have a linear diameter 540,000 times as great as the distance between the earth and the sun.[1]

It is furthermore of interest to compare this nebula's possible attractive force with that exerted by the sun on

[1] Note 44, Appendix.

the earth. It may be computed that if the nebular density is only $\frac{1}{3000000000}$ of the sun's, the nebula will attract the earth as much as the sun does.[1] So the fact that we find no perturbative attraction whatever in the solar system resulting from the nebulæ, proves that, though enormous in size, they are of an almost inconceivable tenuity; in fact, almost without any density whatever.

To the foregoing very recent researches must be added an observation of most ancient date, but one that all the modern theories have failed to explain quite fully. This is the Milky Way, or "galaxy," which shows itself as a band of small stars — star dust — encircling the sky almost like the celestial equator, ecliptic, etc. It has many starless rifts and lanes, and several "holes"; notably the "coal-sack," situated near the south pole of the heavens. It contains numerous star-clusters, but few nebulæ. One of the most interesting is shown in Plate 29, — the "North America" nebula, so named because of its shape.

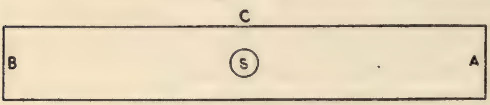


FIG. 104. The Galaxy.

The galaxy, resembling a great circle of the celestial sphere, must of course have two nodes, or points of intersection with the ecliptic circle. These are near the solstices, and the galactic circle makes an angle of about 60° with the ecliptic. According to Sir William Herschel, the stars of the galaxy are spread out in a thin disk, in which our sun is also situated. In Fig. 104, the solar system is shown at S, and the galactic disk is within the rectangle. Outside the rectangle, the stars are fewer and farther asunder. It is evident that if we look in the direction A or B from S, we shall look through the

[1] Note 45, Appendix.

Photo by Barnard.

PLATE 29. The North America Nebula.

thick part of the galaxy, and see an enormous number of stars projected on the celestial sphere; but if we look toward C, we shall see projected on the sky only a thin part of the disk, and the sparser stars, outside it. And the disk, of course, when produced outward to the celestial sphere, will cut out the galactic great circle.

Actual counts of stars have been made by Herschel and others, to ascertain the number per square degree of sky surface at various angular distances from the galaxy. The numbers are found to vary in the following proportions:

Angular Distance from Galaxy	Relative Number of Stars per Square Degree
90°	4
60°	7
30°	18
0°	122

The unexplained difficulty with Herschel's explanation of the galaxy still remains. The extreme minuteness of the galactic stars indicates immense distance, as does also their lack of observable proper motions across the sky. But if so enormously distant, how can the galaxy constitute a single disk-shaped cluster or universe? But perhaps we are here inventing a difficulty, because of our inability even to imagine the scale of the sidereal universe.

However this may be, we may close this part of our subject with a definite statement that there is no evidence in the possession of astronomers to indicate a "central sun" around which all the stars are circulating in their orbits. As already stated, we now believe their motions more nearly resemble the gyrations of the molecules in a gas under the kinetic theory. Gravitation probably takes hold in an appreciable degree, only when a couple of stellar molecules happen to pass near each other, speaking cosmically.

CHAPTER XXI

THE UNIVERSE ONCE MORE

THE reader may recall that we commenced our long explanation of astronomic science in the present volume with a chapter entitled "The Universe." Now that we at last approach the end, let us once again return to the beginning, and reëxamine the evolutionary processes of the cosmic universe, in the light of the astronomic knowledge we have been able to gain.

Cosmogony is a name given to the various theories of the universe and its life-history: there is no subject more enticing to the mind of man; none in which he is more prone to be misled into fields of mere speculation, quite outside the domain of strictest logic, based on irrefragible observational premises.

We have already mentioned (p. 235) the Laplacian nebular hypothesis, with its rotating nebulous sun, forming planets by the successive separation of rings: it will now be proper to inquire a little more closely into the admissibility of Laplace's idea.

It will be well to begin by summarizing the known facts that are favorable to Laplace:[1]

1. The planetary orbits all lie nearly in the same plane; and the direction of orbital motion is the same for all planets, and for the sun's axial rotation.

2. The orbits are all nearly circular.

[1] Laplace, "Exposition du Système du Monde," p. 470, in *Oeuvres de Laplace*, Paris, MDCCCXLVI.

3. With the exception of Uranus and Neptune, the equatorial planes of the planets, and even the orbital planes of their satellites, all coincide approximately with the fundamental plane of all the orbits; and the direction of the satellites' orbital motion is in general also the direction of the planets' axial rotation.

4. We have accepted the Helmholtz theory (p. 294), that the sun's source of heat must be sought in the contraction of its bulk: in that case the sun must have once been incomparably larger than at present, just as the nebular hypothesis requires. Laplace, of course, was not in possession of Helmholtz' calculations when his own theory was published.

Now, as a matter of logic, a correct theory must explain every observed fact within its range. A single contrary observation may destroy logically an entire theory, no matter how many other observations seem to confirm it. Let us then next enumerate some of the objections that have been advanced against the nebular hypothesis.

1. Phobos, the inner satellite of Mars (p. 222), and the inner edge of Saturn's ring (p. 245), revolve in their orbits faster than the axial rotation periods of Mars and Saturn respectively. This will not do: the contracting planet should rotate faster than any satellite revolves around it, just as the inner planets have shorter sidereal periods than the outer ones.

2. The kinetic theory of gases would lead us to expect (pp. 167, 222) that a gas like hydrogen would all be lost into space from each planet in the form of molecules, soon after the ring was thrown off from the sun, on account of the very small gravitational pull of the ring. Yet we still find hydrogen in plenty on the earth.

3. The throwing off of the rings is in itself an hypothesis difficult of acceptance, on account of the presumably extreme rarity of the outer parts of the solar nebula, and consequent lack of cohesion. And why was the process of expelling rings intermittent instead of continuous?

4. The mechanical movements in the system present difficulties. For instance, the total quantity of rotary momentum now belonging to Jupiter is about $\frac{95}{100}$ of the total belonging to the entire solar system included within the orbit of Saturn. Yet Jupiter's mass is only about one-thousandth of the total mass remaining within Saturn's orbit, including the sun. Are we then to suppose that the present sun, at a single moment, parted with so small a fraction of its mass, which yet carried away almost all its rotary power?

Chamberlin and Moulton have endeavored recently to substitute a new and different theory for that of Laplace. They call it the "planetesimal hypothesis"; and they think the recent evolution of stellar systems, and of our solar system, began with the accidental close approach (not collision) of two stars. If we imagine such an approach to have taken place, we must suppose the two bodies revolving for a time in orbits around their common center of gravity. If the approach was not very close, these orbits would be open curves, like the orbits of most comets when they approach the sun (p. 312). The two bodies would separate after a certain time, and would never again pass near each other.

But while they were together, the gravitational effect of each on the other would be tremendous. Doubtless each would eject masses of highly heated gaseous matter, much as the great red hydrogen prominences (p. 293) are ejected from our sun. Upon these ejected gases, and upon the other outer gaseous envelopes of the two stars, gigantic tidal forces

Photo by Keeler.

Spiral Nebula, seen edgewise.

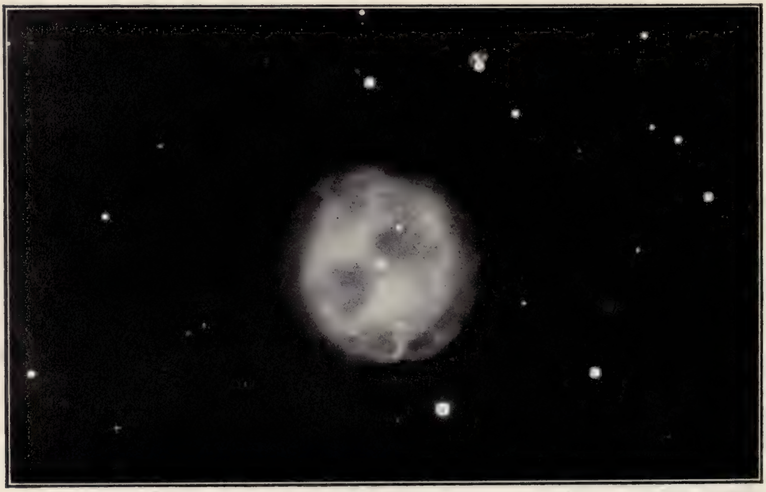


Photo by Hale.

Owl Nebula.

PLATE 30. Nebulae.

would be exerted. Consequent gigantic tidal effects would be produced in each star by the other. Like the tidal crests caused by the moon in terrestrial oceans (p. 252), the ejected masses would travel outwards from each star, directly toward the other star, and directly away from it.

Afterwards, these masses would move in strange orbits under the combined gravitational attractions of the two stars: we can in a way trace out these orbits by computational methods, which have been tried by Moulton. Chamberlin calls these masses "planetesimals"; and it is assumed that they make their appearance in great numbers and at short intervals. Figure 105 shows the probable orbits they would pursue. The dotted lines indicate various orbits; the full lines show the points reached at a given instant of time by the several planetesimals pursuing the dotted orbits.

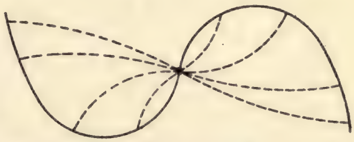


FIG. 105. Planetesimal Hypothesis.

When we look at the result of such a performance, what shall we expect to see? If we assume the instant of time when we make our observation to be that instant when the several planetesimals have just reached the full lines, we shall not see them traveling along their dotted orbits, but we shall see them all lying on the full lines. In other words, we shall see a spiral nebula.

Now whatever strength there may be in this hypothesis, there can be no doubt that Keeler's observations of nebulæ (p. 353) establish the fact that the spiral is the normal type of nebula. For comparison with Plate 2, p. 4, we give in Plate 30 a photograph of a nebula which is doubtless another spiral, but seen edgewise. The lower part of the plate

shows the "owl" nebula in *Ursa Major*, which looks more like a Laplacian nebulous sphere. Plate 31, p. 360, the "trifid" nebula, is a good example of quite irregular shape.

So the new theory gives a notion as to the possibility of spirals resulting from a close approach of two stars, which may have been previously moving about in space aimlessly, after the fashion of the molecules belonging to a rarefied gas (p. 347). Thus the theory goes back to a very early stage of cosmic development, and shows how stars, even dark ones, can be transformed into nebulæ.

But how can the spiral nebula, in turn, develop into a sun and planets such as we have in our solar system? Of course, the sun is simply the remaining material from one of the original stars after the other had left it. If the approach was near enough to lead to a permanent orbital proximity, we might perhaps expect a binary system. And the sun provided, it is easy to imagine the origin of the planets. They are unusually large occasional planetesimals, increased gradually by the accretion of other smaller ones, swept up by them as they moved along in their dotted orbits. When there was no large dominating planetesimal for a long time, a group of minor planets might result. The satellites must be regarded as little planetesimals that were shot out near the larger ones that became planets.

Chamberlin and Moulton have traced out in detail the bearing of their theory upon the various objections that have been enumerated against the Laplacian idea, and with considerable success. The great advantage of their hypothesis is that it gives us an origin antedating the nebular stage; that it makes a cycle of cosmic life and death; and especially a cycle in accord with the actual visible appearance of existing nebulæ. This the Laplacian theory does not do.

Photo by Hale.

PLATE 31. The Trifid Nebula.

The future of the solar system seems fairly clear under either hypothesis. The present state of affairs is one of apparently stable equilibrium; and should continue, unless an accident arrives from outside the system. But even without such accident, the solar system cannot be eternal, because the gradual shrinkage of the sun cannot continue forever. When the time comes for the sun to lose its heat-radiating power, the solar system must become cold and dead. If, after countless ages, it shall ever thereafter be revivified, the cause will be a fresh approach to some other star, dark or brilliant, whose vast disturbing attraction will once more break up the solar matter into a mist: and if a great part of the energy remaining in the system shall be transformed into heat, then that mist will once again be a fire-mist, which may once more pass through all the stages of cosmic life and death.

APPENDIX

The following notes contain explanations omitted in the text, and requiring occasionally a knowledge of elementary algebra, geometry, and trigonometry as far as the solution of plane right triangles.

Note 1. Declination and Right-ascension (p. 35).

Declination corresponds exactly to latitude on the earth; the declination of a star is its angular distance on the celestial sphere measured north or south from the celestial equator. This angular declination, like all angles, must, of course, be measured on some circle; for measuring it we must imagine a circle drawn upon the sky from the star to the equator, and perpendicular to the equator. Such a circle, drawn for the purpose of measuring declination, is called an Hour-circle. The point where the hour-circle meets the celestial equator may be called the foot of the hour-circle. The right-ascension of a star is then defined as the angular distance, measured on the celestial equator, from the vernal equinox point on the equator to the foot of the hour-circle drawn from the star to the equator.

Note 2. Hour-angle, etc. (p. 37).

We may now define also the term "hour-angle," which is closely related to the hour-circle used in measuring right-ascension. The hour-angle of a star is defined as the angular distance, measured like right-ascension on the celestial equator, from the meridian to the foot of the hour-circle drawn from the star to the equator. Thus hour-angle and right-ascension are both arcs measured on the equator; both arcs have one end in common, the foot of the hour-circle; but the other ends are different, being respectively the meridian and the vernal equinox.

All the astronomical terms so far defined are exhibited in Fig. 106. It represents half the celestial sphere, the half which is

above the horizon, and therefore visible to us. The large circle *NESW* represents the horizon; and the celestial hemisphere is shown projected down upon the plane of this horizon. The zenith, or point directly overhead, of course projects down into the center of the horizon circle. The great meridian circle appears as the line *NPZS*, since it must pass through the zenith Z as well as the north and south points of the horizon shown at N and S. The celestial north pole, which is, by definition, in the celestial meridian, will project down to some point P. The celestial equator, everywhere 90° distant from the pole P, will project into the circle *WME*.

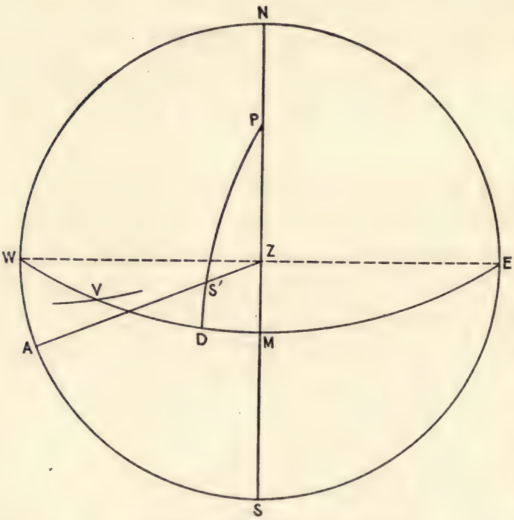


FIG. 106. The Celestial Sphere.

Any star selected at random may be supposed to be projected down at the point S'. Then $S'D$, an arc drawn on the sphere through the star and perpendicular to the equator, is by definition an hour-circle. It is evident that all hour-circles must pass through the pole P. The arc DM on the celestial equator, included between the meridian at M and the foot of the hour-circle at D, is the hour-angle of the star. The arc $S'A$ is the star's altitude, or angular elevation above the horizon. Finally, if we draw a short piece of the projected ecliptic circle, we may take V to be one of its points of intersection with the celestial equator *WME*, the other point of intersection being of course below the horizon. And if we let V be that one of the two points of intersection which we have called the vernal equinox, then the right-ascension of the star S' is the arc VD, measured on the equator, and included

between the vernal equinox V and the foot of the hour-circle at D. The arc DS' is the declination.

The above astronomical terms may be divided into two classes; viz.: those that retain a constant position among the stars on the celestial sphere, and those that are as constantly shifting their positions among the stars on account of the daily seeming rotation of the whole sphere. Thus, for instance, the zenith, which is the point directly overhead, does not partake of the seeming turning of the sphere. The following little table shows the two classes of terms:

Unchanging Positions among the Stars. Rotate with the Sphere	Changing Positions among the Stars. Do not Rotate with Sphere
Celestial Poles,	Zenith
Celestial Equator	Horizon
Ecliptic Circle	Altitude
All hour-circles	Hour-angle
Right-ascension	Meridian
Declination	
The stars, sun, etc.	
Vernal Equinox	

Note 3. Position of Celestial Pole (p. 40).

The relative positions of the celestial pole and the horizon may be made clear by means of a simple diagram. Figure 107 shows a portion of the earth, with its center at C, and the observer on the surface at O. The outer concentric circle $HPZE$ is the celestial meridian, on the celestial sphere. The zenith Z will be directly over the observer at O, on the prolongation of the observer's terrestrial radius CO. The celestial pole must be at some point of the celestial meridian, by definition. Let this point be P. The celestial equator will meet the meridian at E, 90° distant from P. The terrestrial pole will be at P', and E' will be a point of the terrestrial equator. The angle $E'CO$ will be the terrestrial latitude of the point O, since it is the

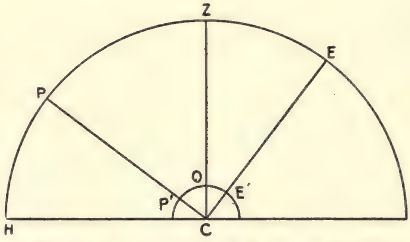


Fig. 107. Position of Celestial Pole.

angular distance of the point O from the terrestrial equator at E'. The angle PCH is the altitude or angular elevation of the celestial pole above the horizon at H. For H, as we know, is the north point of the horizon for an observer at O.

But $\measuredangle ZCE = \measuredangle PCH$, since PC is perpendicular to CE, and HC perpendicular to ZC. Hence we have a demonstration that the altitude of the celestial pole is everywhere equal to the terrestrial latitude of the observer. Thus, as stated in the text, this altitude will be 90° to an observer at the pole of the earth, where the latitude is 90°; and it will be 0° to an observer at the equator, where the terrestrial latitude is likewise 0°.

Note 4. Stars that never Set (p. 43).

It is evident that these stars are the ones whose diurnal circles have an angular distance from the celestial pole less than PH (Fig. 107); *i.e.* less than the observer's terrestrial latitude. These stars will have a declination greater than (90° − latitude).

Note 5. Sidereal Time (p. 67).

The sidereal time, or hour-angle of the vernal equinox, is the arc VM in Fig. 106.

To make the definition of sidereal time perfectly general, astronomers count all hour-angles westward from the meridian, and allow them to increase continuously to 24^h. Thus, an hour-angle 1^h east from the meridian, corresponding to 23^h sidereal time, would be called a west hour-angle of 23^h.

Note 6. Right-ascension of the Meridian (p. 67).

Again recurring to Fig. 106, it is clear from our definitions that the right-ascension of a star on the meridian is the arc VM; and we have seen in Note 5 that this identical arc is also the sidereal time. Therefore the sidereal time and the right-ascension of the meridian at any instant are the same.

The general relation of hour-angle, right-ascension, and sidereal time may also be deduced from Fig. 106. We have from our definitions:

$$VD = \text{right-ascension of star } S',$$
$$DM = \text{hour-angle of star } S',$$
$$VM = \text{sidereal time.}$$

And since, from Fig. 106:

$$VM = VD + DM,$$

it follows that in general:

Sidereal time = right-ascension + hour-angle.

Hour-angle = sidereal time − right-ascension.

The last equation enables us to ascertain the hour-angle of a star at any instant, if we know its right-ascension, and have a correct sidereal clock at hand.

Note 7. Hour-angle of Visible Sun (p. 68).

In Fig. 106, if we let S' be the visible sun at any instant, its hour-angle is the arc DM, measured in hours, minutes, and seconds. This same arc is also the apparent solar time at that instant.

Note 8. Terrestrial and Celestial Meridians (p. 73).

If we imagine a line drawn from the center of the earth to the observer, and thence continued outward to the celestial sphere, it will pierce the sphere at the observer's zenith. The terrestrial meridian, by definition, passes through the north pole of the earth and the observer. The celestial meridian, also by definition, passes through the celestial north pole and the observer's zenith. Therefore the celestial meridian is a projection of the terrestrial meridian outward on the celestial sphere. Figure 108 is like Fig. 106, with the addition of a second celestial meridian. The figure represents the celestial sphere projected down upon the horizon of New York,

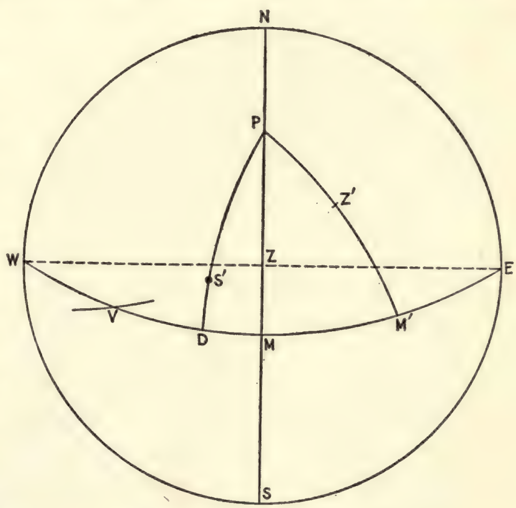


FIG. 108. Time Differences.

of which the zenith appears as before at Z. The projection of the zenith of Greenwich at the same instant is at Z'. Therefore $PZ'M'$ will be the projection of part of the celestial meridian of Greenwich. The sun and vernal equinox are projected at S' and V', as before. Then DM is the sun's hour-angle at New York; DM', its hour-angle at the same instant at Greenwich. MM', which measures the angle between the two celestial meridians, is also the difference of the two hour-angles, or the solar time difference between New York and Greenwich. And this is the same as the longitude difference, measured by the two corresponding terrestrial meridians on the earth inside the celestial sphere.

At the same moment, VM and VM' are the hour-angles of the vernal equinox at New York and Greenwich; and MM' is also the sidereal time difference. Consequently, the sidereal and solar time differences are equal and identical; they are both measured by the same arc MM'.

Note 9. Angle of Gnomon (p. 79).

It is evident that the "factor" in the table is simply the tangent of the latitude. In Fig. 24,

$$bc = ac \tan bac,$$

and if the tabular factor is tan latitude, the construction of the figure will make the angle bac equal to the latitude, as required for the gnomon.

Note 10. Mathematical Principles of the Sundial (pp. 80, 84).

To demonstrate the correctness of the construction given in the text for drawing a sundial, it is necessary to have recourse to the well-known formulas of spherical trigonometry relating to the solution of right-angled spherical triangles. The accompanying Fig. 109 represents the conditions of the problem. The large circle $ZPNQS$ is the celestial meridian. The circle $NIVS$ is the horizon, on the plane of which the dial is to be drawn. The center of the dial is at O; and QP is the axis of the celestial sphere. As the edge of the gnomon is parallel to the axis QP, we may regard it as lying in that axis, because the sun will appear to rotate around the edge of the gnomon (p. 84). So we may consider the edge of

the gnomon to start at O, and to extend a short distance in the direction OP.

Now suppose OIV, situated in the horizon plane $NIVS$, to be the direction in which the shadow falls at four o'clock. Then, remembering that solar time is simply the hour-angle of the sun, we recall that "four o'clock" means that the sun's hour-angle is four hours, or 60°. We may suppose the sun to appear at the point S' at four o'clock. Then, from the definition of hour-angle (p. 363), the sun is then distant 60° from the meridian; or the angle ZPS' is 60°. The opposite angle $NPIV$, being equal to ZPS', is thus also 60°.

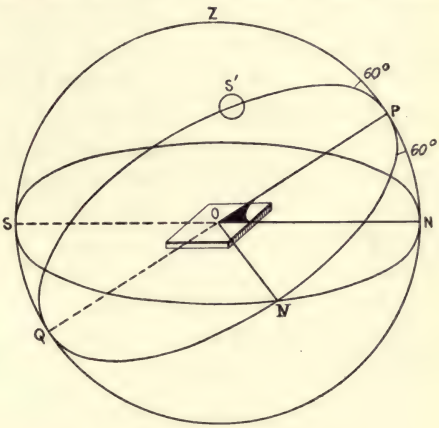


Fig. 109. The Sundial.

Now let us consider the spherical triangle formed on the celestial sphere by the three points P, N, and IV. In it we know the side PN, for it is the elevation of the celestial pole above the horizon, and therefore equal to the latitude of the place where the dial is to be used (p. 365). As we have just seen, we also know the angle $NPIV$, which is 60°. And we know the angle $PNIV$ to be a right angle, because the celestial meridian is perpendicular to the horizon.

According to the principles of spherical trigonometry, if we know one side and one acute angle of a right-angled spherical triangle, we can calculate all the other parts of the triangle. In the present problem, we need only calculate the side NIV. For this measures the angle $NOIV$, which is the dial angle for the four-o'clock line, or the angle which the four-o'clock line makes with the north-and-south line ON.

In the same way, we can calculate the dial angles for the one-o'clock, three-o'clock lines, etc. The twelve-o'clock line, or noon-

line, is of course ON; for at noon the sun is on the meridian, and the shadow of a gnomon pointing at the celestial pole will then fall due north. We might construct the dial by simply laying off the proper computed angles for the various hours from the dial center O.

The trigonometric formula for calculating the side NIV, or the dial angle $NOIV$, is:

$$\tan NIV = \tan NPIV \sin PN.$$

And if we let:

u = dial angle for any hour,
t = corresponding hour-angle of the sun,
l = latitude of the place,

then the general formula is:

$$\tan u = \tan t \sin l.$$

The dial angles calculated by this formula for the latitude of New York are as follows:

XII.	0° 0′
I.	9° 56′
II.	20° 40′
III.	33° 10′
IV.	48° 32′
V.	67° 42′
VI.	90° 0′

It now remains to show that the construction given in the text (Fig. 25) is in accord with the above general formula. In this figure we have really drawn two half-dials, so as to allow for the thickness of the gnomon. To prove the construction of Fig. 25 correct, we have now to show, for instance, that:

$$\tan caI = \tan 15° \sin l.$$

The factors given in the table on p. 80 are the sines of the latitudes l. Therefore, since we made Mc (Fig. 25) equal to ca multiplied by the factor in the table, it follows that:

$$Mc = ca \sin l. \tag{1}$$

We made the angle cMI (Fig. 25) equal to one-sixth of a right

angle, or 15°. Therefore, from the right-angled plane triangle McI, we have:

$$\frac{Ic}{Mc} = \tan 15°,$$

or:

$$Ic = Mc \tan 15°.$$

Substituting the value of Mc from equation (1) gives:

$$Ic = ca \tan 15° \sin l,$$

or:

$$\frac{Ic}{ca} = \tan 15° \sin l. \qquad (2)$$

Now from the right-angled plane triangle cIa, we have:

$$\frac{Ic}{ca} = \tan caI. \qquad (3)$$

Substituting from equation (2) in equation (3), we have

$$\tan caI = \tan 15° \sin l;$$

and the correctness of the construction in Fig. 25 is proved, since the above equation accords with the general form:

$$\tan u = \tan t \sin l.$$

Note 11. Theory of Foucault Experiment (p. 91).

We have explained the conditions of the problem if the experiment were performed at the north pole of the earth. There the point of suspension of the pendulum's wire would of course be situated in the prolongation of the earth's axis, and would consequently not move as a result of the earth's axial rotation, which is the only motion of the earth here requiring consideration. In any other latitude, the point of suspension would go around as the earth rotates: it is therefore necessary to explain further the statement that the direction in space of the pendulum's plane of oscillation tends to remain constant. The fact is that when the point of suspension moves, the plane of oscillation moves also; but it tends to occupy a position constantly parallel to itself. Any one can satisfy himself that this is correct by fastening a small metal ball to a string and letting it oscillate, the end of the string being held in the experimenter's hand. It will be found that the experimenter may walk across the room, carrying the end

of the string; yet the plane of oscillation will remain constantly parallel to itself.

So much being premised, we may now proceed to calculate the rate at which the marks under the pendulum should rotate. Let us suppose we start the pendulum swinging in a north-and-south direction, and therefore directly under the celestial meridian, and in the plane of the meridian. In Fig. 110, let $NABCS$ be the meridian directly over the pendulum when we start it swinging, and suppose it swings between two points in the room corresponding to the points A and B of this meridian. In a second of time the earth's rotation will have brought a new celestial meridian over the swinging pendulum, and the old one will have gone to the position $NA'B'C'S$.

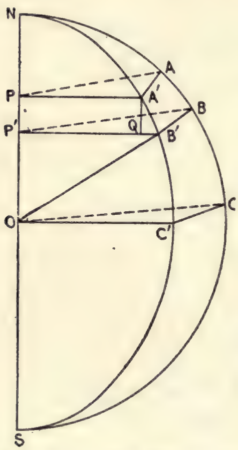


FIG. 110. The Foucault Experiment.

But the pendulum will still swing parallel to the plane of the first meridian, and the rotation shown by the experiment will be equal to the angle between the two meridians. Let us draw Fig. 111 to show this angle. This figure is like a map in a geography book. If the original meridian was AB, and the meridian at the end of one second $A'B'$, the rotation shown by the pendulum will be the angle between these two lines. If we draw $A'M$ perpendicular to BB', the rotation angle will be the angle $MA'B'$. Let us call this angle α.

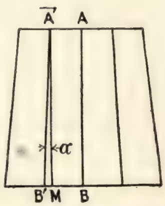


FIG. 111. The Foucault Experiment.

It is well known that on a map of the earth's equatorial regions the terrestrial meridians are practically parallel: there is no "convergence of meridians" there; and there would be no Foucault effect. Near the pole the angle between the meridians is a maximum: there the Foucault effect is also greatest.

In this way we translate our astronomical problem into terms of geometry: it is now merely a question of simple geometry to as-

certain the angle of convergence between two neighboring meridians on the earth in any latitude, such as that of New York, for instance; and this angle will be the Foucault pendulum rate of rotation.

We see at once from Fig. 111 that, in any latitude, we have from the triangle $A'B'M$:

$$\tan \alpha = \frac{MB'}{MA'};$$

and since for very small angles like α the tangent and the arc are equal, we may write:

$$\alpha = \frac{MB'}{MA'}. \qquad (1)$$

Referring again to Fig. 110, which we may now take to represent the earth instead of the celestial sphere, we observe that the latitude arcs AA', BB', and CC' are all arcs of circles whose radii are PA', $P'B'$, and OC'. The last radius OC' is the earth's radius, because we shall consider CC' to be an arc of the equator. Now suppose the point B' to correspond to the terrestrial latitude l'. Then l' is the angle $B'OC'$, for the latitude is the angular distance of B' from the equator. But the line $P'B'$ in the plane of the circle $NA'B'S$ is proportional to the cosine of the angle $B'OC'$. Similarly, the radii of all arcs like AA', BB', etc., are simply proportional to the cosines of the latitudes corresponding to the points A', B', etc.

But the arcs themselves must be proportional to their radii. So it follows that the linear lengths of the arc AA', BB', are also proportional to the cosines of the corresponding latitudes.

We have called l' the latitude corresponding to the point B'. Let us call l the latitude corresponding to A'. Now we have found that the arcs AA' and BB' are proportional in length to the cosines of the latitudes l and l'. Therefore the difference between AA' and BB' must be proportional to the difference of the same cosines, which we may express by the following equation, in which K is simply a constant denoting proportionality:

$$BB' - AA' = K(\cos l' - \cos l).$$

But, from Fig. 111:

$$MB' = BB' - AA'.$$

Consequently, from the preceding equation:

$$MB' = K(\cos l' - \cos l). \tag{2}$$

Now, in Fig. 110, draw the line $A'Q$ perpendicular to $P'B'$, completing a little right-angled triangle $A'B'Q$. (We may regard the short arc $A'B'$ as here equivalent to a straight line.) Then we have:

$$QB' = \cos l' - \cos l,$$

and:
$$QB' = A'B' \sin QA'B'.$$

But:
$$QA'B' = B'OC' = l';$$

therefore:
$$QB' = A'B' \sin l'.$$

But:
$$A'B' = (l - l').$$

Consequently:
$$QB' = \cos l' - \cos l = (l - l') \sin l'.$$

It then follows from equation (2) that:

$$MB' = K(l - l') \sin l'. \tag{3}$$

We also have, obviously, from Fig. 111:

$$MA' = l - l'. \tag{4}$$

Now substituting from equations (3) and (4) in equation (1), we have finally:

$$\alpha = K \sin l'. \tag{5}$$

This simple equation (5) establishes the important principle that the rate of rotation of the Foucault pendulum in one second must everywhere be proportional to the sine of the latitude of the place where the experiment is performed.

It is further obviously indifferent whether the original impulse was given to the pendulum in the direction of the meridian; for whatever angle the original impulse made with the original meridian, at the end of one second of time that angle will have changed by the same quantity α with respect to the meridian.

It is quite easy to find the value of the constant K in equation (5). For at the north pole, $\sin l' = 1$, since $l' = 90°$. Therefore, at the pole, equation (5) becomes:

$$\alpha = K.$$

But we already know that at the pole the pendulum must make one complete revolution of 360° in 24 hours. So it must

there revolve at the rate of 15° per hour, or 15′ per minute. With this value of K we therefore have in any latitude l':

$$\text{Rate of revolution} = (15' \text{ per minute}) \sin l'.$$

In New York, for instance,

$$l' = 40° \, 48', \sin l' = 0.65.$$

$$\text{Rate of rotation} = 9.'75 \text{ per minute.}$$

The above demonstration of the Foucault pendulum theory is not rigorous, but it is sufficiently accurate for ordinary purposes, provided the duration of the experiment is not much greater than one hour.

Note 12. The Torsion Constant (p. 108).

The problem of ascertaining the torsion constant T from the time of oscillation of the torsion balance is quite analogous to the corresponding problem of determining the length of an ordinary pendulum from its time of vibration. It is shown in books on physics that if we let:

t = vibration time of an ordinary simple pendulum,
l = length of the pendulum,
g = the force of gravity on the earth,
π = the ratio 3.1416,

then:

$$t = \sqrt{\frac{l}{g}}.$$

An analogous formula exists in the case of the torsion balance, except that instead of g, the force of gravity, the formula involves T, the torsion constant. We now let l represent the entire length ab of the torsion balance arm (Figs. 32 and 33), and m the mass of either small ball a or b. Then the torsion balance formula is:

$$t = \pi\sqrt{\frac{ml^2}{2T}},$$

and those readers who are acquainted with the science of mechanics will note that $2m\left(\frac{l}{2}\right)^2$ is the "moment of inertia" of the entire balance.

Solving this equation for T gives:

$$T = \frac{\pi^2 ml^2}{2\,t^2};$$

and this equation will make known the value of T for any torsion balance after we have observed its vibration time t, measured the length of the arm ab, and ascertained m by weighing the small balls in an ordinary balance.

Note 13. The Cavendish Experiment (p. 110).

Returning now to Fig. 33, let us use the following notation:

M = mass in grams of either big lead ball,

m = mass in grams of either small ball,

d = measured distance in centimeters from the position of rest b' to B', the center of the big lead ball,

g = the acceleration due to the "force of gravity," as used in physics, or 981 centimeters,

l = length of torsion balance arm, or the distance ab, in centimeters,

f = the force with which both big balls turn the balance.

Now, according to Newton's law, the attractive force between the balls B' and b' is (p. 103):

$$G\frac{Mm}{d^2}, \tag{1}$$

in which formula G is a so-called "gravitational constant," introduced to indicate that the attraction is proportional to $\frac{Mm}{d^2}$, not equal to it.

The distance from B' to a' is:

$$\sqrt{d^2 + l^2};$$

consequently, the attractive force existing between B' and a' is:

$$G\frac{Mm}{d^2 + l^2}. \tag{2}$$

Both forces (1) and (2) tend to turn the torsion balance. They act against each other, however, tending to rotate the balance in opposite directions. And the force (1) is larger than (2); so that it will determine the final direction of rotation.

Furthermore, the entire force (1) tends to turn the balance, while only a small part or "component" of (2) has such an effect. We can easily find this component, which acts from B' upon a' so as to turn the balance. According to the so-called "parallelogram of forces" this component is:

$$G\frac{Mm}{d^2 + l^2} \sin B'a'b',$$

or:

$$G\frac{Mm}{d^2 + l^2} \cdot \frac{d}{\sqrt{d^2 + l^2}},$$

or, finally:

$$G\frac{Mmd}{(d^2 + l^2)^{\frac{3}{2}}}. \tag{3}$$

The effective force tending to rotate the balance, and resulting from the big ball B', will be the difference of (1) and (3). It will be $\frac{1}{2}f$, since, in our notation, f is the force with which *both* big balls tend to rotate the balance. By subtracting (3) from (1) we thus obtain the equation:

$$\tfrac{1}{2}f = GMm\left(\frac{1}{d^2} - \frac{d}{(d^2 + l^2)^{\frac{3}{2}}}\right). \tag{4}$$

For brevity, let us put:

$$D = \frac{1}{d^2} - \frac{d}{(d^2 + l^2)^{\frac{3}{2}}}. \tag{5}$$

Then we have:

$$\tfrac{1}{2}f = GMmD, \tag{6}$$

and, solving for M, we obtain:

$$M = \frac{1}{G} \cdot \frac{f}{2\,mD}. \tag{7}$$

The force f must be determined from observations of the torsion balance, when under the influence of the big lead balls. Transferring these big balls from the position A', B' to the position A'', B'' usually rotates the balance through a very small angle only. It is therefore necessary to measure this angle by very delicate means. For this purpose a small light mirror is attached to the center of the arm ab of the balance, and rotation is measured by allowing a strong beam of light to fall on this mirror, and to be

thence reflected upon a scale at some distance from the apparatus. The rotation of the balance is thus magnified, and can be measured without difficulty.

To introduce these measures into our formulas, let:

α = the total change in centimeters of the light on the scale brought about by changing the big balls from A', B' to A'', B''.

Q = the distance of the scale from the mirror.

To employ the units usual in measures of this kind, we must reduce the motion α to what it would have been on a scale at unit distance from the mirror. This would be $\frac{\alpha}{Q}\cdot$ We must also allow for the well-known fact that a moving reflected beam changes its direction twice as fast as the mirror turns. This reduces the motion on the scale at unit distance to $\frac{\alpha}{2Q}\cdot$ Finally, we must again divide by 2 to obtain the effect corresponding to the half motion bb', instead of the whole motion $b''b'$, since we are calculating the disturbance of the balance from a position of rest, and have measured its motion between two positions of extreme disturbance. This gives the observed motion on the scale, to be used in the further calculations, as:

$$\frac{\alpha}{4Q}. \tag{8}$$

Now it is a principle underlying the torsion or twisting of rods or fibers, a principle verified easily by experiment, that the force required to twist the rod or fiber through any angle is proportional to that angle. For instance, if a certain force would turn the torsion balance through an angle of 10°, it would require just twice as much force to turn it through 20°. It follows from this principle, and from the definition of the torsional constant T, that the force required to turn the balance through the angle (8) is:

$$\frac{\alpha}{4Q}T; \tag{9}$$

where readers familiar with Mechanics will note that T is really the "turning moment" for unit angle applied at unit distance from the center.

This expression (9) is not yet equal to the force f, because f is applied at the ends of the balance arms where the small balls are. The length of this balance arm being $\frac{l}{2}$, we see that (9) must be equal to $f\frac{l}{2}$; and so we may write the equation:

$$\frac{\alpha}{4Q}T = f\frac{l}{2}. \tag{10}$$

From this we have:

$$f\text{ (observed)} = \frac{1}{2}\frac{T\alpha}{Q\,l}. \tag{11}$$

We have already obtained in Note 12 (p. 375) an expression for T as follows:

$$T = \frac{\pi^2 m l^2}{2\,t^2}. \tag{12}$$

With the help of equations (11) and (12) we can compute the observed force f from our observations of α and Q, and the known dimensions, etc., of the parts of the balance.

Next we can establish easily an expression for the attractive force existing between either little ball and the earth. For this purpose, let

R = the radius of the earth, in centimeters,
E = mass of the earth, in grams.

Then we have:

$$\text{Attractive force between small ball and earth} = G\frac{Em}{R^2}. \tag{13}$$

Equation (13) follows directly from Newton's law, if we recall that the earth attracts bodies exterior to it precisely as if the entire mass of the earth were concentrated at its center. Thus the radius of the earth becomes the distance between the earth and the small ball, and its square appears in the denominator of equation (13).

Furthermore, according to the teaching of Physics, the attractive force existing between the small ball and the earth is also measured by the weight of the small ball, since weight is merely the result of such attractive force of the earth. And in physics, the weight

of any object is shown to be equal to its mass multiplied by the force of gravity, g. So we have:

Attractive force between small ball and earth $= mg$. (14)

Equating the right-hand members of equations (13) and (14) gives:

$$mg = G\frac{Em}{R^2}; \tag{15}$$

or:

$$E = \frac{R^2 g}{G}. \tag{16}$$

If we now divide equation (16) by equation (7), we obtain:

$$\frac{E}{M} = \frac{2}{f} \cdot mR^2 gD. \tag{17}$$

We now obtain the value of f from equation (11) by the help of equation (12). This gives:

$$f = \frac{\pi^2 ml\alpha}{4\,t^2 Q}. \tag{18}$$

Substituting from equation (18) in equation (17) gives, finally:

$$E = \frac{8\,t^2 R^2 gD}{\pi^2 l} \cdot \frac{Q}{\alpha} \cdot M. \tag{19}$$

Equation (19) enables us to calculate the mass of the earth, E, in terms of the mass of either big lead ball, M. It will be noted that the only quantities used in equation (19) and actually observed in the Cavendish experiment are α and Q. Most of the other quantities are ascertained by measurements and weighings before the torsion balance is put together. The time of vibration, t, is found in seconds by observing the combined duration of a considerable number of oscillations, made with the big lead balls entirely removed.

In the actual apparatus mounted for use in the astronomical lecture-room at Columbia University, New York, the following numerical data exist:

t = 281.5 seconds,
d = 5.3 centimeters,
l = 3.6 centimeters,
g = 981 centimeters,
π = 3.1416 centimeters,
M = 2750 grams,

and the radius of the earth is:

$$R = 6.371 \times 10^8 \text{ centimeters.}$$

With these numbers we obtain from equation (19):

$$E = 0.30 \times 10^{27} \cdot \frac{Q}{\alpha} \text{ grams.}$$

In an actual experiment the writer found:

$$Q = 189 \text{ centimeters,}$$
$$\alpha = 10.86 \text{ centimeters.}$$

Therefore:

$$Q = 17.4,$$

and:

$$E = 5.22 \times 10^{27} \text{ grams.}$$

The present accepted value of the earth's mass is:

$$6 \times 10^{27} \text{ grams;}$$

so that the result of the above lecture-room experiment is fairly satisfactory.

Note 14. Linear Distances from Angles Alone (p. 119).

The simple Figure 112 shows the correctness of the principle stated in the text. Suppose, for instance, that the three angles of a triangle are given, and it is required to draw the triangle. It is not possible to do so; because, with the given angles, we do not know whether we should make it of the size A, or the size B, or any other size. To know the triangle fully, we must know the length of at least one side. Angles alone enable us to draw a figure which is geometrically *similar* to the required figure, but they do not enable us to draw the figure itself to scale.

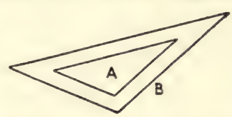


FIG. 112. Distance from Angles.

Note 15. Calendar Rule (p. 144).

To demonstrate this rule, we begin by assuming that our era commenced with a year numbered 0, so that 1913 was the 1914th year of the era. Of course there was not really an initial year 0, but we can imagine the calendar extended to that time. Then the principle on which our rule is based consists in calculating the number of days from January 1 of the year 0 to the date under in-

vestigation, and ascertaining how many weeks elapsed in the interval.

It happens that January 1 of the year 0 was a Sunday. Let us next compute the number of days between January 1 of the year 0 and March 1 of any year, such as 1913. We select March 1, because it is desirable for the moment to use a date that follows the possible extra day inserted as February 29 in leap-years.

Let us indicate the year number, such as 1913, by the letter y, and the century number, such as 19 in the year 1913, by the letter c. The total number of days from January 1 in the year 0, to March 1 of the same year, is 59, for the year 0 was not a leap-year. Consequently, if there were no leap-years, we should have:

No. of days from Jan. 1, year 0, to Mar. 1, year $y = 365\,y + 59$.

As each leap-year adds one day, we must increase this by the number of leap-years from the year 0 to the year y, and including the year y, if it be a leap-year. To find this number, let us divide c and y by 4, and call the remainders after the division r_1 and r_3. Then it is clear that under the Gregorian rule the number of leap-years will be:

$$\tfrac{1}{4}(y - r_3) - c + \tfrac{1}{4}(c - r_1).$$

Furthermore, this number will be a whole number, because we can prove easily that $y - r_3$ and $c - r_1$ are both divisible exactly by 4, without remainder.

The proof of this is as follows: If we divide any number whatever, N, by some other number D, and find from the division a quotient Q and a remainder R; then, if we divide $N - R$ by D, we shall again find the same quotient Q, but the remainder will now be 0. Thus, if we divide 1913 by 4, we find the quotient Q is 478 and the remainder R is 1. If we now subtract this remainder 1 from the original number 1913, we have for $N - R$, 1912. This being divided by the same divisor 4, gives the old quotient Q as 478, but the remainder is now 0. This shows that our expression for the number of leap-years is a whole number, as it should be.

We then have, by addition of the number of leap-years:

Total no. of days from Jan. 1, year 0, to Mar. 1, year y

$$= 365\,y + 59 + \tfrac{1}{4}(y - r_3) - c + \tfrac{1}{4}(c - r_1).$$

Now, if March 1 in the year y is a Sunday, like the first day of the era, the above number must be divisible exactly by 7. But if March 1 in the year y is Monday, one day later than Sunday, we can make the above number divisible by 7 if we subtract 1 from it; and 1 is the week-day number for Monday, *minus* 1. Similarly, for Wednesday, for which the week-day number is 4, we would subtract 3. In general, let us indicate by w the week-day number of March 1, whatever it may be in the year y, and subtract $w - 1$ from the above total number of days. This gives:

$$365\,y + 59 + \tfrac{1}{4}\,(y - r_3) - c + \tfrac{1}{4}\,(c - r_1) - (w - 1),$$

and this number is now always divisible exactly by 7.

Our real problem is to determine $(w - 1)$ from the fact that the number just obtained is thus divisible exactly by 7. In doing this we may evidently increase or diminish our number by any exact multiple of 7 without impairing its divisibility by 7 or affecting the value of w. We shall introduce two new remainders r_2 and r_4, by dividing the century number c, and the year number y, by 7, just as we have already divided them both by 4.

This having been done, we may correct our total number of days as follows, noting, of course, that each number added or subtracted is divisible exactly by 7. We shall add:

	$\tfrac{7}{4}(y - r_3) + \tfrac{7}{4}(c - r_1),$
subtract	$364\,y + 56,$
add	$7\,r_1 + 7\,r_3,$
subtract	$3\,(y - r_4) + (c - r_2),$

and so our total number becomes:

$$3 + r_2 + 5\,r_3 + 5\,r_1 + 3\,r_4 - (w - 1).$$

This number is now made up of remainders only. It will be a comparatively small number, as no remainder is larger than 6; and it is still divisible exactly by 7. It is therefore clear that $(w-1)$ is simply the remainder that will occur in the division by 7 of the number:

$$3 + 5\,r_1 + r_2 + 5\,r_3 + 3\,r_4,$$

and thus $(w - 1)$ is determined for March 1 in the year y.

But we need to find $(w - 1)$ for any day in the year y, not merely for March 1. To accomplish this for any other day in March, say the 3d, for instance, we have merely to add 2 to the above number, before dividing by 7, because March 3 comes two days later than March 1. In general, if we indicate by d the date in March for which the week-day is required, we must add $(d - 1)$ to the above number. This gives, for March d^{th}:

$$3 + 5\,r_1 + r_2 + 5\,r_3 + 3\,r_4 + (d - 1),$$

or:

$$2 + 5\,r_1 + r_2 + 5\,r_3 + 3\,r_4 + d;$$

and this number being divided by 7 will give the $(w - 1)$ of March d for a remainder.

The same expression will hold for April, if we add 31, because there are 31 days in March. Adding 31, and deducting 28, an exact multiple of 7, gives for April:

$$5 + 5\,r_1 + r_2 + 5\,r_3 + 3\,r_4 + d.$$

A similar expression holds for each month, a difference occurring only in the number at the beginning of the expression. If we indicate that month-number by m, we may write for any month:

$$m + 5\,r_1 + r_2 + 5\,r_3 + 3\,r_4 + d.$$

The values of m for the various months may then be written in a little table (see Rule, p. 144). In forming this table it is necessary to remember that there will be a slight difference between the m's for leap-years and ordinary years. We have started with the formula for March 1, in order to avoid this difference as much as possible. After that date in the year there is no difference. But in January and February the leap-year m's are smaller by 1 than those for ordinary years, on account of the interpolated February 29.

The entire rule may be arranged in the accompanying tabular form. That part of the formula which does not vary in a whole century, namely, $5\,r_1 + r_2$, we have designated by K. In the Julian calendar K is evidently always 0, because there is no century exception in the leap-year rule of that calendar. For the sake of symmetry, we have here indicated the final remainder $(w - 1)$ by r_5.

APPENDIX

CALCULATION OF WEEK-DAY, GREGORIAN OR JULIAN CALENDAR

FORMULA (d=Day of the Month)			TABLE OF m			WEEK-DAY NOS.	
Divide	by	and call the remainder		Ord'y Year	Leap Year		
Century No.	4	r_1	Jan.	6	5	1	Sunday
Century No.	7	r_2	Feb.	2	1	2	Monday
Year No.	4	r_3	March	2	2	3	Tuesday
Year No.	7	r_4	April	5	5	4	Wednesday
$\left.\begin{array}{r}5r_3+3r_4+K\\+m+d\end{array}\right\}$	7	r_5	May	0	0	5	Thursday
			June	3	3	6	Friday
			July	5	5	7	Saturday
			Aug.	1	1		
			Sept.	4	4		
			Oct.	6	6		
			Nov.	2	2		
			Dec.	4	4		

where $K = 5\,r_1 + r_2$, Gregorian;

$K = 0$, Julian.

(Gregorian $K = 20$, from 1900 to 1999.)

Week-day No. $= r_5 + 1$.

Note 16. Gauss' Rule for Easter (p. 148).

To demonstrate the rule, we shall consider the Julian calendar first, and then modify our results to accord with the present Gregorian calendar.

The lunar month of chronology, or the period of the moon's orbital revolution around the earth, is approximately $29\frac{1}{2}$ days long. In making the ecclesiastical calendar it was therefore decided to have lunar months of 29 and 30 days occur alternately as a general rule. But for a reason to be explained in a moment, an extra lunar month of 30 days is inserted at the end of every third year for six successive periods of three years each, or eighteen years in all. Then, one year later, at the end of the nineteenth year, an additional extra lunar month of 29 days is further inserted in the calendar.

The lunar calendar for nineteen years then stands as follows:

3 years (36 months) alternating 29 and 30 days, total . .	1062 days
Extra months of 30 days	30 days
The above repeated five times more (1092 × 5)	5460 days
The 19th year of 12 months alternating 29 and 30 days . .	354 days
The final extra month of 29 days	29 days
Total	6935 days

The above calculation takes no account of leap-years, which occur every fourth year in the Julian calendar. To get these leap-years into the lunar calendar, too, the ecclesiastical chronologists adopted the simple plan of putting an extra day into the lunar month of February, whenever it is put into the civil month of February. In 19 years this will happen five times when any one of the first three years of the 19 is a leap-year; but if the fourth year of the 19 is a leap-year, it will happen four times only. Thus, on the average:

19 years will have $6935 + 5$, or 6940 days three times, and
19 years will have $6935 + 4$, or 6939 days once.

The mean of these figures is $6939\frac{3}{4}$ days; and this is the average number of days in 19 lunar years, according to accepted chronologic rules.

Now the length of a Julian tropical or calendar year is $365\frac{1}{4}$ days. Consequently, 19 Julian years will contain $365\frac{1}{4} \times 19$, or $6939\frac{3}{4}$, days, agreeing exactly with the lunar figures just found. This agreement is evidently not accidental, but is the result of the above conventional and arbitrary rules for the extra lunar months.

One important thing follows from this agreement: if we write the calendar dates of full-moon for a period of 19 years, these calendar dates will then be repeated in the next and in every subsequent period of 19 years. Now it so happens that the year 0, or the year next preceding the year 1 of our era, was the first year of a 19-year cycle. Consequently, the year 1 was the second of the 19-year cycle, the year 2 the third, and the year 19 the first of the next cycle. It is clear that, in general, if we divide the year number y by 19, and call the remainder r_6, then $r_6 + 1$ will be the position of the year y in a 19-year cycle.

The next step is to find for any year the date of the Easter full-moon, which, according to the Nicene council's decree, is the first to fall on March 21 or thereafter. Let us call the date of this full-moon March 21 + P, and suppose dates in March to be carried over into April, so that April 1 will be called March 32. Now it so happens that in the year preceding the year 1, the Easter full-moon, Julian calendar, fell on March 36, so that P was then 15. As there are 354 ($12 \times 29\frac{1}{2}$) days in a lunar twelve-month, it is clear that in the year 1 Easter full-moon must have occurred 11 (which is 365–354) days earlier. And in each succeeding year of the 19-year period, Easter full-moon must have occurred either 11 days earlier than in the preceding year, or 19 (which is 30–11) days later. Of course the occasions when it occurs 19 days later are accounted for by the extra 30-day months inserted every three years. The following table exhibits the above state of affairs:

YEAR	r_6	P	
0	0	15	
1	1	4	11 days earlier than year 0
2	2	23	19 days later than year 1
3	3	12	11 days earlier than year 2
4	4	1	11 days earlier than year 3
5	5	20	19 days later than year 4,
etc.			etc.

It is clear that we shall have, in general, if we let v and x be two unknown whole numbers:

$$P = 15 + 19\,x - 11\,v,$$

or:

$$P = 15 + 19\,(x + v) - 30\,v.$$

It is further clear that in this equation:

$$x + v = r_6,$$

because, to get P in the table above, we have always added 19 r_6 and then subtracted the largest possible value of 30 v. So we may write:

$$P = 19\,r_6 + 15 - 30\,v.$$

From this it appears that P is simply the remainder occurring in the division of $19\,r_6 + 15$ by 30. If we call this remainder r_7 we can therefore find the date of Easter full-moon in the Julian calendar thus:

Divide	by	and call the remainder	
Year No., y,	19	r_6	And the date of Easter full-moon,
$19\,r_6 + 15$	30	r_7	Julian calendar, is March $21 + r_7$.

The above method of calculation not only applies to the first period of 19 years, but is entirely general; because, as we have seen, subsequent 19-year periods simply repeat the same dates of full-moon.

We must now pass to the Gregorian calendar. It is evident that the two calendars are in accord at the beginning of the era, and do not diverge until the year 100, when the Gregorian calendar omits a Julian leap-year. This will of course change P by one day, and this same difference of one day will continue from the year 100 to the year 199. From 200 to 299 there will be a difference of two days, etc.

It is clear that we can allow for this cause of difference between the two calendars by varying the number 15 that occurs in the quantity $19\,r_6 + 15$. Let us call this variable number M. Then, in both calendars, M is 15 from the beginning of the era to the year 99. In the Gregorian calendar M increases by 1 each century thereafter, except that for every fourth century this increase is omitted because of the Gregorian leap-year exception. Using our former notation, in which c is the century number, we have for the Gregorian calendar:

$$M = 15 + c - \tfrac{1}{4}(c - r_1).$$

But this value of M, thus corrected for the Gregorian leap-year, is not yet quite right. A further last correction is still necessary on account of a slight inaccuracy in the lunar period of 19 years. A lunar month is not exactly $29\frac{1}{2}$ days long; its true length is 29.530586 days. So the 235 lunar months of a 19-year period

really amount to 235×29.530588 days; or 6939 days, 16 hours, 31 minutes, and not $6939\frac{3}{4}$ days, as already obtained.

The error of $1^h\ 29^m$ amounts to a day in 308 years. But the framers of the ecclesiastical calendar assumed this error to reach one day in $312\frac{1}{2}$ years, or 8 days in 2500 years. So they directed that a correction be made, such that M be diminished by 1 seven times successively at the ends of 300-year periods, and an eighth time at the end of a 400-year period, or 8 times in 2500 years. The last period of 2500 years terminated at the end of the year 1799, and the correction was then 5; new corrections are therefore required in 2100, 2400, 2700, 3000, 3300, 3600, 3900, all at intervals of 300 years. But the next following correction does not come until 4300 instead of 4200, on account of the eighth period being one of 400 years. This condition will be satisfied for all time if we divide $8\,c + 13$ by 25, call the remainder r_8, and subtract from M the correction:

$$\frac{8\,c + 13 - r_8}{25}.$$

This may be verified readily by drawing up a table of this correction, when it will be found to have the value 5 for $y = 1799$, and to increase thereafter forever in the proper way. We have, then, finally, for the Gregorian calendar:

$$M = 15 + c - \tfrac{1}{4}\,(c - r_1) - \tfrac{1}{25}\,(8\,c + 13 - r_8);$$

and when M is greater than 30, we may subtract from it, if we choose, the largest possible exact multiple of 30. And in the Gregorian calendar the date of Easter full-moon is now March $21 + r_7$, where r_7 is the remainder resulting from the division by 30 of the number $19\,r_6 + M$.

Having thus found a method of calculating the Gregorian date, March $21 + r_7$, of the Easter full-moon, we must now find the date of the Sunday next following, which will be Easter Sunday. We need therefore only calculate the week-day of the date March $21 + r_7$, to know the date of Easter. Referring to our former civil calendar formulas, we shall find the remainder r_5 for the Easter full-moon date, March $21 + r_7$, which remainder we shall call r_9 for this special case, if we divide by 7 the quantity:

$$5\,r_3 + 3\,r_4 + K + 2 + r_7.$$

Now if r_9 comes out 0, the Easter full-moon comes on Sunday, and Easter is 7 days later, according to the Nicene decree. If r_9 is 1, the full-moon day is Monday, and Easter is 6 days later. In general, we obtain the date of Easter Sunday by adding to March 21 + r_7 the number:

$$7 - r_9.$$

Collecting all our formulas, we can now find the date of Easter Sunday as follows; and thus the rule given on p. 148 is demonstrated.

Divide	by	and call remainder	
Century No., c	4	r_1	$K = 5\,r_1 + r_2$, Gregorian calendar.
Century No., c	7	r_2	$K = 0$, Julian calendar.
Year No., y	4	r_3	$M = 15 + c - \frac{1}{4}(c - r_1)$ $- \frac{1}{25}(8\,c + 13 - r_8)$, Greg. calen-
Year No., y	7	r_4	dar. $M = 15$, Julian calendar.
$8\,c + 13$	25	r_8	Easter Sunday is then March
Year No., y	19	r_6	$28 + r_7 - r_9$;
$19\,r_6 + M$	30	r_7	or April $r_7 - r_9 - 3$.
$\left.\begin{matrix}5\,r_3 + 3\,r_4 + K \\ + 2 + r_7\end{matrix}\right\}$	7	r_9	The following are values of K and M for the Gregorian calendar: 1800–1899, $K = 14$, $M = 23$, 1900–1999, $K = 20$, $M = 24$.

As an example, let us calculate the date of Easter Sunday for 1913. We have:

$$r_3 = 1,\ r_4 = 2,\ K = 20,\ M = 24,\ r_6 = 13,\ r_7 = 1,\ r_9 = 6;$$

Easter Sunday is on March 28 + 1 − 6 = March 23.

We must now explain the two exceptions that occur in the Gregorian calendar only (p. 149). The first of these happens when $r_7 = 29$. The formulas have been deduced on the supposition that the March and April full-moons occur at an interval of 30 days. But that interval may be 29 days only. The framers of the calendar have assumed, rather arbitrarily, that if there is a full-moon on March 19, or earlier in March, the April full-moon

will occur 30 days later. But if the March full-moon is on the 20th, or later, the April full-moon will happen 29 days later. Thus the ecclesiastical April full-moon will happen on the same day, no matter whether the March full-moon comes on the 19th or 20th.

As this cannot occur in reality, the framers of the calendar have directed that when the March full-moon happens on the 20th, which occurs whenever $r_7 = 29$, then r_7 shall be diminished arbitrarily by 1. That is, we must use 28 instead of 29, or move the April moon back one day. But the diminution of r_7 by 1 will ordinarily also diminish r_9 by 1. Consequently, $r_7 - r_9$ will remain unchanged, and so will the date of Easter Sunday, which depends on $r_7 - r_9$. Only when $r_7 = 29$ and $r_9 = 0$ will the change of r_7 from 29 to 28 have any effect. For when $r_9 = 0$, a diminution by 1 will change it into 6, and $r_7 - r_9$ will be diminished by 7, making Easter exactly one week earlier. But when $r_7 = 29$ and $r_9 = 0$, the rule always makes Easter come on April 26. Therefore the exception is as stated: whenever Easter comes by the rule on April 26, use April 19 instead. There will be an example of this in 1981.

Unfortunately, the above exceptional case introduces another complication. The change of r_7 from 29 to 28 does not make it impossible for the value $r_7 = 28$ to occur again under the general rule, and during the same 19-year period. This might make two full-moons occur on the same date twice in a single 19-year period, which is, in fact, impossible. To avoid this, the framers of the calendar have ruled, again arbitrarily, that there shall be a second exception. Under this exception, 28 is changed to 27, whenever, in the same 19-year period, the first exception occurs.

We must therefore investigate when the first exception can occur. In determining r_7 we performed a division by 30. Let us indicate the quotient of this division by v. Then we have, if r_7 is 29, according to the first exception:

$$19\,r_6 + M = 30\,v + 29.$$

Now multiply this equation by 11, and add 11 to each member. This gives:

$$209\,r_6 + 11\,M + 11 = 330\,v + 319 + 11 = 330\,v + 330.$$

The right hand member is now divisible exactly by 30; therefore the left-hand member is also so divisible. But the division of $209\,r_6$ by 30 will leave a remainder of $29\,r_6$. To make this disappear, the remainder in the division of $11\,M + 11$ by 30 must be r_6. But r_6 is always less than 19 by its definition. Therefore the first exception will occur whenever, in the division of $11\,M + 11$ by 30, the remainder is less than 19.

But again, as in the case of the first exception, the change of r_7 from 28 to 27 will make no difference in the date of Easter, unless $r_9 = 0$. When $r_7 = 28$ and $r_9 = 0$, Easter, according to the rule, comes on April 25. The change of r_7 moves this date to April 18. Therefore the second exception reads: whenever, in the division of $11\,M + 11$ by 30, the remainder is less than 19, and if $r_7 = 28$ and $r_9 = 0$, Easter Sunday is on April 18, instead of April 25, as given by the rule. An example of this will occur in 1954.

To complete this subject it is necessary to remark that r_7 can never be 29, 28, and 27 within a single period of 19 years. Therefore no further exception is necessary on account of the possibility that the above two exceptions might, by acting together, produce two cases of $r_7 = 27$ in a single 19-year period.

Note 17. The Sextant (p. 154).

To prove the fundamental principles of the sextant, that the angle between the mirrors is half the altitude of the sun, imagine the plane of the paper to be the plane of the circle of the sextant. Then, in Fig. 113, the plane of the circle is supposed to be held vertically, in such a way that it will pass through the sun at S. The navigator sees the horizon with the upper part of the telescope through the unsilvered part of the mirror m; and he sees the sun along the line $TmMS$ after reflection from both mirrors. The angle MTm is the altitude of the sun above the horizon; the angle at P is the angle between the mirrors. It is necessary to prove that:

$$\text{Angle } MTm = 2 \text{ angle } P.$$

The lines MP' and mP' are drawn perpendicular to the mirrors M and m. Then, according to the optical principles governing

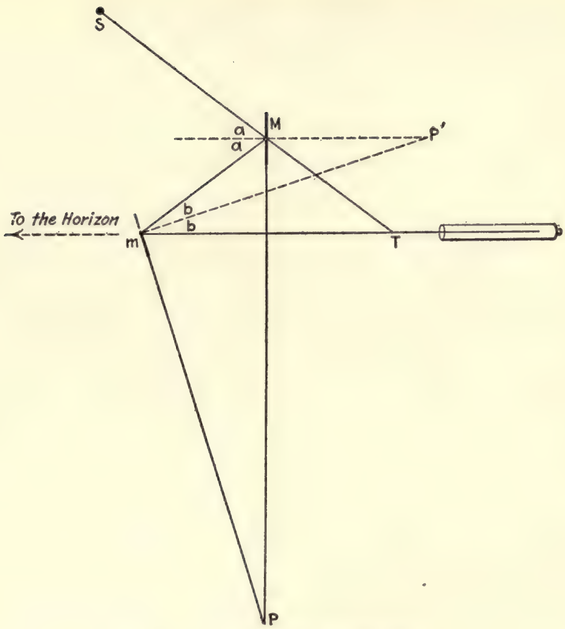


FIG. 113. Theory of Sextant.

the reflection of light from plane mirrors, the two angles a are equal and so are the two angles b. Furthermore, the angle SMm, or $2a$, is an exterior angle to the triangle mMT. Consequently:

$$\text{Angle } 2a = \text{angle } 2b + \text{angle } MTm,$$

or:

$$\text{Angle } MTm = 2\,(\text{angle } a - \text{angle } b).$$

Similarly, from the triangle mMP':

$$\text{Angle } P' = \text{angle } a - \text{angle } b.$$

But angle P' = angle P, because their sides are perpendicular, each to each. Therefore,

$$\text{Angle } P = \text{angle } a - \text{angle } b.$$

And it follows that:

$$\text{Angle } MTm = 2 \text{ angle } P.$$

Q. E. D.

Note 18. Longitude Determination (p. 158).

A reference to Fig. 106 (p. 364) will show how the longitude may be computed from the sun's altitude, measured with the sextant. In the figure, if S' is the sun, the arc AS' is the measured altitude. If this be subtracted from 90°, we have the arc ZS', or angular zenith distance of the sun. The arc $S'D$ is the sun's declination, and may be ascertained for the date of the observation from the nautical almanac. Subtracting this declination from 90° makes known the arc PS', or the angular polar distance of the sun.

The ship's latitude is also supposed to be known; without it, the longitude cannot be computed. But the ship's latitude always is known, because the navigator will have determined it at noon, and can easily allow for any slight change in the ship's latitude since the last noon observation, since he knows the compass course he is steering, and the approximate speed of the ship.

But the latitude is the arc PN in the figure, or the altitude of the celestial pole above the horizon. This latitude being subtracted from 90°, gives the arc ZP, or the angular distance from the celestial pole to the zenith. Thus these three subtractions from 90° make known the three sides of the spherical triangle ZPS'.

It is a principle of trigonometry that any spherical triangle can be solved completely, and all its parts made known, if we know its three sides. Thus we find the spherical angle $S'PZ$, of which the vertex is at the pole, and which is measured by the arc DM on the celestial equator. But DM is by definition the hour-angle of the sun S'; and the sun's hour-angle is the local apparent solar time. This need merely be corrected by applying the equation of time (p. 134) to obtain the local mean solar time of the ship, ready for comparison with the Greenwich time taken from the face of the chronometer by an assistant at the instant when the sun was observed for longitude by the navigator.

Note 19. Moon's Distance (p. 169).

Figure 114 shows how the moon's distance is determined. We shall assume, as a sufficiently close first approximation, and to make the problem easy to understand, that the two observatories are situated on the same meridian of terrestrial longitude, but very widely separated in latitude. One should be in the northern

Also for stars

hemisphere; the other in the southern. The observatories of Greenwich, England, and the Cape of Good Hope, for instance, satisfy these conditions quite closely.

In Fig. 114, then, O and O' are the two observatories, the circle representing the earth. The arc OO' is known, for it is simply the latitude difference of the two observatories. The angle OCO' is equal to the arc OO'; and the lines CO and CO' are each known radii of the earth. Therefore, by simple trigonometry, we can solve the triangle OCO', and gain a knowledge of the distance OO', which is to be our base-line, and of the two angles COO' and $CO'O$.

Fig. 114. Moon's Distance.

We next measure at both observatories simultaneously, with suitable astronomic instruments, the exact lunar altitude, or angular elevation of the moon above the horizon, at the instant when the diurnal rotation has brought the moon to the celestial meridian. These simultaneous observations will be possible, because the moon will reach the meridian of both places at the same instant, since we have imagined our two observatories lying on the same meridian of terrestrial longitude, and therefore having the same celestial meridian over them in the sky.

Having measured the moon's altitude above the horizon, we can at once find its angular distance from the zenith. For the latter point is always 90° distant from the horizon; so that we obtain the angular zenith distance of the moon by simply subtracting its measured altitude from 90°.

Those two angular zenith distances, thus known from the measured altitudes, are the angles MOZ and $MO'Z'$ in Fig. 114. Next we subtract these angles from 180°, giving us the angles MOC

and $MO'C$. From these we again subtract the angles COO' and $CO'O$, found above, thus obtaining values of MOO' and $MO'O$. These now make possible a trigonometric solution of the triangle MOO', of which we now know the base OO' and the two adjoining angles. Thus we get OM and $O'M$ in miles. After that we can solve the two triangles COM and $CO'M$, since we know the length of the two sides CO and OM, as well as the included angle COM; and in the other triangle we know CO' and $O'M$ as well as the included angle $CO'M$. A solution of either triangle gives us CM, the distance from the center of the earth to the moon.

It is scarcely necessary to add that the ideal condition here assumed as to location of observatories does not exist in fact. But a slight divergence from this condition in no way impairs the principle of the method; it merely adds a certain additional complexity to the trigonometrical calculations.

Note 20. Lunar Parallax (p. 169).

Figure 42 shows that the moon's parallax and distance are connected by a very simple trigonometric formula:

$$\sin \text{parallax} = \frac{AC}{AM} = \frac{\text{radius of earth}}{\text{distance of moon}}.$$

This formula shows that we can calculate the parallax if we know the distance, or the distance if we know the parallax. The two are closely related; astronomers frequently speak of measuring the parallax of the moon or other heavenly body, when they merely mean a measurement of its distance.

Note 21. The Moon's Mass (p. 175).

Figure 115 is intended to make this matter clear. S is the sun; the circle is the annual terrestrial orbit. When the center of gravity is at C_1, the earth at E_1, and the moon at M_1, the sun will appear from the earth projected in the direction S_1. This is exactly the same as would be the case if there were no moon, for then the earth would itself be at C_1. But when the center of gravity is at C_2, the earth will be at E_2; and the sun will be seen projected in the direction S_2', instead of S_2, which is its direction as seen from C_2, and which would be its direction from the earth if there were no moon.

Thus the sun will be seen a certain angular distance in advance of its proper position; and a half-month later it will be similarly retarded. The total range is 12″, so that the angle S_2SS_2' is 6″. Therefore, in the triangle C_2SE_2, we know the angle C_2SE_2 to be 6″; and we know the two sides C_2S and E_2S, the distance from the earth to the sun, which can be measured. Solving the triangle, we find the side C_2E_2 to be about 2880 miles. We then form the proportion:

$E_2C_2 : M_2C_2 =$ moon's mass : Earth's mass;

from which we can compute the lunar mass, since the other quantities in the proportion are now all known.

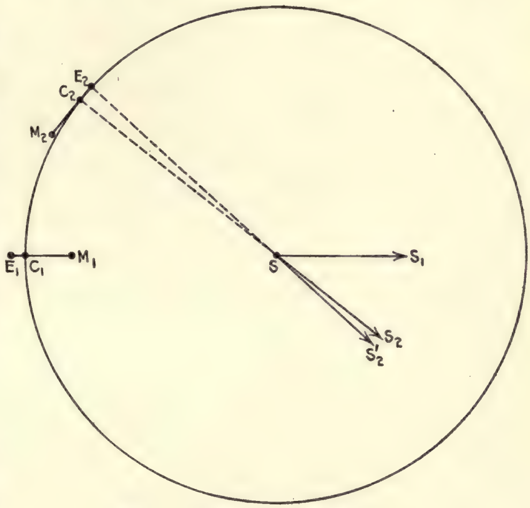


FIG. 115. Mass of the Moon.

Note 22. Concavity of Moon's True Orbit with Respect to the Sun (p. 181).

We can test this question by means of Fig. 116. It is evident from a mere glance at Fig. 46 (p. 181) that there is no doubt as to the concavity of the moon's orbit toward the sun at the time of full-moon, shown at M_3. Difficulty arises only in the case of the new-moon phase, shown at M_1 and M_5. Therefore, in Fig. 116, we shall examine especially the new-moon phase. Let E_1, M_1, and S be positions of the earth, moon, and sun at the time of new-moon. Let E_1E_2 be a portion of the earth's orbit around the sun; and let the small circles represent the lunar orbit around the earth. While the earth moves from E_1 to E_2, we may suppose the moon to move around the earth from C to M_2. In other words, if the moon did

not revolve around the earth, it would be at C when the earth reached E_2. Designate the angle E_1SE_2 by the letter θ, and let r_e and r_m represent radii of the earth's orbit around the sun and the moon's orbit around the earth. Finally, let E_1T be a tangent to the earth's orbit at E_1; draw E_2P perpendicular to E_1S; E_2B parallel to E_1S; and M_2A parallel to E_1T. If we let θ be a small angle, M_1M_2 will be a small part of the moon's path near new-moon: it will evidently be concave towards the sun if M_2 is farther from the tangent E_1T than is M_1.

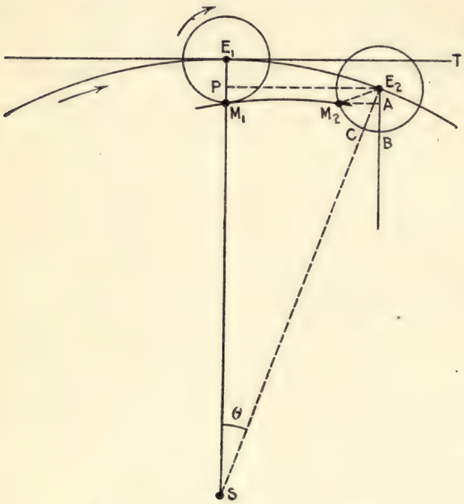


FIG. 116. Moon's True Orbit.

While the moon was moving from M_1 to M_2 the entire lunar orbit fell away from the tangent the distance E_1P; but the moon rose toward the tangent a distance nearly equal to AB. Therefore the moon recedes from the tangent a total distance of $E_1P - AB$. Now we have, evidently:

$$E_1P = r_e - r_e \cos \theta, \tag{1}$$

$$AB = r_m - r_m \cos M_2E_2A. \tag{2}$$

But:

$$M_2E_2A = M_2E_2C + CE_2A.$$

Also:

$M_2E_2C = 13\,\theta$, because the moon's angular motion in its orbit is about 13 times as fast as the earth's (p. 161); and:

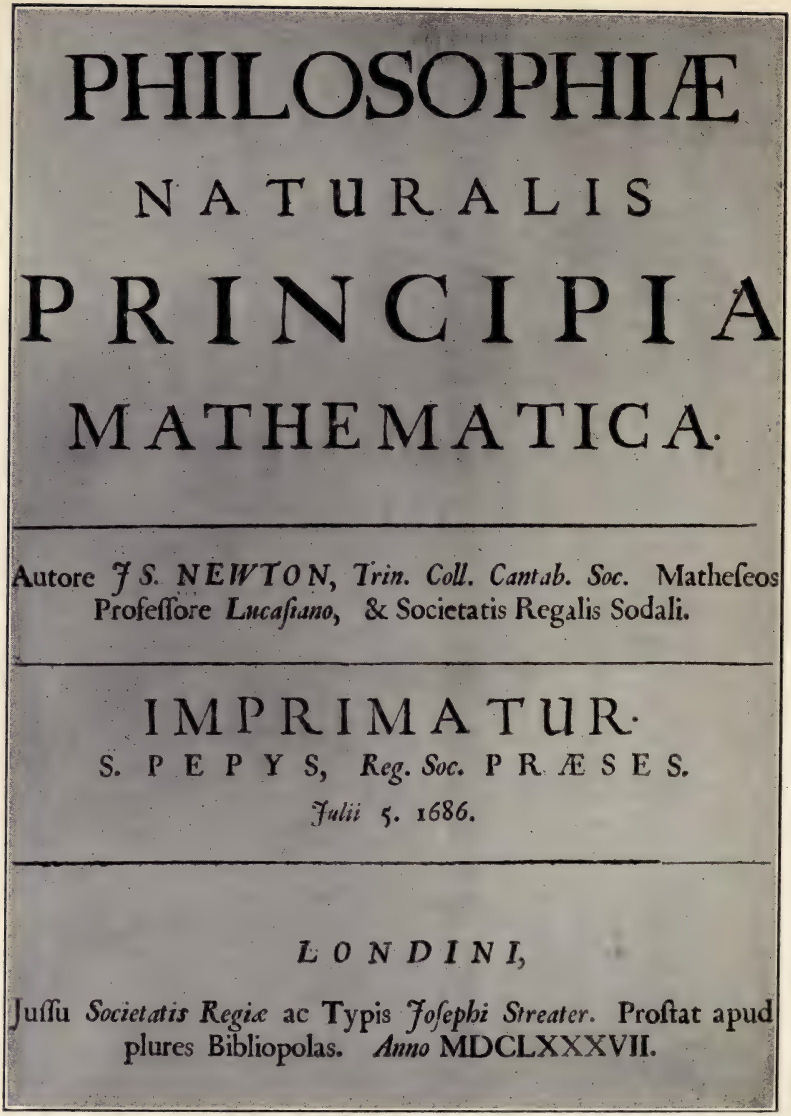
$$CE_2A = \theta, \text{ because } AB \text{ is parallel to } E_1S.$$

It follows that:

$$M_2E_2A = 14\,\theta;$$

and, from equation (2):

$$AB = r_m - r_m \cos 14\,\theta. \tag{3}$$



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

PLATE 32. Title-page of Newton's Principia.

A simple calculation, using the value $\theta = 1°$, $r_m = 240{,}000$, $r_2 = 93{,}000{,}000$, gives:

$$E_1P = 16{,}000 \text{ miles},$$
$$AB = 7130 \text{ miles}.$$

It follows that the moon recedes from the tangent about 8870 miles in one day, while the earth is moving about 1° in its orbit around the sun. This proves that the moon's true orbit is concave towards the sun, even at the time of new-moon.

Note 23. Law of Areas (p. 186).

Figure 117 shows how the point P_3' is found. Draw P_3P_3' parallel and equal to P_2P_2'. Then the actual motion of the planet in the second second will take place along the diagonal P_2P_3' of the parallelogram $P_2P_3P_3'P_2'$. This theorem of the "parallelogram of forces" is demonstrated in works on elementary physics:[1] perhaps the easiest way to understand it is to notice that P_3' is point to which P_2 must go, if it actually completes separately the two motions P_2P_2', and $P_2'P_3'$ equal and parallel to P_2P_3.

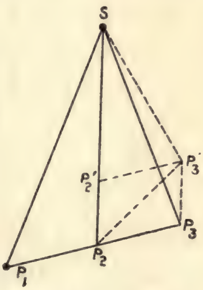


FIG. 117. Law of Areas.

Note 24. Law of Areas (p. 186).[2]

We have still to prove the triangles SP_1P_2 and SP_2P_3' equal in area. Referring again to Fig. 117, we see that the triangles SP_2P_3' and SP_2P_3 are equal, since they have the same base SP_2, and their altitudes are equal because their vertices P_3 and P_3' lie on the line P_3P_3', which is parallel to P_2S. And we have already found the triangle SP_2P_3 equal to SP_1P_2. Therefore the triangle SP_2P_3' is also equal to SP_1P_2.

[1] Figure 117 may be found in the first edition of Newton's immortal *Principia*, of which the title-page is reproduced as Plate 32. The president of the Royal Society, whose name appears on the title-page as having authorized the printing, is the famous diarist. On p. 13 of the *Principia* appears Corol. I: "Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore describere, quo latera separatis."

[2] On p. 37 of the same work of Newton appears Prop. I, Theorem I: "Areas quas corpora in gyros acta radiis ad immobile contrum virium ductis describunt . . . esse temporibus proportionales."

Note 25. Harmonic Law (p. 188).

It would carry us too far afield in mathematical astronomy to give here the demonstrations by which Kepler's three laws may be derived from Newton's single law; but there is little difficulty in considering by elementary methods the special case of a circular planetary orbit. The circle is, in fact, a close approximation to the actual planetary orbits in the solar system: none of these orbits are very much flattened from the circular form.

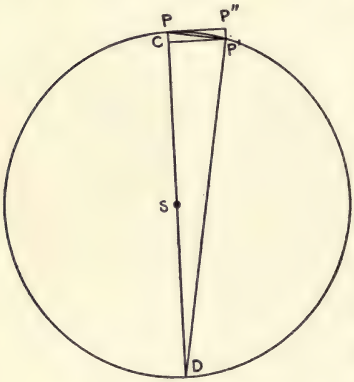


Fig. 118. Solar Attraction.

We must first investigate the nature of the solar attractive force. In the case of a circular orbit this force is necessarily constant under Newton's law, because the planet is always at the same distance from the sun. Now consider the accompanying Fig. 118. Let PP' be a very short arc of a circle, whose center is at S. Draw the diameter PD and the chord $P'D$; and let fall the perpendicular $P'C$ upon PD. Draw the chord PP', the tangent PP''; and let fall the perpendicular $P'P''$ upon PP'' from P'. Then, from the similar right-angled triangles $PP'C$ and $PP'D$, we have:

$$PP' : PC = PD : PP',$$

or:

$$PC = \frac{\overline{PP'}^2}{PD}.$$

Now let our circle be a planetary orbit, with the planet at P, the sun at S; and suppose that in one second of time the planet would move along the orbit to P'. We may consider this very short arc PP' coincident with its chord PP'.

From the principle of the parallelogram of forces (p. 399), the actual motion PP' may be regarded as the resultant of two motions: PP'', which would be the planet's actual motion from P in a second if the solar attraction were to cease suddenly; and

PC, which would be the planet's actual motion in a second if attraction toward the sun operated alone.

Now PP' is the planet's velocity in its orbit per second, which we shall call V; and PD is twice the radius of the orbit, which latter we shall call r. Let us also designate the distance PC by the letter x, and consider all distances to be measured in miles. Then, from the geometry of the figure, as we have just seen:

$$x = \frac{V^2}{2r}. \tag{1}$$

But as we have said, PC or x is the distance the planet would move or "fall" toward the sun in a second, if the solar attraction acted alone, without any additional orbital or tangential impulse derived, perhaps, from the original catastrophe by which the planet was brought into existence. The question now is: How great must be the solar attractive force to cause a planet to fall from a position of rest at P through the distance PC or x in a second?

This raises the question of how forces are measured. What is a suitable unit of force? Now the solar attraction is not applied suddenly as a single impulse; it is applied continuously. Consequently, the planet would fall the short distance x toward the sun with a uniformly increasing velocity, faster and faster, but beginning with zero velocity at P. Its average velocity would be attained halfway between P and C. But the actual distance it would move in a second is of course the same as if it traveled constantly with its average velocity. And as it would fall a distance x miles in a second, its average velocity must be x miles per second. Therefore it would be moving with the velocity x miles per second when halfway between P and C; and upon reaching C its velocity would have increased to $2x$ miles per second. But in astronomy, as in mechanics, our units are so chosen that force is always measured by the quantity of velocity accumulated in a second, multiplied by the quantity of mass in the moving body. The velocity thus accumulated in a second is called "acceleration"; and as the velocity of the falling planet increased from zero to $2x$ miles per second, the acceleration produced by the solar attractive

force must be represented by the number $2x$. Calling this acceleration f, we thus have:

$$f = 2x;$$

and this, combined with equation (1), gives:

$$f = \frac{V^2}{r}. \tag{2}$$

Now the whole circumference of the circular orbit is $2\pi r$; and the planetary orbital velocity V is of course equal to the circumference divided by the period of orbital revolution. It follows that if we call this period t, expressed in seconds of time, we shall have:

$$V = \frac{2\pi r}{t};$$

and, therefore, from equation (2):

$$f = \frac{4\pi^2 r}{t^2}. \tag{3}$$

If we now apply equation (3) to two separate planets, indicating by subscript numbers quantities belonging to the first and second of the two planets, we shall have:

$$f_1 = 4\pi^2 \frac{r_1}{t_1^2}, \qquad f_2 = 4\pi^2 \frac{r_2}{t_2^2};$$

or:

$$\frac{f_1}{f_2} = \frac{r_1}{r_2} \cdot \frac{t_2^2}{t_1^2}. \tag{4}$$

But we know from Newton's law that the attractive forces exerted by the sun on two different planets, if of equal mass, will be inversely proportional to the squares of the distances separating those planets from the sun. This may be written thus:

$$f_1 : f_2 = r_2^2 : r_1^2;$$

or:

$$\frac{f_1}{f_2} = \frac{r_2^2}{r_1^2}. \tag{5}$$

Equating the right-hand members of equations (4) and (5) gives:

$$t_1^2 : t_2^2 = r_1^3 : r_2^3.$$

This is the third (or harmonic) law of Kepler, which is therefore thus demonstrated as a consequence of Newton's law in the case of circular orbits. A similar proof is possible, by the aid of the higher mathematics, without this assumption as to the form of the orbit; but a small correction is always required, because we have taken the planets to have equal masses.

Still retaining our circular orbit formulas as a sufficient first approximation, we are now in a position to understand Newton's famous test as to whether the force of gravitation observable on the earth also extends outward as far as the moon. We shall present this test here in a somewhat modernized form, based on the formulas just obtained. Resuming our equation (3), we have the acceleration exerted by the earth upon the moon:

$$f = 4\,\pi^2 \frac{r}{t^2}, \tag{6}$$

in which r is now the distance from the earth to the moon, and t the moon's sidereal period (p. 161). This equation is correct, if the Newtonian law of gravitation extends to the moon, and not otherwise.

Newton's test now consists in comparing the value of f calculated by means of equation (6) with its value easily obtained by another method. It was known from laboratory experiments, even in the time of Newton, that the earth attracts an object situated on its surface with a force which is called the "force of gravity," and which produces an acceleration designated by the letter g in physics. It is also known that the earth's attraction upon any object exterior to it acts as if the entire mass of the earth were concentrated at its center.[1]

Now the distance from the earth's center to an object on its surface is equal to the earth's radius, and may be designated by R; while the distance from the earth's center to the moon is r. It follows that if the earth's attraction varies inversely as the square of its distance from the object attracted, as postulated by Newton, we may write the following simple proportion involving

[1] This was demonstrated by Newton.

f, due to the earth's attraction upon the moon, and g; due to the earth's attraction upon surface objects:

$$f : g = \frac{1}{r^2} : \frac{1}{R^2};$$

from which we have at once:

$$f = g\frac{R^2}{r^2}. \qquad (7)$$

The values of f in equations (6) and (7) must be equal, if both g and f result from the same identical law of Newtonian gravitation. Equating these quantities gives:

$$g\frac{R^2}{r^2} = 4\,\pi^2\frac{r}{t^2};$$

or:

$$g = 4\,\pi^2\frac{r^3}{t^2R^2}. \qquad (8)$$

In this equation, r is the moon's distance from the earth, which we here suppose expressed in feet; and t is the moon's sidereal period, in seconds of time. Let us then calculate g, and ascertain whether it agrees with its known value derived by physicists from laboratory experiments. The moon's sidereal period is $27^d\ 7^h\ 43^m\ 11.5^s$, or 2360591.5 seconds. The moon's distance, r, is 238,840 miles, or 1,261,075,200 feet. The earth's radius is 3858.8 miles, or 20,902,464 feet. The value of π is 3.1416. Making the calculation by means of logarithms, the above data give, by the aid of equation (8):

$$g = 32.5,$$

which is in very close accord with the value of g found directly by experiment in the physical laboratory. It is a most astounding thing that a series of quantities can thus be brought together, as it were, from various parts of the solar system: the moon's distance determined by astronomic observations at Greenwich and the Cape of Good Hope (p. 395); the earth's radius by triangulation measures (p. 97); the moon's period by noting the interval between distant full-moons (p. 162), — it is astounding that these heterogeneous quantities, thus determined by direct observation, can be combined by a simple formula based on that wonderful law

of Newton, and made to produce the identical value for g which we obtain by terrestrial laboratory observations, quite without using astronomic material. There could not be a more striking proof of the unity of science; nor can any doubt remain that the same force of gravity which controls experiments on the earth, also controls the moon's orbital motion.

Note 26. Planet's Mass (p. 205).

Let us suppose once more that orbits are all circular. Considering the satellite orbit, we found, when discussing Newton's test of the law of gravitation by means of the moon, that the acceleration due to the attractive force toward the center of the orbit may be represented by the equation (p. 403):

$$f = 4\,\pi^2 \frac{r}{t^2},$$

where r is now the radius of the satellite's orbit in miles, and t its period of revolution. In this equation, f is due to a continuously acting attractive force toward the planet situated in the center of the orbit, supposed circular.

It is easy to obtain another expression for this force. We have at once, from Newton's law of gravitation (p. 376), that the attraction existing between the planet and the satellite is proportional to the product of their masses, and inversely proportional to the square of the orbital radius. If we let M indicate the planet's mass, and m that of the satellite, this force is:

$$G\frac{Mm}{r^2},$$

where G is a constant depending on the units adopted for linear distances, etc.

Now this Newtonian force produces the acceleration f in the planetary mass m; and since force is measured by the acceleration produced, multiplied by the mass moved, it follows that:

$$fm = G\frac{Mm}{r^2},$$

or:

$$f = G\frac{M}{r^2},$$

and this is the acceleration due to the attraction of the planet on the satellite.

In an exactly similar way, we can show that the satellite produces an acceleration of the planet equal to:

$$G\frac{m}{r^2};$$

so that the total acceleration existing between the two bodies is:

$$G\frac{M+m}{r^2}.$$

If we now equate this value of the acceleration to that given in the equation for f, we have:

$$G\frac{M+m}{r^2}=4\pi^2\frac{r}{t^2},$$

or:

$$M+m=\frac{4\pi^2}{G}\frac{r^3}{t^2}.$$

Let us next apply this equation to two planets, each having a satellite, and indicate by subscript numbers quantities belonging to the two planets. We thus easily obtain the proportion:

$$\frac{M_1+m_1}{M_2+m_2}=\frac{\frac{r_1^3}{t_1^2}}{\frac{r_2^3}{t_2^2}},$$

or:

$$M_1+m_1 : M_2+m_2=\frac{r_1^3}{t_1^2}:\frac{r_2^3}{t_2^2}.$$

With the help of this general proportion, we can now find the planet's mass as compared with that of the earth. We need only let the subscript 1 refer to the earth and moon, and the subscript 2 to the planet and satellite. Then everything is known in the proportion except $M_2 + m_2$, if we have determined by direct observation the distance and period of the satellite with respect to its planet. It is to be noted that this method gives only the sum of the masses of the planet and its satellite, not the mass of the planet alone. But this is of minor importance, since the satellites

are almost always very small compared with their planets: and, in any case, it is the combined mass of the system, including both planet and satellite, that we really need to know. For it is this combined mass which pulls upon other bodies in space; and it is the pulling force upon such other bodies which must be used in any further calculations relating to orbits, etc.

When a planet has no satellite, as in the case of Venus and Mercury, we cannot employ the above simple and accurate method. We must then have recourse to a mathematical discussion of the slight perturbations the planet produces in the observed motions of other bodies in the solar system. These perturbations, of course, depend on the planet's mass, being greater for a massive planet than for a small one; and consequently the planetary masses must admit of numerical evaluation from the observed perturbative effects they produce. Unfortunately our knowledge of the mass of Mercury is still incomplete; that of Venus, however, is known with some precision.

The mass of a planet once determined, it is easy to calculate the force of gravity on the planet's surface, its Superficial Gravity, as it is called. If we designate by P the planet's radius in terms of the earth's radius, and by g the planetary superficial gravity, analogous to the customary designation of the force of gravity on our earth's surface, we have at once, from Newton's law of gravitation:

$$g = \frac{M}{P^2},$$

where M is the planet's mass in terms of the earth's mass.

To ascertain the planet's density in comparison with that of our earth, we proceed thus: We know, in general:

$$\text{Mass} = \text{Volume} \times \text{Density}.$$

Therefore we have for the earth's mass M_e:

$$M_e = V_e \Delta_e,$$

where Δ_e represents the terrestrial density, and V_e the earth's volume.

And for the planet we have:

$$M_p = V_p \Delta_p.$$

Consequently:

$$\frac{\Delta_p V_p}{\Delta_e V_e} = \frac{M_p}{M_e}.$$

But, again using P to indicate the planet's radius:

$$\frac{V_p}{V_e} = P^3.$$

Therefore, if we take the mass of the earth as unity:

$$\Delta_p = \frac{M_p}{P^3} \Delta_e.$$

If we wish the actual specific gravity of the planet, compared with water, we must substitute for the Δ_e the value 5.53, as determined by means of the Cavendish experiment (p. 110).

Note 27. Synodic and Sidereal Periods (p. 209).

Let us indicate by J_{sid} and E_{sid} the sidereal periods of Jupiter and the earth, each expressed in mean solar days. E_{sid}, for instance, is then $365\frac{1}{4}$, approximately. Then, regarding both orbits as circular, and the motions uniform, the earth in one day will pass over a fraction of its total orbit represented by $\frac{360°}{E_{\text{sid}}}$; and Jupiter will pass over a fraction represented by $\frac{360°}{J_{\text{sid}}}$. These two fractions are not equal: if we take the difference:

$$\frac{360°}{E_{\text{sid}}} - \frac{360°}{J_{\text{sid}}},$$

this quantity will be the angle by which the earth and Jupiter fail to lie in a straight line, as seen from the sun at the end of one day after the beginning of Jupiter's synodic year (see Fig. 55, p. 208).

This quantity is therefore, by definition, Jupiter's daily synodic motion. But if Jupiter's entire synodic period be represented by J_{syn}, its daily synodic motion will also be $\frac{360°}{J_{\text{syn}}}$. Equating this with the above value of the same quantity, we have:

$$\frac{360°}{J_{\text{syn}}} = \frac{360°}{E_{\text{sid}}} - \frac{360°}{J_{\text{sid}}};$$

or:
$$\frac{1}{J_{\text{syn}}} = \frac{1}{E_{\text{sid}}} - \frac{1}{J_{\text{sid}}}.$$

By means of this equation, Jupiter's synodic period may be calculated from his sidereal period, and *vice versa;* for E_{sid} is known to be $365\frac{1}{4}$ days.

Note 28. Periods of Inferior Planet (p. 210).

The synodic motion, as in the case of a superior planet, again depends on the earth's orbital motion as well as on that of the planet. As before, the daily sidereal motions of Venus and the earth may be represented by $\frac{360°}{V_{\text{sid}}}$ and $\frac{360°}{E_{\text{sid}}}$. The difference will be the daily synodic motion of Venus, supposed seen from the sun. This quantity is:
$$\frac{360°}{V_{\text{sid}}} - \frac{360°}{E_{\text{sid}}}.$$

Thus the formula for the daily synodic motion of an inferior planet is perfectly analogous to that for a superior planet, except that the terms are now interchanged. This is of course due to the fact that the superior planet has a slower angular motion around the sun than the earth, while the inferior planet has a faster angular motion. But, as before, if V_{syn} be the synodic period of Venus, the daily synodic motion will be $\frac{360°}{V_{\text{syn}}}$; and we have:
$$\frac{360°}{V_{\text{syn}}} = \frac{360°}{V_{\text{sid}}} - \frac{360°}{E_{\text{sid}}},$$
or:
$$\frac{1}{V_{\text{syn}}} = \frac{1}{V_{\text{sid}}} - \frac{1}{E_{\text{sid}}}.$$

It follows that for any planet whatever the reciprocal of the synodic period is always equal to the difference between the reciprocals of the planet's sidereal period and the earth's sidereal period of $365\frac{1}{4}$ days.

Note 29. Table of Periods (p. 211).

It will be of interest to calculate some of the numbers in the table (p. 211) by means of the formulas in Notes 27 and 28. We find:

	Reciprocal of Sidereal Period	Reciprocal of Earth's Sidereal Period	Difference	Synodic Period
Mercury	.011364	.002738	.008626	116
Mars	.001456	.002738	.001282	780
Uranus	.000033	.002738	.002705	370

In computing the numbers in this table, all periods have been reduced to days; and the numbers in the last column are reciprocals of those in the column headed "Difference." It is at once clear from this little calculation how the peculiarities of the table of periods arise. As the sidereal periods of the outer planets increase, the reciprocals of these periods must diminish. Consequently, these reciprocals must gradually approach zero, and the numbers in the column "Difference" must approach the value .002738. So the numbers in the final column of synodic periods must approach the value $365\frac{1}{4}$, or the earth's period. This is just what we should expect. For the outermost planets remain practically stationary for many days among the fixed stars, and must therefore have a conjunction every time the earth goes around its orbit, or very nearly so. The effect of their own slow orbital motion on their synodic motion is necessarily very slight.

Note 30. Greatest Elongation, Mercury and Venus (p. 212).

In Fig. 58 (p. 212) the triangle SEV is right-angled at V. We can therefore calculate the angle SEV, which is the required greatest elongation angle, by means of the formula:

$$\sin SEV = \frac{SV}{SE},$$

or:

$$\text{sin of greatest elongation} = \frac{\text{distance of planet from sun}}{\text{distance of earth from sun}}.$$

Let us make the calculation for Mercury. The orbit of this planet is more flattened than any other in the solar system: the approximate distance of Mercury from the sun varies from 28.5

to 43.5 million miles. Obviously, the greatest elongation will be larger if it happens when the planet is in that part of its orbit which is farthest from the sun. We shall therefore make the calculation twice, using the two values just given for the distance from Mercury to the sun. We have:

	LEAST	GREATEST
Distance of Mercury	28.5	43.5
Distance of earth	93.0	93.0
Log distance of Mercury	1.4548	1.6385
Log distance of earth	1.9685	1.9685
Log sin greatest elongation	9.4863	9.6700
Greatest elongation	17°51′	27°53′

From this calculation we see that Mercury can never attain an angular distance from the sun greater than 28°, as seen projected on the sky from the earth; and ordinarily its greatest elongation will be much less than 28°.

Note 31. Temperature of Mars (p. 226).

The distance from Mars to the sun is about $1\frac{1}{2}$ times that from the earth to the sun. Therefore, if we assume the heat radiated by the sun to diminish with the square of the distance, Mars receives only $\frac{1}{(1.5)^2}$ as much heat as the earth, or $\frac{4}{9}$ as much. We may also assume that, on the average, all planets radiate annually the same amount of heat they receive; otherwise they would become continuously hotter or colder. Now we have a law of physics known as Stefan's law, which gives us an estimate of the quantity of heat a body will radiate at different temperatures. According to this law, calling the quantity of radiated heat Q, and the temperature F (Fahrenheit), we have:

$$\text{for the earth, } Q_e = (458° + F_e)^4,$$
$$\text{for Mars, } Q_m = (458° + F_m)^4.$$

But if each planet radiates as much heat as it receives,

$$\frac{Q_e}{Q_m} = \frac{9}{4}.$$

Therefore: $$\frac{(458° + F_e)^4}{(458° + F_m)^4} = \frac{9}{4}.$$

Now for the average temperature of the earth, we may put

$$F_e = 60°.$$

Therefore:

$$\frac{(518°)^4}{(458° + F_m)^4} = \frac{9}{4}, \quad (458° + F_m)^4 = \tfrac{4}{9}(518°)^4,$$

$$458° + F_m = \sqrt[4]{\tfrac{4}{9}}(518°) = 0.82 \times 518° = 425°.$$

So that:

$$F_m = -33° \text{ Fahrenheit.}$$

This result is of course uncertain, because we cannot be sure that Stefan's law is really reliable in the case of Mars and the earth. It has been tested in the laboratory only, and for a black body radiating its heat freely.

Note 32. Saturn's Ring (p. 245).

We have already found (p. 402) a formula for the acceleration toward the center of an orbit. It is:

$$f = \frac{V^2}{r}.$$

But, according to Newton's law, f is inversely proportional to r^2; so that V^2 must be inversely proportional to r. Therefore if the rings are really a mass of satellites, the squares of their linear velocities are inversely proportional to their distances from the planet. In other words, the outside of the ring should revolve more slowly than the inside.

The outside radius of the ring has been measured by the usual methods (p. 203) to be 86,500 miles; the inner, 55,700. The square roots of these numbers are in the ratio of 1 to 1.24; while the observed linear velocities are in the ratio of 1 to 1.25. There is therefore a surprisingly close agreement; and there can be no doubt that the various parts of the rings rotate in accordance with Kepler's harmonic law, and are composed of satellite swarms.

Note 33. Halley's Transit of Venus Method (p. 269).

We must first show how to calculate the length of the chord in seconds of arc. In Fig. 119, let S, V_1, and E be the positions of the

sun, Venus, and the earth at the moment of inferior conjunction. Let P be the synodic period (p. 208) of Venus, in days. Then Venus gains a whole revolution of 360° on the earth in P days, from the definition of the synodic period. In one day Venus gains $\frac{360°}{P}$. Therefore, if we let the arc V_1V_2 represent the synodic gain of Venus on the earth in a day, as seen from the sun, we have:

FIG. 119. Halley's Method.

$$\text{Angle } S = \frac{360°}{P}.$$

But in the plane triangle SEV_2, we have:

$$\sin S : \sin E = V_2E : V_2S,$$

since the sines of the angles of any plane triangle are proportional to the opposite sides.

Therefore:
$$\sin E = \frac{V_2S}{V_2E} \sin S.$$

But the ratio $\frac{V_2S}{V_2E}$ is known from the known *relative* lengths of the radii of the two orbits belonging to Venus and the earth (cf. p. 262). The angle S being also known, as has just been shown, it follows that we can calculate the angle E, which is the angular distance through which Venus advances across the face of the sun in one day, as seen from the earth. This angle is transformed into seconds of arc; and the observers having found the fraction of a day required by Venus to traverse the observed chords, we find at once by proportion the lengths of the chords in seconds of arc.

As soon as the lengths of the two chords SP and sp (Fig. 73, p. 269) thus become known in seconds of arc, the further proceedings are simple. For the angular semi-diameter, or radius, of the sun's disk is of course known also in seconds of arc (p. 118); consequently, it is possible to calculate the distances Sa and Sb (Fig. 73) in seconds of arc, and also their difference ab. We also know (Fig. 73) the ratio of the lines VA and Va, because we know

the *relative* distances of Venus and the earth from the sun. Va is 0.723 if Aa is 1.000. Therefore:

$$Va : VA = 723 : 277;$$

and ab, in miles, is $\frac{723}{277} AB$, provided, of course, that the distance AB is perpendicular to the plane of Venus' orbit. If not, it is easy to calculate the necessary correction. Now, knowing ab, on the sun, both in miles and in seconds of arc as seen from the earth, we easily obtain the distance of the sun. The simple Fig. 120 shows how this is done. Calling r the radius of the earth's orbit, or the distance from the earth to the sun, we have, from the right-angled triangle Eab, in which the line ab is on the sun, as usual:

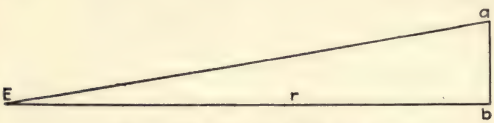


FIG. 120. Halley's Method.

$$\tan ab(\text{seconds of arc}) = \frac{ab(\text{miles})}{r(\text{miles})},$$

or

$$r(\text{miles}) = \frac{ab(\text{miles})}{\tan ab(\text{seconds})}.$$

Note 34. Solar Parallax from the Aberration of Light (p. 271).

Let us study somewhat in detail the action of light aberration. In Fig. 121, suppose that an observer at t has his telescope pointed in the direction tT; that the earth, carrying the observer and telescope, is for the moment moving in its annual orbit in the direction tt', with the velocity v miles per second. Now suppose light from a star at S reaches T at the moment when the telescope is in the position tT. And suppose this light travels with a velocity of V miles per second in the direction ST. Now indicate by α the angle tTt'. Then we may say, as it were,

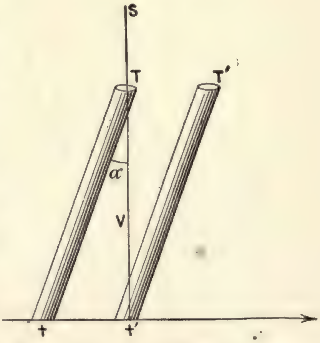


FIG. 121. Solar Parallax from Aberration of Light.

that if the velocities v and V are properly proportioned to fit the angle α, the light will "stay in the telescope tube" while the tube is moving from tT to $t'T'$. We shall then have:

$$\tan \alpha = \frac{v}{V}.$$

This equation signifies that a star at S will really appear projected on the sky in the direction $t'T'$. In other words, the aberration of light displaces the apparent position of the star on the sky through the angle α.

And there is no difficulty in measuring this angle α: for the displacement of the star is always in the direction of the earth's motion, here tt'. And as that motion takes place in a nearly circular orbit, the displacement α must be in opposite directions at intervals of half a year (cf. p. 137). For the earth's orbital motion is, of course, reversed in direction at opposite points of the orbit. We have therefore merely to determine by observation the apparent declination of a star on the sky at intervals of six months. If a suitably located star is selected, the declination will be found to vary by twice the angle α; about 41″ of arc.

From this we easily compute the solar distance. For the velocity of light, V, is known from laboratory experiments. With V and α both known, we can compute v with the equation just obtained, and v is the earth's linear velocity in its orbit. Thus it has been found that v is about 18.5 miles per second. This we have now to multiply by the number of seconds in a year, to get the linear circumference of the earth's orbit. Finally, dividing by 2π, we have the orbital radius, or the solar distance.

Note 35. Sun's Mass (p. 291).

To ascertain this quantity, we resume the formula which expresses the acceleration which the sun gives a planet. It is (p. 402):

$$f = \frac{V^2}{r} = \frac{(\text{velocity of planet in orbit})^2}{\text{radius of planetary orbit}}.$$

In the case of the earth r is 93,000,000 miles. Assuming the orbit approximately circular, we can find its circumference by the formula

$$\text{Circumference} = 2\pi r;$$

and this being divided by the number of seconds in a sidereal year, we find V, the linear orbital velocity of the earth, in miles per second. It is approximately $18\frac{1}{2}$. Now, calculating f, we find:

$$f = 0.233 \text{ inch.}$$

If we now let g, as usual, represent the constant of terrestrial gravity, we may write a simple proportion by the aid of Newton's law:

$$f : g = \frac{\text{sun's mass}}{(\text{sun's distance})^2} : \frac{\text{earth's mass}}{(\text{earth's radius})^2}.$$

This proportion is a direct consequence of Newton's law, which makes attractive forces proportional to masses, and inversely proportional to squares of distances. The earth's radius becomes the distance for terrestrial gravity g, because the earth attracts as if its mass were concentrated at its center; and the radius is the distance from the center to the surface, where gravity acts.

In the proportion everything is known but the solar mass: we can therefore readily calculate it.

Note 36. Angle at Earth's Center for Possible Eclipse (p. 300).

To find the size of the angle M_1cS in Fig. 84, we consider the triangle $O'M_1c$, taking the point M_1 as the point of tangency of the moon at M_1 with the line $O'O$. Then, in the triangle $O'M_1c$:

$$\frac{\sin M_1cO'}{\sin M_1O'c} = \frac{M_1O'}{M_1c}.$$

But, as the sines of these small angles are proportional to the angles themselves, we may write:

$$\frac{M_1cO'}{M_1O'c} = \frac{M_1O'}{M_1c}.$$

But $\quad M_1O' = O'O - M_1O = 93{,}000{,}000 - 240{,}000$, very nearly;

$$M_1c = 240{,}000.$$

$$\therefore \quad \frac{M_1cO'}{M_1O'c} = \frac{93000000 - 240000}{240000} = 386.$$

But $\quad M_1O'c =$ solar parallax $= 8''.8.$

$$\therefore \quad M_1cO' = 8''.8 \times 386 = 57';$$

also $\quad O'cS =$ sun's angular radius as seen from the earth $= 16'$, approximately.

Therefore:

$$M_1cS = M_1cO' + O'cS = 57' + 16' = 1\tfrac{1}{4}°, \text{ approximately.}$$

And if we now consider M_1 to be at the center of the moon, the angle M_1cS will be increased by the moon's angular radius as seen from the earth, or 16′. So that, finally, the angle at c between the centers of the sun and the moon at M_1 is $1\frac{1}{4}° + 16'$, or $1\frac{1}{2}°$, approximately.

Note 37. Draconitic Period (p. 305).

We have seen (p. 299) that the moon's node makes a complete circuit of the ecliptic in 19 years. Therefore, in one year it moves $\frac{360°}{19}$, or 18.5°. In one month it will move about $\frac{18.5°}{12}$, or 1.54°. The moon itself moves 13° per day, as a result of its orbital motion around the earth. Therefore it will move $\frac{13°}{24}$, or 0.54° per hour. So the moon will require about $\frac{1.54}{.54}$ hours, or about three hours, to move the distance traveled by the lunar node in a month. Hence the difference of three hours between the draconitic and sidereal lunar periods.

Note 38. Stellar Magnitudes (p. 324).

It is possible to express the light-ratio relations by means of very simple formulas.

Let M_1, N_1 be the brightness, or luminosity, of stars of the mth and nth magnitudes; and let n be the larger number, belonging to the fainter star.

Then:
$$\frac{M}{N} = [\sqrt[5]{100}]^{n-m};$$

or, passing to logarithms:

$$\log \frac{M}{N} = (\log \sqrt[5]{100})(n - m) = 0.4(n - m).$$

From this we also obtain:

$$n - m = 2.5 \log \frac{M}{N}\cdot$$

These two equations enable us to calculate the light-ratio from the difference of magnitudes, and *vice versa.*

Note 39. Stellar Photometry (p. 325).

To understand how this is done, we shall first consider the following interesting question. What are the faintest stars that can be seen with a telescope of given size? The answer here depends on the diameter of the object-glass, because this determines its area; and the area, or light-gathering surface, in turn determines the light-gathering power. Now it has been found, by experiment, that the faintest star visible in a telescope having an object-glass one inch in diameter is of the ninth magnitude. An object-glass of diameter d inches will have an area d^2 times as great, and will therefore gather d^2 times as much light. Consequently, it will just show a star sending us a quantity of light equal to:

$$\frac{\text{the light of a ninth-magnitude star}}{d^2}.$$

If we assume this star to be of the nth magnitude, we can apply the last equation of Note 38. We then have, putting $m = 9$:

$$M = \text{light of a ninth-magnitude star},$$

$$N = \frac{\text{light of a ninth-magnitude star}}{d^2}.$$

And then our equation gives:

$$n - 9 = 2.5 \log \frac{M}{N} = 2.5 \log d^2;$$

or, $$n = 9 + 2.5 \log d^2.$$

This simple equation tells us the magnitude n, of a star just visible in a telescope of which the object-glass has a diameter of d inches. And it also enables us to calculate the magnitude of a star just visible through a diaphragm of which the aperture similarly has a diameter of d inches.

Note 40. Light emitted by Vega (p. 326).

As we have stated, rough measurements show that the entire quantity of starlight received by an observer on the earth is equal to that of 2000 Vegas. This has also been estimated as being

equivalent to $\frac{1}{33000000}$ of sunlight. Therefore, we receive from Vega:

$$\frac{\text{sunlight}}{33000000} \times \frac{1}{2000}, \text{ or } \frac{\text{sunlight}}{66000000000};$$

and the word "sunlight" here means the quantity of light received from the sun. Then, since the intensity of light diminishes proportionately to the square of our distance from its source, Vega must emit:

$$\frac{\text{light emitted by sun}}{66000000000} \times \frac{(\text{distance of Vega})^2}{(\text{distance of sun})^2}.$$

But Vega is one of the stars whose distance has been measured, approximately. It has been found that:

$$\frac{\text{Vega's distance}}{\text{sun's distance}} = 1820000.$$

Therefore, Vega must emit:

$$\text{Light emitted by sun} \times \frac{1820000 \times 1820000}{66000000000},$$

or, approximately: $\text{Light emitted by sun} \times 49.$

Note 41. Motion of Solar System (p. 338).

Figure 122 may make this matter clearer. The solar system is for the moment imagined stationary, and the stars all moving with parallel annual velocities represented by the arrows SS_1.

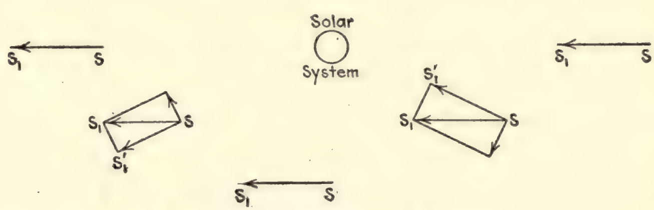


FIG. 122. Motion of Solar System.

On each of these arrows a parallelogram is constructed, having one side SS_1', directed toward the solar system, or away from it. In the two parallelograms shown in the figure, the diagonal velocity SS_1 may be regarded as equivalent to, and it may be replaced by, the two smaller velocity arrows forming the sides of the parallelo-

grams (cf. p. 399). Only the part SS_1' affects the velocity of approach or recession with respect to the solar system. The entire arrow SS_1 indicates approach on the right-hand side of the figure, and recession on the left-hand side. At the lower edge of the diagram appears a star none of whose real velocity SS_1 will appear as either approach or recession.

We might satisfy the above observations if all the arrows SS_1 were replaced by a single parallel arrow, starting from the solar system, and pointing toward the right. A study of radial velocities all around the sky must therefore prove one of two things: either a stream of stars is passing us in a definite direction, or the solar system is moving with an equal velocity in the opposite direction. The latter hypothesis is, of course, the more probable.

Note 42. Distance of Vega (p. 341).

Figure 123 shows the sun, the earth, and Vega. The parallax angle, $0''.11$, is, by definition, the angle EVS, subtended at Vega by the radius of the earth's orbit around the sun. As usual, we can solve the right-angled triangle ESV, in which we know the angle at V and the side ES. We have:

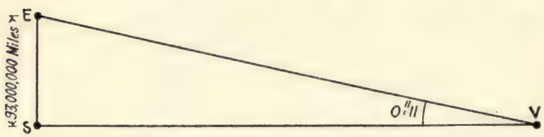


FIG. 123. Distance of Vega.

$$\tan 0''.11 = \frac{ES}{SV}, \text{ or } SV = \frac{ES}{\tan 0''.11}.$$

But $\tan 0''.11$ is, approximately:

$$\frac{0.11}{200000};$$

and so SV, the distance of Vega, is:

$$\frac{93000000 \times 200000}{0.11}.$$

Note 43. Mass of Binary Star (p. 350).

Referring to Note 261 (p. 405), the formula in the case of a binary system is:

$$\text{Mass of system} = \frac{Sa^3}{8\,t^2},$$

where S is the sun's mass, a the linear diameter of the binary orbit in terms of the distance earth-to-sun as a unit, and t the binary orbital period in years.

Note 44. Size of Andromeda Nebula (p. 353).

Figure 124 will make this clear. S is the sun; E, the earth; and N, the center of the nebula. C and C' are points on the circumference of the nebula. The angle ENS is the parallax of the nebula, here assumed to be $0''.01$; since it is by definition the angle subtended by ES, the radius of the earth's orbit, to a supposed observer at the nebula. The angle CSC' is the angular diameter of the nebula, seen from the solar system, and it is 1.5°. Therefore the linear distance CC' must be greater than ES in the approximate ratio $\frac{1.5°}{0''.01}$, or 540,000.

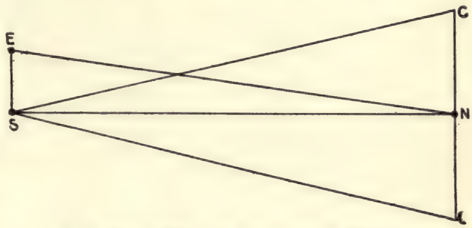


FIG. 124. Size of Andromeda Nebula.

Note 45. Attraction of Andromeda Nebula (p. 354).

Regarding the nebula as approximately a globe 540,000 × 93,000,000 miles in diameter, and the sun as a globe 1,000,000 miles in diameter, the volume of the nebula equals $(540{,}000 \times 93)^3$ times the sun's volume. Let us imagine the sun and nebula to have equal densities. Then their masses will be in the above ratio of their volumes. But with a parallax of $0''.01$, the nebula is 20,000,000 times as far away from us as the sun. Therefore the relative attractions of nebula and sun on the earth are:

$$\frac{(540000 \times 93)^3}{(20000000)^2}, \text{ or, approximately, } 310000000.$$

It follows that the nebular density may be as slight as $\frac{1}{310000000}$ of the solar density, and yet the earth be attracted by the nebula as much as by the sun.

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